Modern Optics, Prof. Ruiz, UNCA doctorphys.com
Chapter D. Mirrors

D1. The Plane Mirror

"When I was a child, I thought it was so cool when one day my dad turned his movie camera toward a mirror. We were at Clementon Park, an amusement park in New Jersey during the late 1950s. My dad, a printer by trade, was into home movies and interested in science. The event was inspiring." Prof. Ruiz, February 2002.

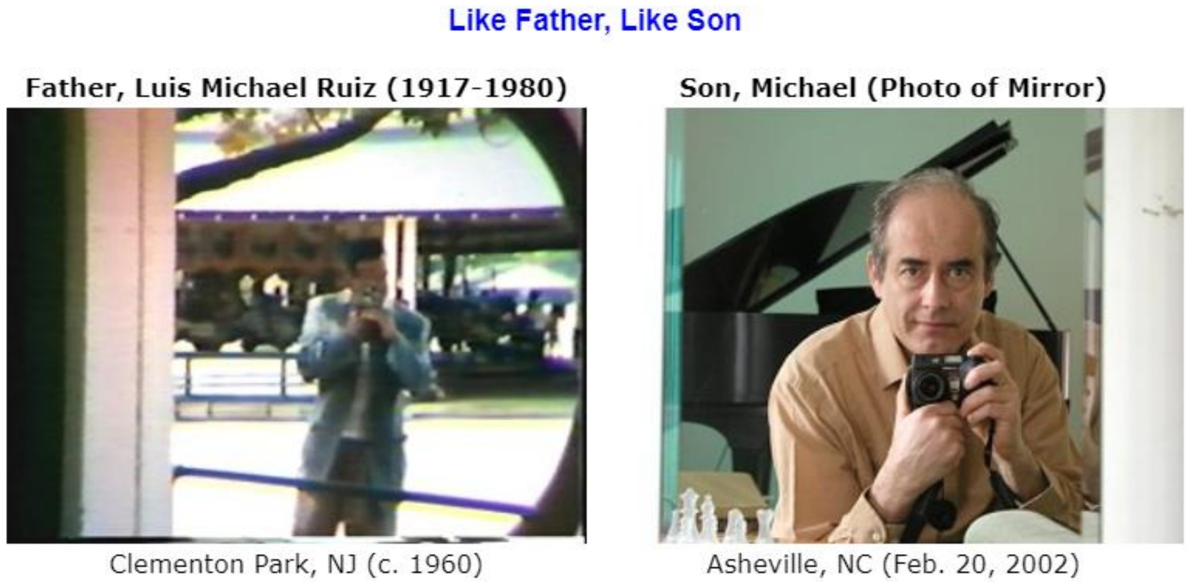


Fig. 1. Dad and Son pointing camera at plane mirror.
See the 1-minute video: https://youtu.be/LG5P4Cvq4do

Say you are in a department store trying on a coat in front of a dressing mirror. Can you answer the following subtle question?

What size mirror do you need to see your entire body?
Video Answer: https://youtu.be/o6YaokJfP98

What are the two principles of mirrors are described in the video?

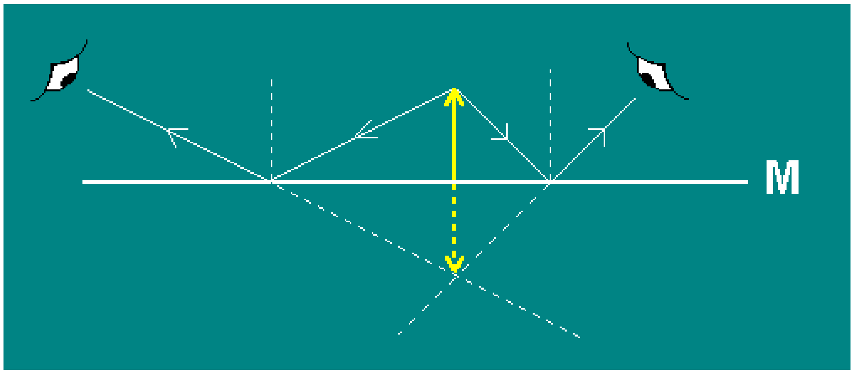


Fig. 2. Standing on a Mirror.

We can use the law of reflection to determine the location of images formed by plane mirrors. We consider two cases. First, we place an object on a plane mirror and draw two rays from the tip of the object to the mirror. Any two

rays are sufficient. We then sketch in normals where the light rays hit the mirror and apply the law of reflection for each. We place an eye to receive each of the reflected rays.
To locate the direction of the image, we move backwards along each reflected ray. We see that we must extend the rays backwards, behind (i.e., underneath) the mirror. We are careful to use dotted rays in this case since the light does not really go there. The intersection of the dotted rays marks the location of the tip of the image, both observers agreeing on this apparent origination point. Since the object is vertical, we can easily complete the sketch for the image, which image we indicate as dotted also. We call such an image formed at a place where light does not really go, a ***virtual image***. Drawing rays of geometrical optics is called **ray tracing**.

Below we have the usual case where a subject stands in front of a vertical mirror. The image is again virtual and behind the mirror, although upright this time. Since our subject is at one fixed distance from the mirror, the image is similarly at one fixed distance behind the mirror. However, the sense of left and right is reversed. Ignore this aspect for now and focus on the fact that the image is behind the mirror. This latter feature is difficult for some to understand, as they insist that the image is on the mirror. After all, don't we know that nothing is behind the mirror, except perhaps for a medicine cabinet?

