Modern Optics, Prof. Ruiz, UNCA **SOLUTIONS** **Fall 2020**

Exam 2 Closed Book, Closed Notes, Closed Everything, HONOR CODE
NOTE EXCEPTION: For this specific exam, a calculator is permitted.

[30 pts] Multiple Choice GRE Style.

MC1. If the wavelength doubles, then the wave number

(A) becomes 1/4 of its original value
(B) halves
(C) remains the same
(D) doubles
(E) quadruples

The wave number $k = \dfrac{2\pi}{\lambda}$. Since $k \sim \dfrac{1}{\lambda}$, if $\lambda \to 2\lambda$, then $k \to \dfrac{k}{2}$.

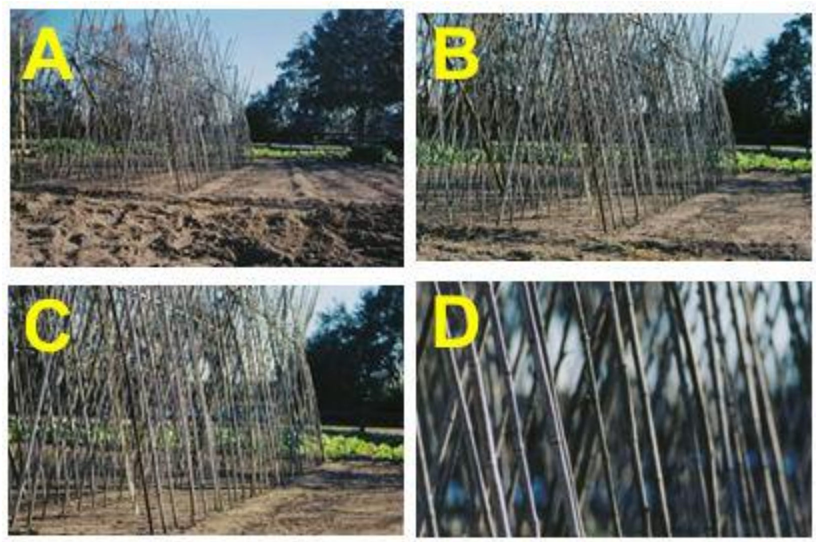
MC2. Three different lenses, chosen from lenses with focal lengths 50 mm, 100 mm, 400 mm, and 2000 mm, were used to take the photos below.

Photo C was taken with the lens having focal length

(A) 28 mm **(B) 50 mm** (C) 100 mm (D) 400 mm (E) 2000 mm.

The magnification of A relative to C is 2x. Therefore $f_A = 2\ f_C$. The only focal lengths having a 2:1 ratio are the 100 mm and the 50 mm. Therefore, C has to be the 50 and A has to be the 100.

MC 3. Photo B below was taken with the normal 50-mm camera lens and two of the other photos with 28-mm and 70-mm lenses.



The fourth photo was taken with a lens with a focal length of

(A) 8 mm

(B) 24 mm

(C) 35 mm

(D) 210 mm

(E) 700 mm.

It is given that f_B = 50 mm. Since A has a wider angle of view, $f_A < f_B$ and therefore f_A = 28 mm. Photo C has a lightly narrower field of field when compared to B, meaning the $f_C > f_B$, but not by too much. This observation leads to f_C > 70 mm. Photo is telephoto which points to 210 mm or 700 mm. Comparing C and D, D appears to be magnified 3 x, which gives f_D = 3 x 70 = 210 mm. Choice E = 700 mm is a far too extreme telephoto.

MC4. What is the net focal length if a 20-cm focal lens is placed together with another 20-cm lens? (A) 0 cm (B) 5 cm **(C) 10 cm** (D) 20 cm (E) 40 cm

Each of these lenses are 100/20 = 5D, i.e., 5 diopters. Since diopters add, the combination is 10D, which gives a focal length f = 100/10 = 10 cm.

MC5. What is the visual acuity of an eye that can see the 20/30 line at 40 feet?

(A) 20/10 **(B) 20/15** (C) 20/16 (D) 20/25 (E) 20/40

We know 30/40 is relevant, i.e., 3/4. But say I forgot the formula 20/20(a/b), where a = 30 and b = 40. I figure that the 20/30 line should be read at 30 feet for the normal eye and this eye is doing better with the 40 feet. The vision is therefore above average. To get an above average value I need (3/4) 20 = 15 and NOT (4/3) 20, which latter case would be bad vision. So the answer is 20/15.

MC6. What is the prescription for a myopic eye having a far point equal to 200 cm?

