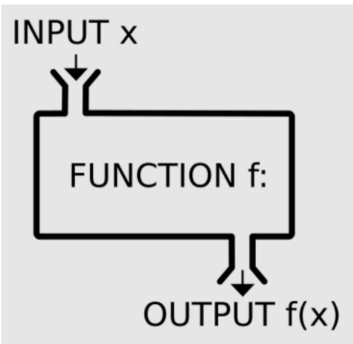
To get us started, we begin with the concept of a function.

1. Function.



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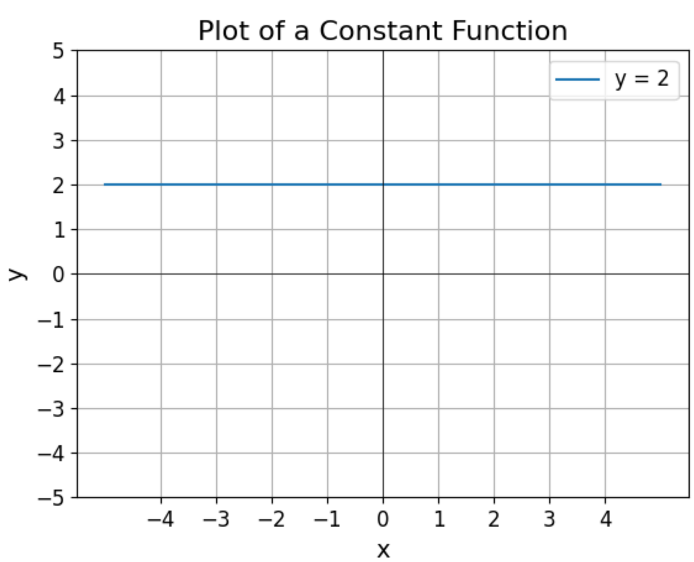
A function takes in a value, which we will call x here and sends out an output value, which we will can $y = f(x)$. The f stands for function. Think of it as a machine that transforms an input into an output.

$$y = f(x)$$

An important point is that we get one output for each input. While two different inputs may give the same output, one input cannot result in two outputs. The function always sends out one output value for each input.

2. Constant Function. The simplest function I can think of is a constant function. For any input value the function sends the same output value. Let's consider the function $y = f(x) = 2$, so that the constant output value is 2. We can make a little table to list the input and output values for some inputs. We can group input x and output y as (x, y), which we call an ordered pair.

Input x	-4	-3	-2	-1	0	1	2	3	4
Output y	2	2	2	2	2	2	2	2	2



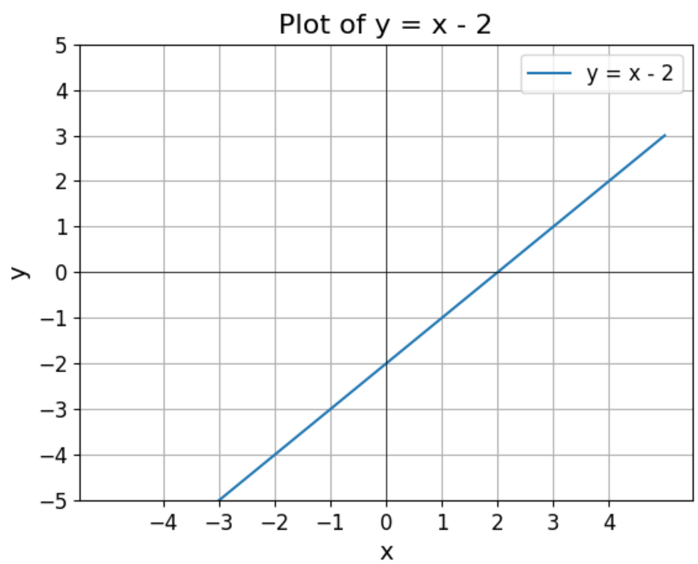
The order is important in an ordered pair, the x is listed first and then the y. We can also make a plot of the input and output ordered pairs (x, y). It is understood that the first entry is the input and the second is the output.