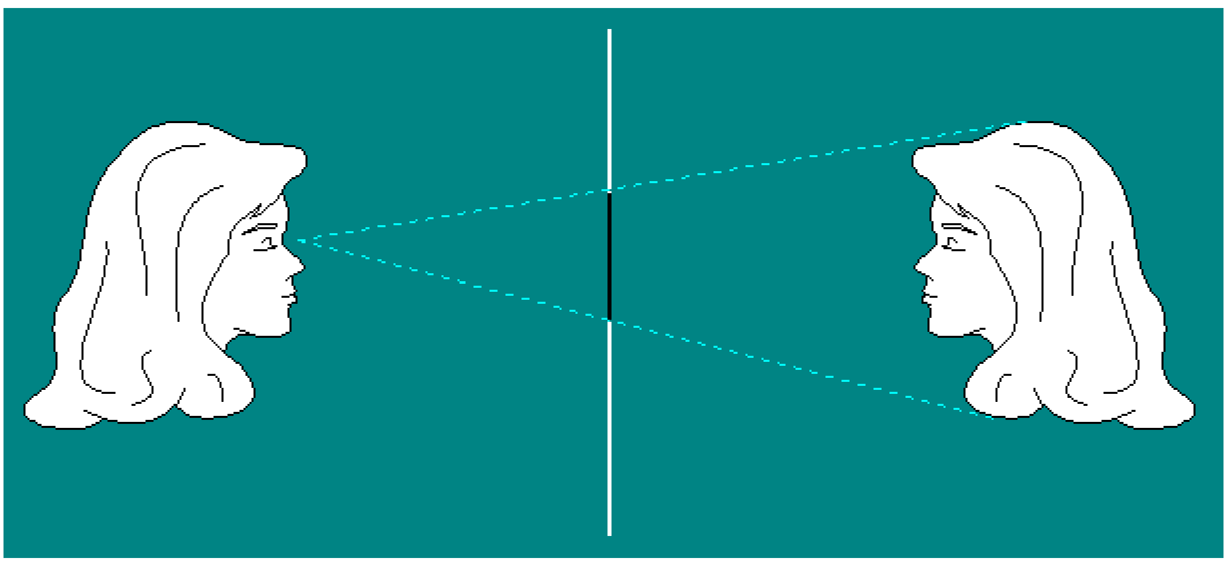


Fig. 3. Physics major traces her face on a fogged mirror. She finds her face diameter is half size at the mirror surface. How does she conclude her reflected image is twice the distance from her to the mirror? Hint: She considered similar triangles.

Being an astute student, our physics major in Fig. 3 concludes that her image is twice as far away from her as the mirror. She imagines a vertical line drawn at the image behind the mirror in order to complete the larger triangle. The smaller triangle which ends at the mirror is a scaled down version of the larger one which includes the image of her head. Your instructor recounts below the difficulty he had in explaining this phenomenon to his father-in-law, the Colonel.

> "One day long ago when I was in grad school, and during my early years with the Light course, the topic of reflection by a mirror came up. My father-in-law, mother-in-law, and wife

were present in our apartment in graduate housing at the University of Maryland. My father-in-law Dan Boyle, a Colonel in the U.S. Air Force and masters graduate from the University of Maryland in business, strongly took issue with my statement that the image is behind the mirror. He insisted that the image is on the mirror surface. It cannot be behind the mirror. I attempted a ray diagram and sketch of the 'shrunken head,' but these did not convince him at all. He was not able to relate to a theoretical analysis. I felt frustrated as I intended to be a college teacher and yet I was failing in my attempts to explain a basic principle.

"I thought that I must find another way. Then I remembered the new present, a Canon AE-1 camera, my dad had received for Christmas from one of his sons. He was very proud of this camera, carrying it along with him always and often taking pictures. I motioned to the camera he had beside him and asked him if he trusted his camera's indication of subject distances. He quickly replied in the affirmative. So I said then let's go in front of a mirror and focus the camera on our image. It should give the distance from us to the image. He readily agreed to this experiment, having confidence in his camera as a measuring device. He stood 3 feet in front of the mirror and focused the camera on his own reflected image. I then said look at the distance. He did, saw the value 6 ft and instantly exclaimed 'Your are right.'

I was taken aback by his sudden admission that I was correct in our debate. My father-in-law didn't typically agree with me during those early years. But he trusted his camera. The camera was his apparatus in doing a physics experiment. Although he did not follow the theoretical arguments, he was convinced by his experimental measurement, which he knew how to make as a photographer, and he trusted those results. One can arrive at understanding physics in many ways, where good theory and experiment support each other." Prof. Ruiz (Spring 2001) reflecting on the mid 1970s.

Here is how you can proceed with a friend if a camera is not available, or even a mirror for that matter.

"You should be able to convince anyone that the image is behind the mirror by asking her or him to pretend to be your image, imagining a fictitious vertical mirror between you. I have tried this with great success with second graders. I had to come up with another method in dealing with very young students. I try to choose students with the same height and similar features. They always get the physics right, with the class correcting our star actors if a mistake should occur. When one student places a hand on the mirror, the other student does the same from behind the mirror." Prof. Ruiz, Spring 2001.

