Theoretical Physics
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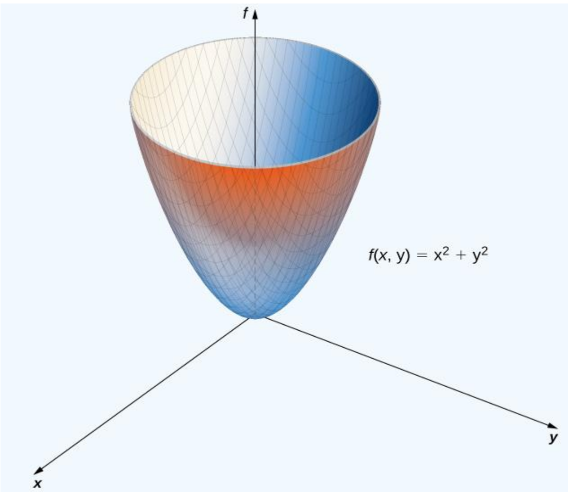
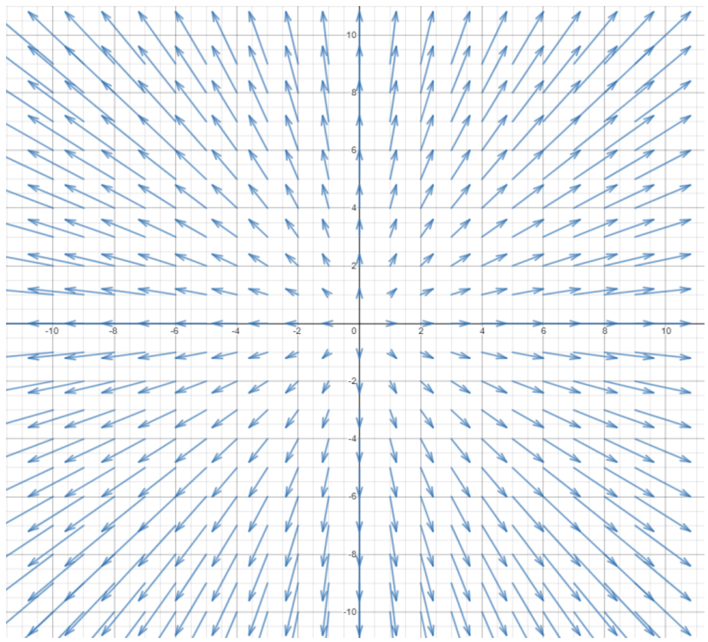
Chapter E Homework. Differential Form for the Maxwell Equations

HW-E1. Gradient. A representation of the function $g(x,y) = x^2 + y^2$ is shown as the paraboloid in the left figure below. You do not need the height representation as you could imagine the value at each (x, y) point in the plane being given by $g(x,y) = x^2 + y^2$. The right figure below is a representation of the gradient of $g(x,y)$.





(a) Determine $\vec{A}(x,y) = \nabla g(x,y)$ where $g(x,y) = x^2 + y^2$. The vector $\vec{A}(x,y) = \nabla g(x,y)$ is the gradient of $g(x,y)$.

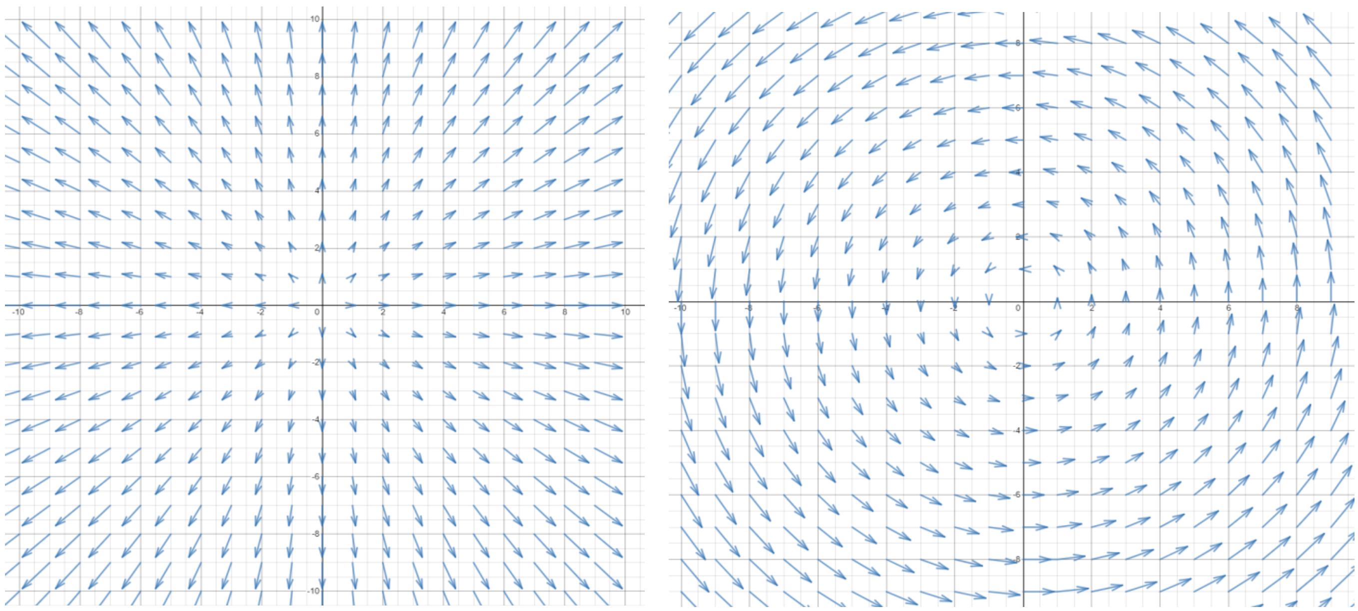
(b) Calculate $\vec{B}(x,y) = \nabla f(x,y)$ where $f(x,y) = x^2 y$.

(c) Find $\vec{B}(3,2) = \nabla f(3,2)$.

(d) Let $\hat{u} = u_x \hat{i} + u_y \hat{j}$ be a unit vector, i.e., $u_x^2 + u_y^2 = 1$. The directional derivative along $\hat{u}$ is by definition $D_u f(x,y) \equiv \nabla f(x,y) \cdot \hat{u}$. Calculate the directional derivative at the point $(x,y) = (3,2)$ for a direction parallel to the vector $\vec{V} = \hat{i} + 2\hat{j}$.

Credits: Mathematics at LibreTexts for the paraboloid, desmos.com graphing for the vector figure, and Duane Q. Nykamp for gradient applications at mathinsight.org.

HW-E2. Divergence and Curl. The vector field $\vec{A}(x, y) = x\,\hat{i} + y\,\hat{j}$ appears in the left figure and $\vec{B}(x, y) = -y\,\hat{i} + x\,\hat{j}$ appears in the right figure below.



Courtesy desmos.com graphing.

Calculate the divergence and curl for each of these vector fields.

Give a description in words for the divergence and curl based on your results and the visualizations of the two fields.