(A) -0.5D (B) -1.0D (C) -1.5D (D) -2.0D (E) -2.5D

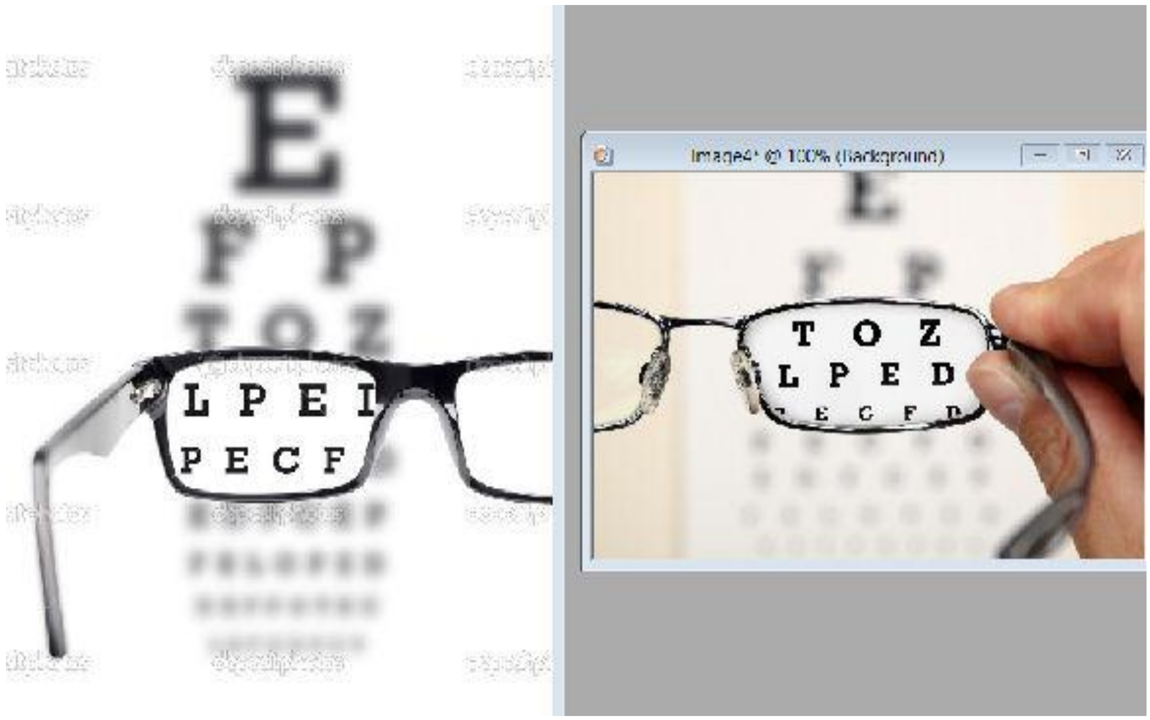
We need to bring the distant mountains to 200 cm. So we want a diverging lens with focal length f = - 200 cm. The prescription is P = 1/f = -1/2 D. Or you can use P = 100/[f (in cm)] = -100/200.

MC7. What is the prescription for a hyperopic eye having a near point equal to 200 cm?

(A) +1.0D (B) +1.5D (C) +2.5D (D) +3.0D **(E) +3.5D**

The normal eye can focus at 25 mm, which means an extra 4D. This eye can at best see as close as 200 cm, which indicates a dioptric power of 100/200 = 0.5D. We need to prescribe power to bring this up to 4.0D. So we prescribe 4 – 0.5 = 3.5D.

MC8. Each lens below is held up to an eye chart where the eye chart is the same distance away for each photo.



Which statement below is true?

(A) The glasses in the left photo are stronger than the glasses in the right photo.
(B) The glasses in the left photo and the glasses in the right photo are the same strength.
(C) The glasses in the right photo are stronger than the glasses in the left photo.

Images Courtesy depositphotos.com and imgkid.com

If the glasses were neutral at 0D, i.e., flat pane glass window, the letters through the window would be the same size as the letters on the chart. Since letters are smaller in each case, we have two pairs of glasses with diverging lenses. But since the virtual images are smaller for the right case, those glasses are stronger.

MC9. Sixteen-year-old Sonny has –0.5D glasses to correct his refractive error of 0.5D. With his glasses, he has no refractive error and is thus 20/20. His mother Agnes has +1.5D glasses to correct her refractive error of –1.5D. Mom Agnes Fedak Ruiz is pictured below with Sonny.

Agnes removes her glasses and puts on Sonny's, her refractive error becoming

(A) –2D

(B) –1.5D

(C) 0D

(D) 1.5D

(E) 2D.