The inspiration for the above intuitive demonstration comes from legendary Groucho Marx in his movie ***Duck Soup*** (1933). There we find the classic "mirror sequence" where Groucho's brother, Harpo, runs into a mirror and breaks it. Harpo then acts as the virtual image of Groucho, giving the impression that the mirror is still there. We excuse the 'artistic license' whereby the pieces of glass somehow disappear from the scene after the mirror breaks.

D2. The Concave Mirror

A spherical mirror appears in Fig. 4. The left surface of the mirror is the concave side, while the right surface is convex, Since the light approaches the concave side, the mirror is said to be concave. First we list some features.

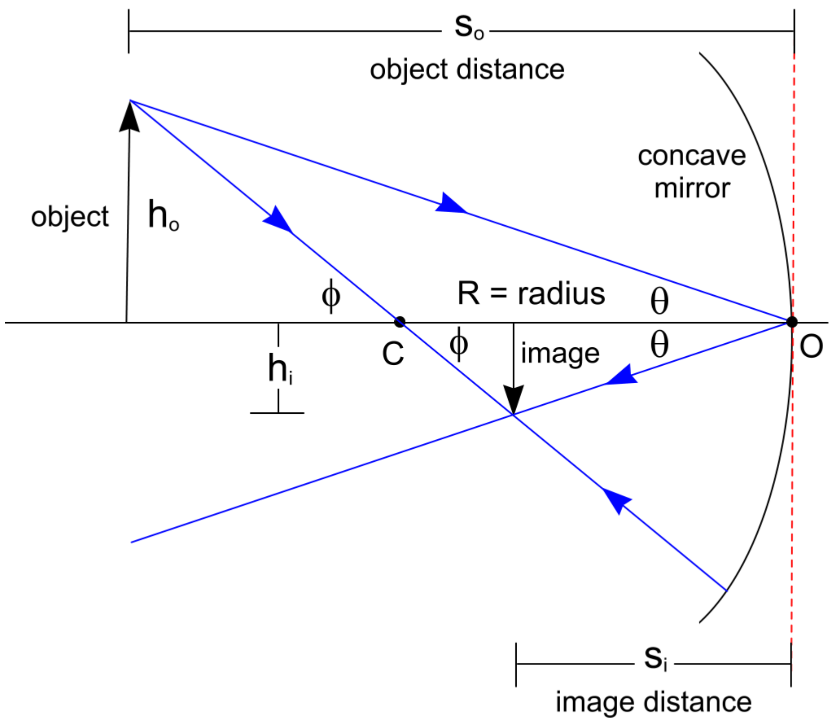


Fig. 4. Concave Mirror.

Point O – he center of the mirror.

Point C – the center of curvature. If you continued drawing a complete circle for the mirror, C is the center of the circle.

Radius R – the radius (C to O).

Optic Axis – the horizontal line that passes through C and O.

Object – the vertical arrow from which light will reflect off the mirror.

Image – the image of the object formed by the reflected light.

Distance s_o – the distance from the object to the "mirror" (the red dotted vertical line).

Distance s_i – the distance from the image to the "mirror" (the red dotted vertical line).

Height h_o – the height of the object, **Height h_i** – the height of the image.

Two rays are drawn from the top of the object arrow to the mirror. The upper ray hits the center of the lens and reflects so that the angle of incidence θ is equal to the angle of reflection θ. Note that angles of incidence and reflection are always measured relative to the normal. In this case the normal at the point of reflection is given by the optic axis. The other ray passes through C on the way to the mirror. Since this ray is normal to the mirror, the angle of incidence is zero and ray reflects right back. From the geometry we have

$$\tan\phi = \frac{h_o}{s_o - R} = \frac{h_i}{R - s_i} \quad \text{and} \quad \tan\theta = \frac{h_o}{s_o} = \frac{h_i}{s_i}.$$

We now proceed to derive two basic formulas for the concave mirror. It would be nice to be able to figure out the image distance s_i and the magnification. First we derive the image distance formula. Returning to

$$\frac{h_o}{s_o - R} = \frac{h_i}{R - s_i} \quad \text{and} \quad \frac{h_o}{s_o} = \frac{h_i}{s_i},$$

we can write

$$\frac{R - s_i}{s_o - R} = \frac{h_i}{h_o} \quad \text{and} \quad \frac{s_i}{s_o} = \frac{h_i}{h_o}.$$

$$\frac{R - s_i}{s_o - R} = \frac{s_i}{s_o}$$

$$s_o(R - s_i) = s_i(s_o - R)$$

$$s_o R - s_o s_i = s_i s_o - s_i R$$

$$s_o R + s_i R = 2 s_i s_o$$

$$s_o + s_i = \frac{2 s_i s_o}{R}$$

$$\frac{s_o + s_i}{s_i s_o} = \frac{2}{R}$$

Finally, $$\frac{1}{s_i} + \frac{1}{s_o} = \frac{2}{R} \quad \text{and} \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{2}{R}.$$

We can get the magnification by comparing the height of the image to the height of the object. The magnification of the image relative to the object is found from $\frac{s_i}{s_o} = \frac{h_i}{h_o}$. We can write

$$M = -\frac{s_i}{s_o},$$

where the minus sign is included since the image is upside down. Now notice that when the object distance $s_o \to \infty$, the image distance $s_i \to \frac{R}{2}$. Also note that the size of the image goes to zero since $M = -\frac{s_i}{s_o} \to \frac{s_i}{\text{large}} \to 0$ as $s_o \to \infty$. From these observations we conclude that light from infinity focuses at $\frac{R}{2}$, the point midway between C and O. Let's call this point F, the focal point and the distance $\frac{R}{2} = f$, the focal length. The formula becomes $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$. You have probably seen concave satellite dishes that focus light coming from far away. Fig. 5 below shows two radio dishes at the Pisgah Astronomical Research Institute (PARI) in Rosman, North Carolina. Radio waves come in from so far that we can take the source object to be at infinity. The radio waves reflect to the focal point, where a detector (receiver) is placed.