See the left figure for a graph or plot of the function $y = f(x) = 2$. The table shows 9 pairs but they are connected by a straight line in the graph since the y-value, i.e., the height, is the same value $y = 2$ for all the x-values.

3. General Linear Function. A linear function is one where the graph of plot is a straight line. But since we cannot get multiple output values for one input, a vertical straight line is not allowed. However, a vertical straight line is, as we have seen with our constant function. While the constant function is a linear equation, it is a special case where the slope is zero. In general, we can have a graph with a finite slope. Let's consider the function $y = f(x) = x - 2$. A table of ordered pairs for this function is provided below.

x	-3	-2	-1	0	1	2	3	4
$y = x - 2$	-5	-4	-3	-2	-1	0	1	2

In the old days we would put little dots on graph paper at the locations for the ordered pairs and then sketch a line or curve through the points. Since this graph is a straight line, you could just can draw a straight line through the points. Today we use graphing software, computer coding, or even a spreadsheet. The result is shown in the plot at the right.



We call the y-value where the plot crosses the y-axis, i.e., vertical axis, the y-intercept. In this case, the y-intercept is -2. The slope is given by the rise over the run. Refer to the graph to understand that the rise/run is 1. For each step along the x-axis (the run), you go up one step along the y-axis (the rise). We can generalize $y = f(x) = x - 2$ for a case where the y-intercept is b and the slope is m by writing

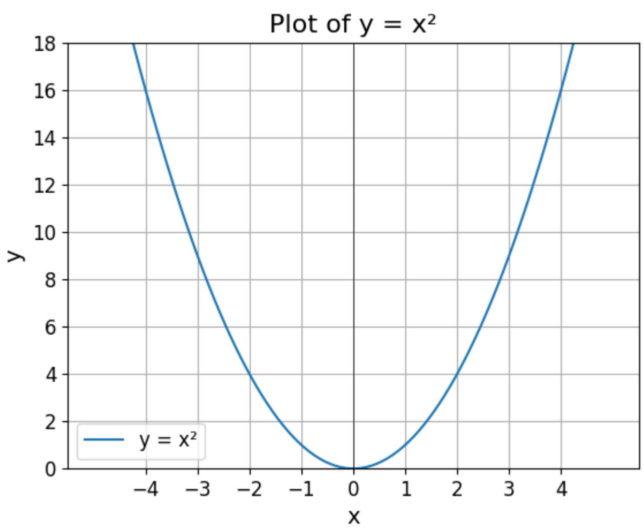
$$y = f(x) = mx + b.$$

If you want the x-intercept, i.e., where the line crosses the x-axis, you set $y = 0$ to put yourself at the elevation for the x-axis, which is zero in this case. The solution is given below.

$$y = f(x) = mx + b = 0 \quad => \quad mx = -b \quad => \quad x = -\frac{b}{m}$$

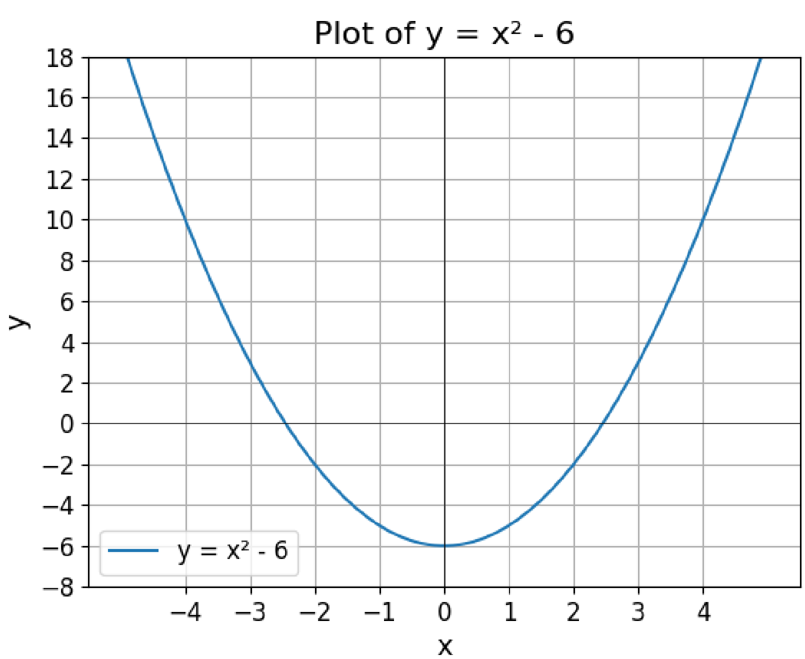
4. Quadratic Function. A quadratic function is one where the highest power of x is x^2. The general form is $y = f(x) = ax^2 + bx + c$. We will consider the simple case $y = f(x) = x^2$. Some ordered pairs for this function are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2$	16	9	4	1	0	1	4	9	16