Mom has a refractive error of -1.5D without her glasses. If she puts Sonny's -0.5D glasses on, she will make her situation worse giving her a -0.5D + (-1.5D) = -2.0D error. She will erroneously conclude that Sonny's glasses are very strong, when in fact there are not.

MC10. Below, the letter n stands for index of refraction. To correct for spherical aberration

(A) the cornea is less curved at the edges and crystalline lens has less n at the edges,
(B) the cornea is less curved at the edges and crystalline lens has greater n at the edges,
(C) the cornea is more curved at the edges and crystalline lens has less n at the edges,
(D) the cornea is more curved at the edges and crystalline lens has greater n at the edges,

In spherical aberration, the marginal rays bend too much. To counteract this too-much refraction, make the cornea flatter at the margins (edges) and crystalline lens weaker there (less index of refraction).

[30 pts] P1. Teleconverter. B&H Photo (https://www.bhphotovideo.com/) sells a high-quality Nikon 1.4 x teleconverter. A professional photographer buys one and inserts it between her 50-mm Nikon camera lens and the camera body. The engineer designed the teleconverter so that when the teleconverter is inserted between the 50-mm lens and camera body, the teleconverter sits 50 mm from the film for distant photography. Find the focal length in mm of the teleconverter that the optical engineer designed for the lens and its associated design distance in mm between the 50-mm lens and the teleconverter when in use. You may use thin-lens physics for your calculation, where the back focal length and effective focal length of the combination are given by the following formulas we derived in class. The parameter d is the distance between the two thin lenses.

$$f_b = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} = L = 50 \text{ mm}$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

Give your answers to 2 significant figures.

Start by dividing $f = \frac{f_1 f_2}{f_1 + f_2 - d}$ **by** $f_b = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}$ **to simply things.**

$$\frac{f}{f_b} = \frac{f_1 f_2}{f_1 + f_2 - d} \cdot \frac{f_1 + f_2 - d}{f_2(f_1 - d)} = \frac{f_1 f_2}{f_2(f_1 - d)} = \frac{f_1}{f_1 - d}$$

Now plug in numbers to find d.

$$f_1 = f_b = 50 \text{ mm} \qquad f_b = 50 \text{ mm} \qquad f = 1.4 f_1 = (1.4)50 \text{ mm}$$

$$\frac{f}{f_b} = \frac{f_1}{f_1 - d} \quad \text{=>} \quad \frac{(1.4)50}{50} = \frac{50}{50 - d} \quad \text{=>} \quad \frac{1.4}{50} = \frac{1}{50 - d}$$

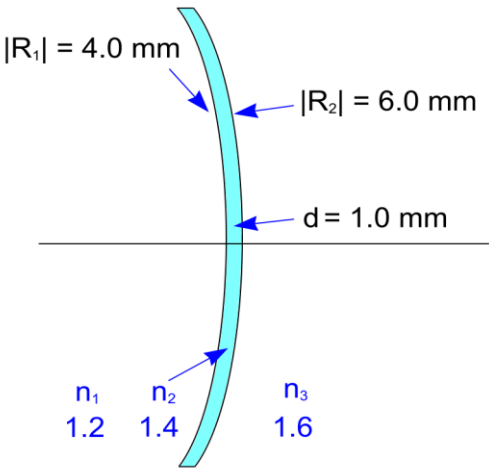
$$50 - d = \frac{50}{1.4} \quad \text{=>} \quad d = 50 - \frac{50}{1.4} = 50 - 35.71 = 14.286 \quad \text{=>} \quad \boxed{d = 14 \text{ mm}}$$

Find f_2. $f = \frac{f_1 f_2}{f_1 + f_2 - d}$ => $(1.4) f_1 = \frac{f_1 f_2}{f_1 + f_2 - d}$ => $1.4 = \frac{f_2}{f_1 + f_2 - d}$

$$1.4 = \frac{f_2}{50 + f_2 - 14.286} \quad \text{=>} \quad (1.4)50 + 1.4 f_2 - (1.4)14.286 = f_2$$

$$0.4 f_2 = (1.4)(14.286 - 50) \quad \text{=>} \quad 0.4 f_2 = -50.00 \quad \text{=>} \quad \boxed{f_2 = -130 \text{ mm}}$$

[20 pts] P2. Dioptric Power. The generic dioptric power formula for a lens where the thickness d is incorporated is given below.