Fig. 5. Concave Radio Telescope Dish with Receiver at the Focal Point.
Photo by Doc, June 13, 2002, at PARI, Rosman, NC

Now can use the formula or sketch a graph with rays using two key rays. Rays indicate that we are doing geometrical optics. Light travels in straight lines. See Fig. 6.

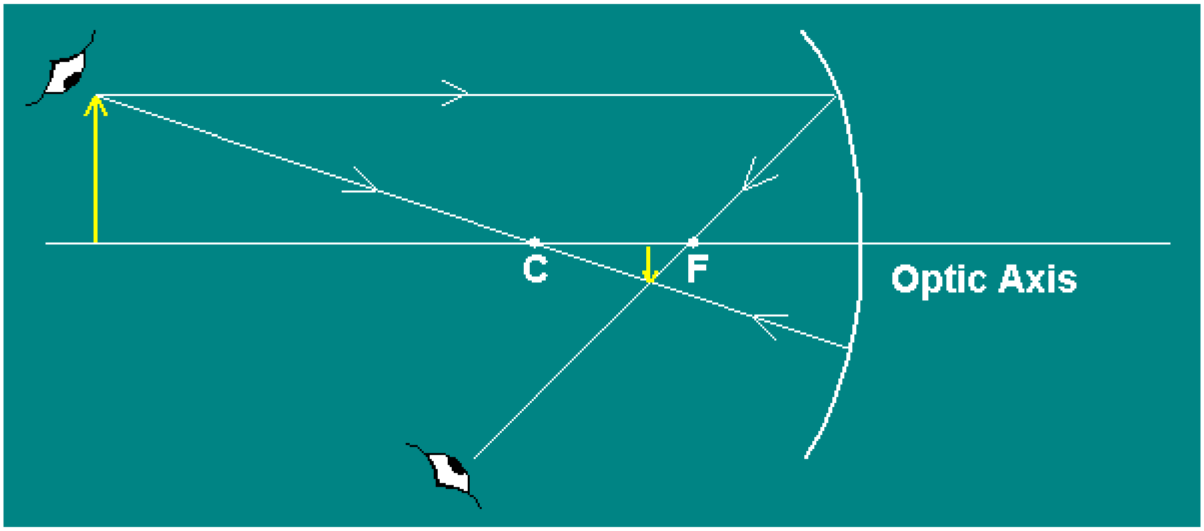


Fig. 6. Ray Diagram with Two Key Rays to Locate Image.

The ray rules are given below.

Ray 1 - the parallel ray – a parallel rays reflects to F
Ray 2 - the central ray – a ray through C bounces right back
Ray 3 – Ray 1 backwards – go through F first and then reflect parallel.

The third ray is included in Fig. 7.

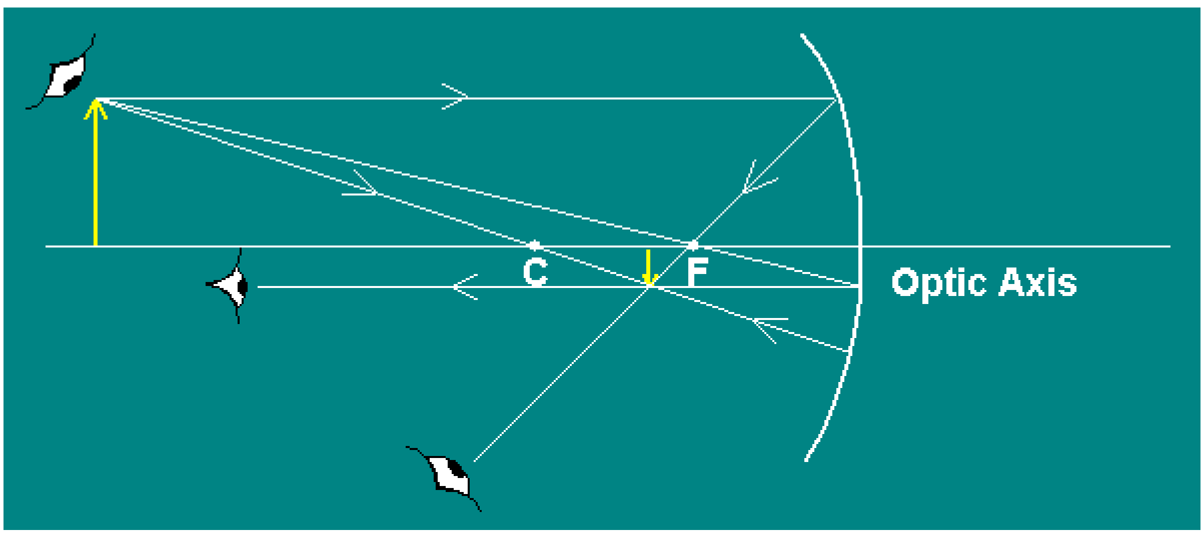


Fig. 7. Ray Diagram with Three Key Rays.