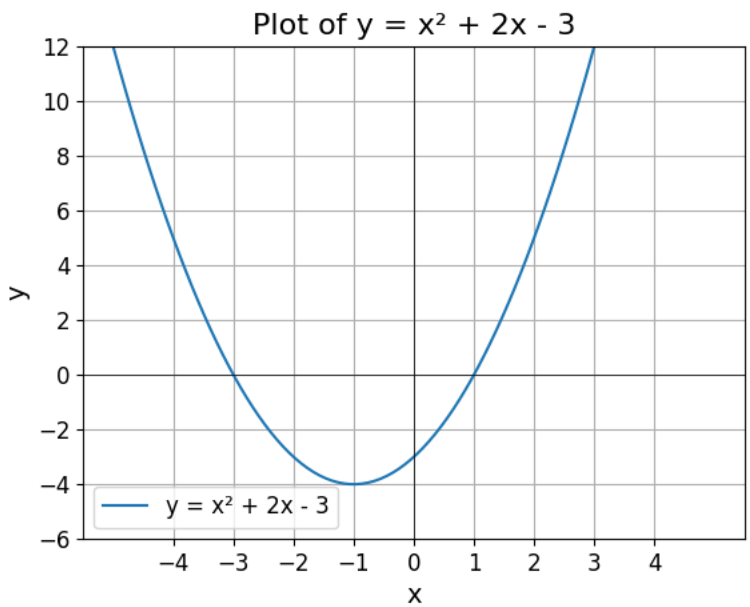
The corresponding plot for $y = f(x) = x^2$ is given in the figure at the left. Note that the graph is symmetric about the y-axis. This symmetry is due to the fact when we square a negative number we get the same value as squaring the positive number that has the same absolute value. This feature is explicitly shown in the table as well as the plot. Note that the graph has a minimum at $x = 0$. The slope there is zero. We will be calculating the slope at an arbitrary point for this graph in a later chapter.

Next consider the function $y = f(x) = x^2 - 6$. This function simply moves the previous one down by 6.



Now consider $y = f(x) = x^2 + 2x - 3$. A table of some values are below. Verify that the table is correct. After the table find the plot.

x	-4	-3	-2	-1	0	1	2	3
$y = f(x) = x^2 + 2x - 3$	5	0	-3	-4	-3	0	5	12



The roots for this equation are found by setting y to zero.

$$y = x^2 + 2x - 3 = 0$$

We can factor the left side.

$$(x+3)(x-1) = 0$$

The solutions are $x = -3$ and $x = 1$.

The minimum occur halfway between at $x = \frac{-3+1}{2} = \frac{-2}{2} = -1$. The y-value for this minimum is

$$y = f(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4.$$

Let's return to the general quadratic form $y = f(x) = ax^2 + bx + c$. The roots are found by solving

$$ax^2 + bx + c = 0.$$

I have a YouTube video that derives the quadratic formula,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

which gives the solutions.

Here is the link: https://youtu.be/UisAIJog42U?si=6CJOxfRPSp2XbHaq