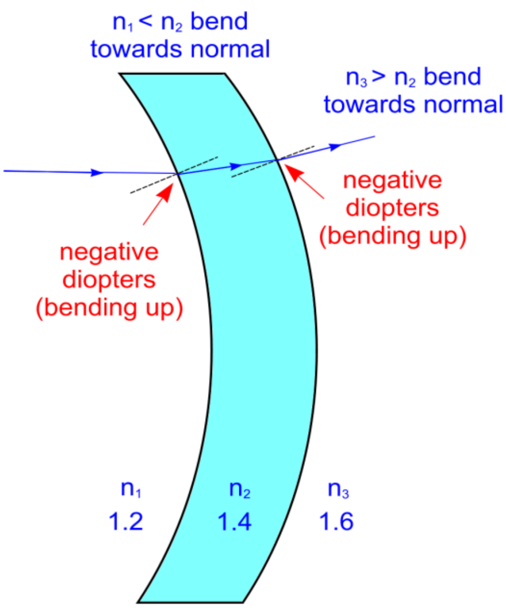


$$\frac{1}{f} = \frac{(n_2 - n_1)}{R_1} + \frac{(n_3 - n_2)}{R_2} - \frac{d}{n_2}\frac{(n_2 - n_1)}{R_1}\frac{(n_3 - n_2)}{R_2}$$

Apply this formula to the left figure and calculate the dioptric power in diopters for the optical system.

You are recommended to give the above three power components in the master formula separately first for partial credit. If you choose to do this option, clearly indicate each of these intermediate results by drawing a box around each and labeling the pieces 1, 2, and 3.

Give your final answer, i.e., the sum, also boxed off, to 2 significant figures.



For the partial credit, break it up so

$$P_1 = \frac{(n_2 - n_1)}{R_1} \qquad P_2 = \frac{(n_3 - n_2)}{R_2}$$ **and the last term is** $-\frac{d}{n_2}P_1P_2$ **.**

Optional Check: First check the signs for P_1 and P_2. See the sketch at the left. Both surfaces have negative power. The sign conventions R_1 = - 4.0 mm and R_2 = -6.0 mm are consistent with this result.

$$P_1 = \frac{(n_2 - n_1)}{R_1} = \frac{1.4 - 1.2}{-0.004} = -50.00D$$

$$P_1 = \frac{(n_2 - n_1)}{R_1} = \frac{1.4 - 1.2}{-0.004} = -50.00D \qquad P_2 = \frac{(n_3 - n_2)}{R_2} = \frac{1.6 - 1.4}{-0.006} = -33.33D$$

$$-\frac{d}{n_2}P_1P_2 = -\frac{0.001}{1.4}(-50.00)(-33.33) = -1.190D$$

$$\frac{1}{f} = -50.00 - 33.33 - 1.190 = -84.5D$$

$$\boxed{P = -85D}$$

[20 pts] P3. Group Velocity. In semi-classical quantum physics, a free particle moving with mass m and velocity v is taken to have kinetic energy given by $E = \frac{1}{2}mv^2$. The momentum $p = mv$ is related to a wavelength by the de Broglie relation $\lambda = \frac{h}{p}$, giving the particle a matter-wave status. The wave number has the same definition we use in class, namely, $k = \frac{2\pi}{\lambda}$. Energy is also related to the frequency with the same formula used for photons: $E = hf$ or $E = \hbar\omega$, where $\hbar = \frac{h}{2\pi}$. Use these relations as needed to calculate the group velocity per our formula $v_g = \frac{d\omega}{dk}$ and express your answer in its simplest form.

We need to find $\omega = \omega(k)$ and then take the derivative $v_g = \frac{d\omega}{dk}$.

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad \textbf{and} \quad E = \hbar\omega \quad \Rightarrow \quad \frac{p^2}{2m} = \hbar\omega$$

$$\lambda = \frac{h}{p} \quad \Rightarrow \quad p = \frac{h}{\lambda} \quad \textbf{and} \quad k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \frac{1}{\lambda} = \frac{k}{2\pi}$$

$$p = \frac{1}{\lambda}h = \frac{kh}{2\pi} = \hbar k \quad \Rightarrow \quad \frac{p^2}{2m} = \hbar\omega \quad \Rightarrow \quad \frac{\hbar^2 k^2}{2m} = \hbar\omega \quad \Rightarrow \quad \omega = \frac{\hbar k^2}{2m}$$

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{\hbar k^2}{2m}\right) = \frac{\hbar}{2m}\frac{d}{dk}(k^2) = \frac{\hbar}{2m}2k = \frac{\hbar k}{m}$$

Since $p = \hbar k$, we have $v_g = \frac{p}{m} = \frac{mv}{m} = v$, the velocity of the particle.

$$\boxed{v_g = v}$$