You now have a mathematical way and a graphical way to locate images. And a third way is to actually do the experiment. See Fig. 8 for the real deal.

Fig. 8. Photo by Doc of Himself. Small Real Image.

Since light actually reaches the image and passes through it, the image is said to be real, i.e., a real image. What about getting close to a concave mirror? See Fig. 9. What’s with that?

Fig. 9. Photo by Doc of Himself. Large Virtual Image.

1. Ray Diagram for Big Face. Here is the ray diagram for the big face effect. We are talking make-up mirror. You can buy them. Small concave make-up mirrors are very popular. You get to see a magnified image of your face and you can better do detailed work.

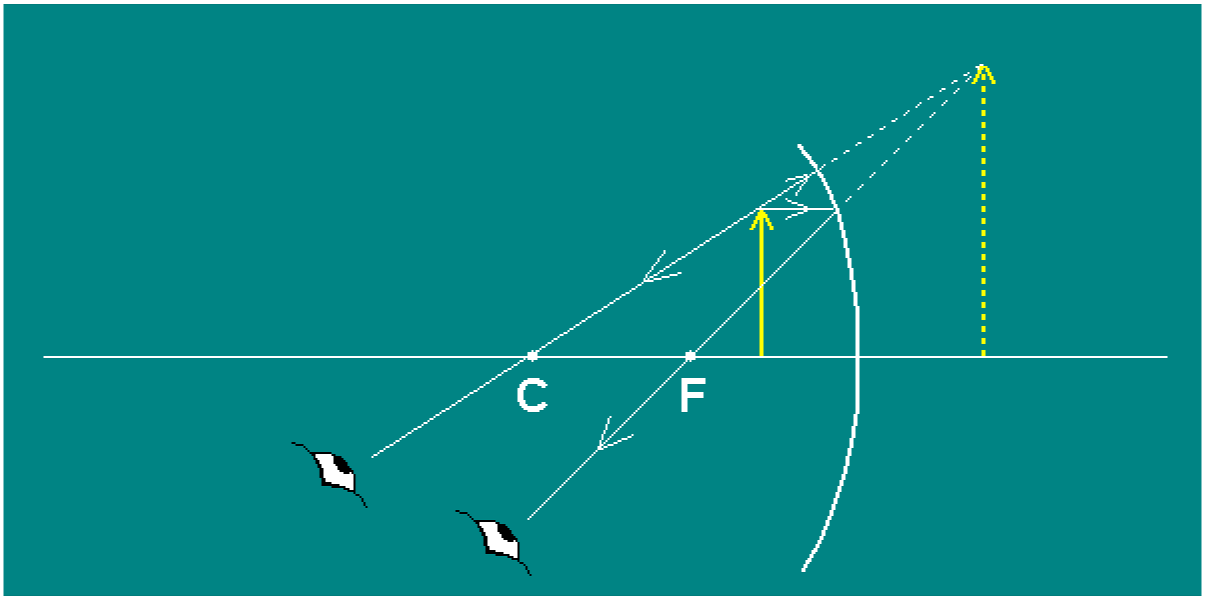


Fig. 10. Ray Diagram for Virtual Image Formation.

2. Math for Big Face. Say f = 50 cm and that yellow object is at s_o = 30 cm. Recall that the formula

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{2}{R} \text{ with } f=\frac{R}{2}, \text{ gives us}$$

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}.$$

Then $\frac{1}{30}+\frac{1}{s_i}=\frac{1}{50}$ and $\frac{1}{s_i}=\frac{1}{50}-\frac{1}{30}=\frac{30-50}{(50)(30)}=\frac{-20}{1500}=-\frac{2}{150}$, leading to

$$s_i=-\frac{150}{2}=-75 \text{ cm} \text{ with magnification } M=-\frac{s_i}{s_o}=-\frac{(-75)}{30}=2.5.$$

The math is teaching us. The minus sign must mean behind the mirror and the magnification comes out positive, meaning upright. And it is larger. Notice that the ray diagram looks a little off. The magnification does not appear 2.5. See Fig. 11 for a more precise ray diagram using graph paper.

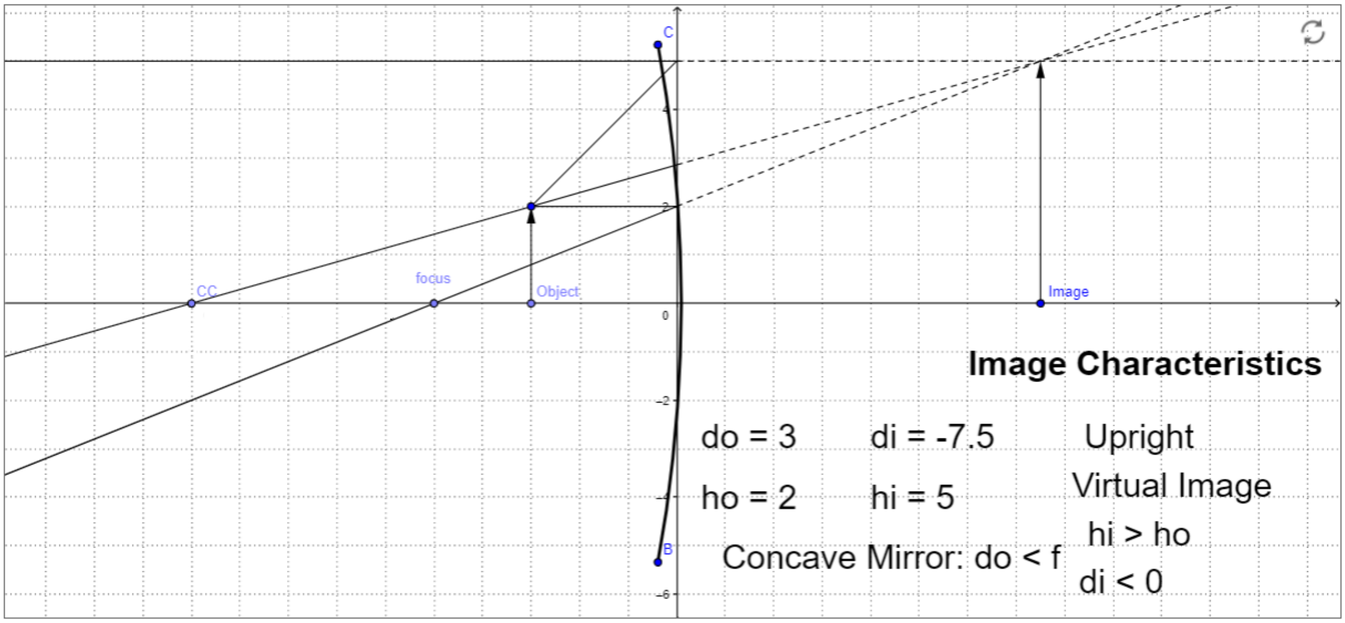
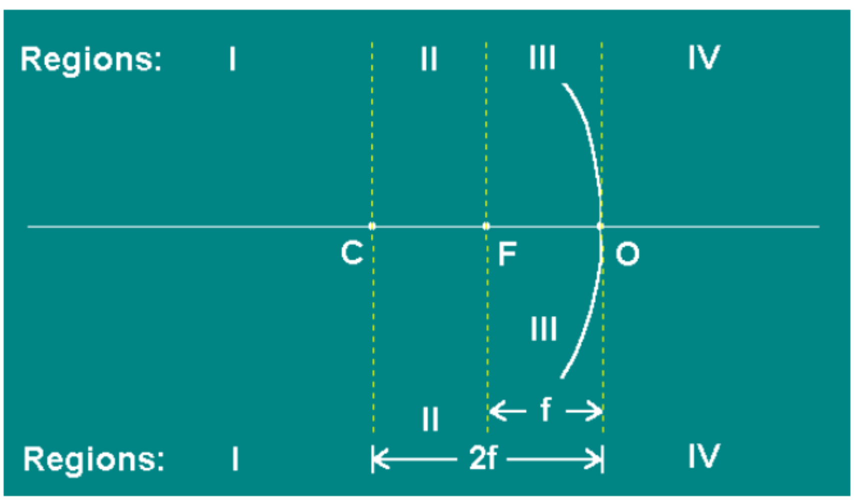


Fig. 11. Ray Diagram with Graph Paper.

Fig. 11 indicates the 2.5 magnification. Note that Ray 3, one aligned with F (the focus) and going out parallel, is included. Also note that the rays are drawn to the vertical line at the mirror. Can you imagine in your mind what happens when the object arrow starts at infinity and marches in to finally end up against the mirror? The answer is sketched out in Fig. 12.



Object Location	Image Characteristics			
	Size	Location	Type	Orientation
Infinity	Zero	At F	Real	(Inverted)
Region I	Smaller	Region II	Real	Inverted
Point C	Equal	Point C	Real	Inverted
Region II	Larger	Region I	Real	Inverted
Point F	Undefined			
Region III	Larger	Region IV	Virtual	Upright
Point O	Equal	Point O	Virtual	Upright

Fig. 12. Regions and Outline of All Cases.

Here is something to think about. When the inverted image runs off to infinity at the left as you approach F, right on the other side of F the image is at infinity upright. It is as if the left infinity met the right infinity and a flip took place. Then as you approach the mirror to touch it, the virtual image on the right side comes toward you decreasing in height until it meets you at the mirror at the same size. When you touch a flat mirror, isn't the virtual image right up at your hand and the same size?

D3. The Convex Mirror

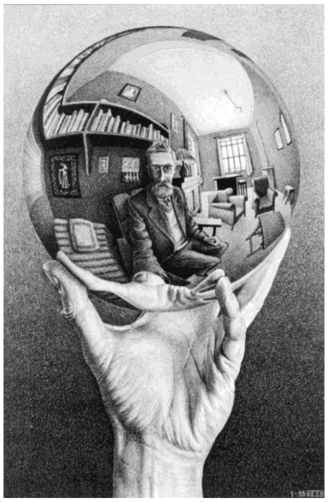


Fig. 13. M. C. Escher, *Hand with Reflecting Sphere*. Lithograph, 1935. The Walker Collection, The Netherlands.
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A convex mirror is similar to the reflecting globe in the Escher lithograph of Fig. 14. Note that all images are smaller except the image at the surface where Escher is touching the reflecting sphere.

Therefore, this mirror is less complicated when compared to the concave mirror with its variety of image formations we found in the previous section. See Fig. 14 below for 2 photos of Doc and his daughter Christa with convex mirrors taken in Robinson Hall 119, a lab room where physics majors usually hang out.

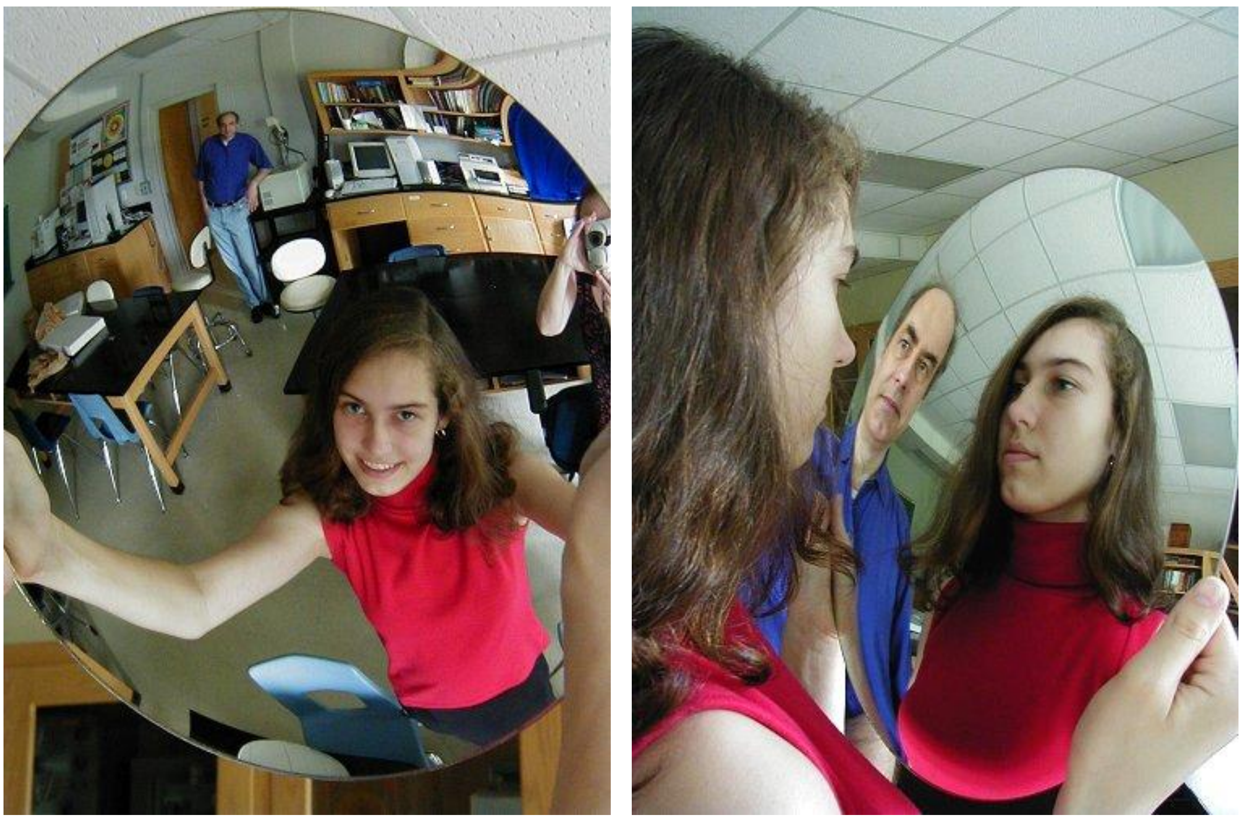

Fig. 14. Convex Mirror with Doc and Daughter Christa. Photo by Wendy Newman

We will derive the convex mirror formula in a similar we did for the concave mirror. Fig. 15 is our starting point.

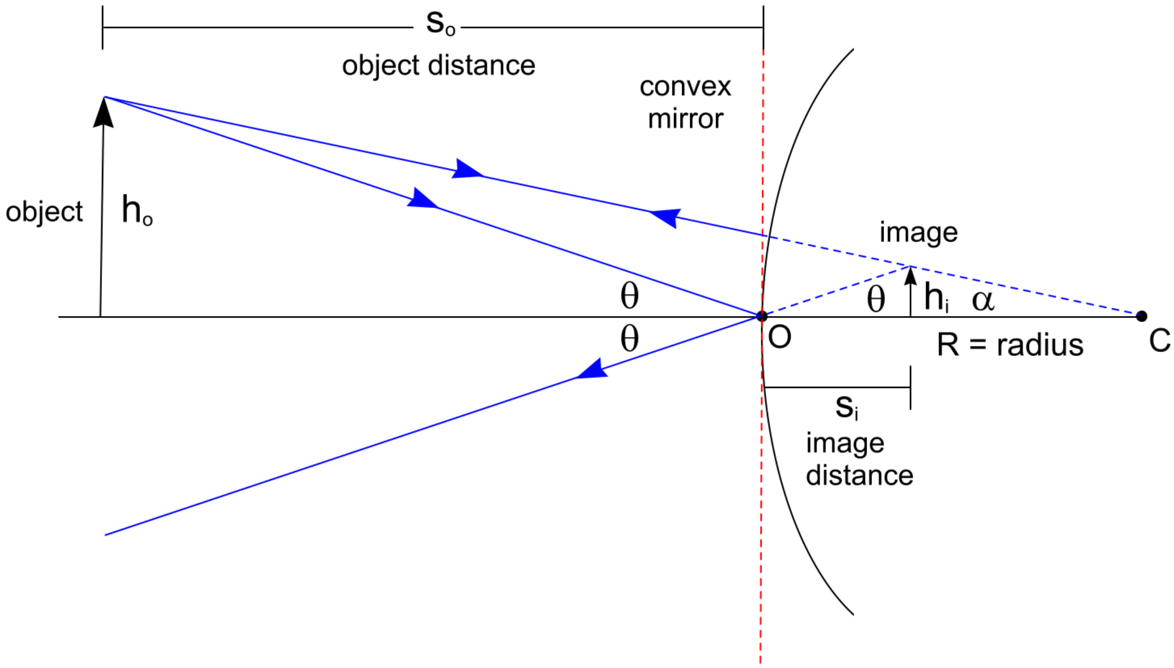

Fig. 15. Convex Mirror.

From the geometry in Fig. 15 we have

$$\tan\alpha = \frac{h_o}{s_o + R} = \frac{h_i}{R - s_i} \quad \text{and} \quad \tan\theta = \frac{h_o}{s_o} = \frac{h_i}{s_i}.$$

We can write

$$\frac{R - s_i}{s_o + R} = \frac{h_i}{h_o} \quad \text{and} \quad \frac{h_i}{h_o} = \frac{s_i}{s_o}.$$

$$\frac{R - s_i}{s_o + R} = \frac{s_i}{s_o}$$

$$s_o(R - s_i) = s_i(s_o + R)$$

$$s_o R - s_o s_i = s_i s_o + s_i R$$

$$s_o R - s_i R = 2 s_i s_o$$

$$s_o - s_i = \frac{2s_i s_o}{R}$$

$$\frac{s_o - s_i}{s_i s_o} = \frac{2}{R}$$

$$\frac{1}{s_i} - \frac{1}{s_o} = \frac{2}{R}$$

$$\frac{1}{s_o} - \frac{1}{s_i} = -\frac{2}{R}$$

The magnification of the image relative to the object is found from $\frac{s_i}{s_o} = \frac{h_\mathrm{i}}{h_o}$. Let's keep the minus from before.

$$M = -\frac{s_i}{s_o},$$

Now notice that when the object distance $s_o \to \infty$, the image distance $s_i \to R/2$ behind the mirror. Also note that the size of the image goes to zero since $M = -\frac{s_i}{s_o} \to \frac{s_i}{\text{large}} \to 0$ as $s_o \to \infty$. From these observations we conclude that light from infinity has a virtual focus at $R/2$, the point midway between C and O, since the light does not really go there. The reflected light appears to come from there. Let's call this point again F, the focal point (virtual) and the distance $\frac{R}{2} = f$, the focal length. The formula becomes $\frac{1}{s_o} - \frac{1}{s_i} = -\frac{1}{f}$. But, using our convention that $s_i < 0$ behind the mirror, if we define $f < 0$ for a convex mirror, the formulas are the same for concave and convex mirrors:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \text{and} \quad M = -\frac{s_i}{s_o}.$$

Note that the minus in the magnification formula indicates our convex-mirror image is upright.

D4. Limitations. The parabola is the shape that focuses parallel light to a focal point. However, a circle approximates a parabola as long as you do not make the circular section too big. In fact all four basic conic sections match the shape for a small region if the circle. See Fig. 16.

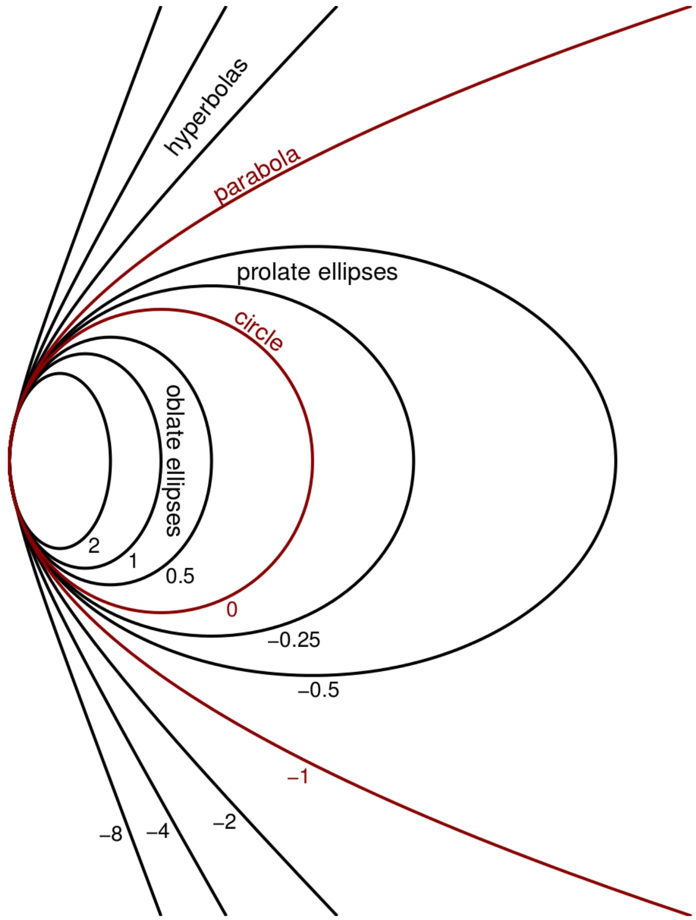

Fig. 16. The parabola matches well the circular shape for the small spherical section near the optic axis.
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