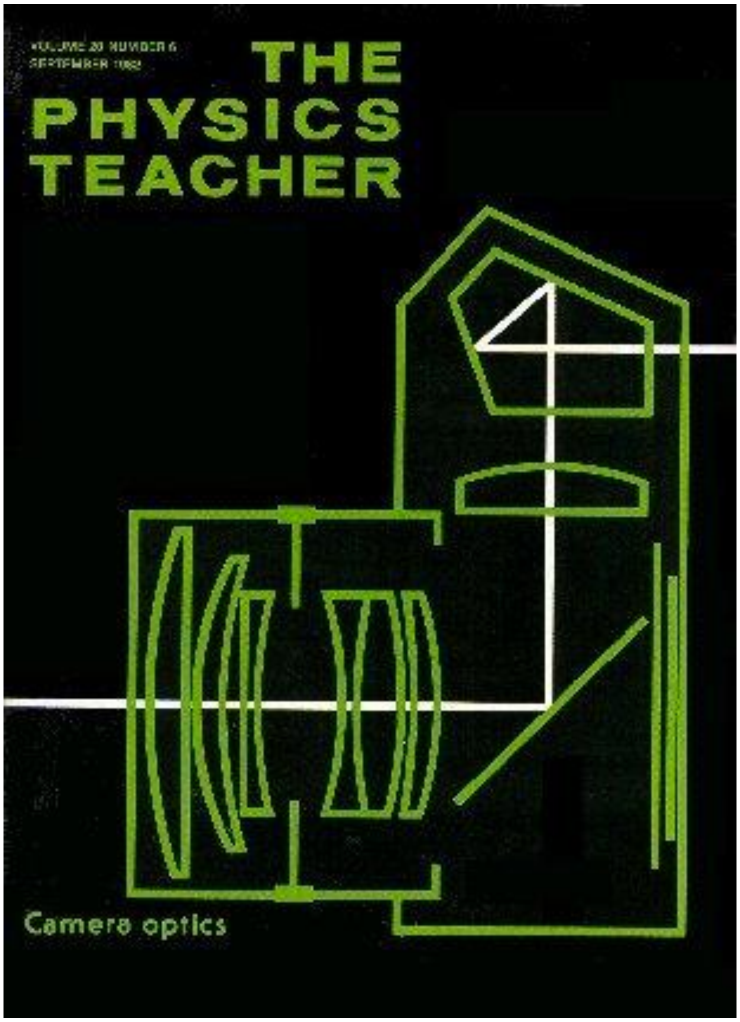
Modern Optics, Prof. Ruiz, UNCA **doctorphys.com**
Chapter H. Camera Lenses

H0. Lens Design.



To the left is a schematic of your instructor's camera featured on the cover of the September 1982 issue of ***The Physics Teacher***. It alerted readers to the issue's main article, one on "Camera Optics" written by me.

I traced the cover scheme and then it was professionally reproduced by Dean Hines, former Director of the ***UNCA Graphics and Publications Office***. That was a long time ago.

M. J. Ruiz, "Camera Optics," *The Physics Teacher* **20**, 372 (September 1982), an invited article and **cover** article. **pdf**

Note how the light ray is shown to come through the lens, reflect off the diagonal mirror, reach the viewfinder where the image is formed, and proceed through the pentaprism to the photographer's eye.

The mirror flips up when the photo is taken. The reflection due to this mirror has led to the word “reflex ”being used in the description of the camera: the R in SLR stands for “reflex,” i.e., refection.

The sketch shows that a camera lens can consist of 6 pieces of glass, i.e., 6 internal lenses. These act together to form one lens. What's with all the lenses? Don't we need just one? The answer is simple - to correct for aberrations. We will discuss these later. Can you find the achromatic doublet in the configuration? This takes care of reducing chromatic aberration.

The multiple lens elements are needed to reduce spherical aberration and the off-axis aberrations: coma, curvature of field, and astigmatism. Finally, two lens groups are necessary with the aperture in the center in order to prevent distortion. Such an arrangement is the result of much analysis and experimentation. Textbooks do not give details on lens curvatures and indexes of refraction as specific design characteristics tend to be kept secret by the manufacturing companies.

H1. Close-Up Photography.

Photos by Doc, May 5, 2020

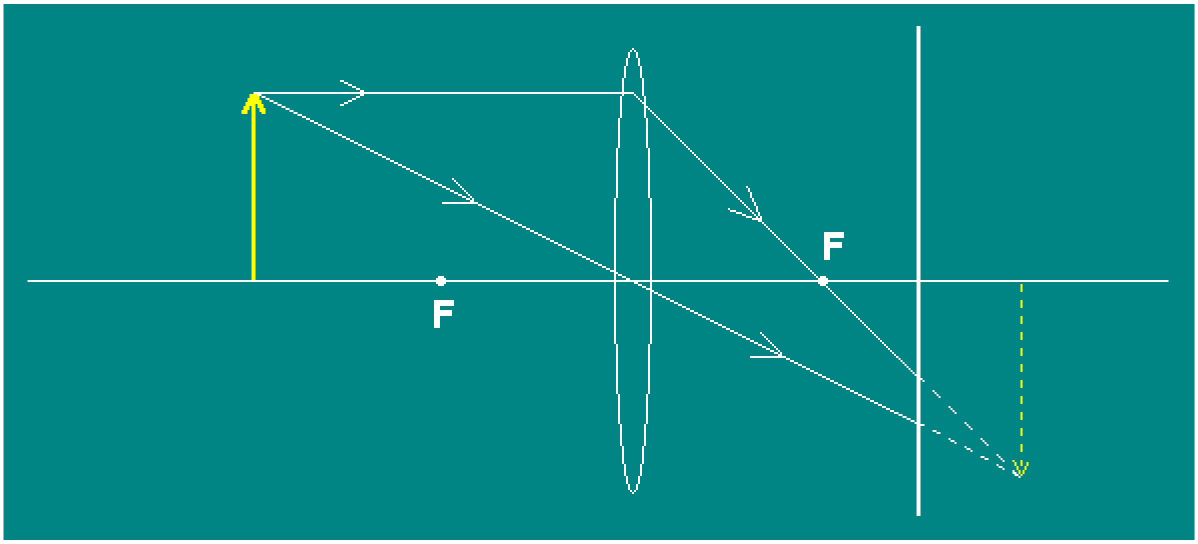
A 50-mm lens provides the normal angle of view for a 35-mm camera. The lens is therefore 50 mm from the film plane when the camera is focused at infinity. A typical minimum object-to-lens distance for such a lens is about 0.5 meter (roughly 1.5 feet). When focused at this closest point, the lens-to-film distance is almost 60 mm. The range of movement for the lens as the focusing barrel is turned is almost 10 mm. For objects closer to the lens, the lens cannot refract the rays enough to place the image on the film. Using $f = 50\text{ mm}$, the max lens-to-film distance of $s_i = f + 10 = 50 + 10 = 60\text{ mm}$, then for these particular values

$$\frac{1}{s_o} + \frac{1}{60} = \frac{1}{50}, \quad \frac{1}{s_o} = \frac{1}{50} - \frac{1}{60} = \frac{60-50}{50 \cdot 60} = \frac{10}{3000} = \frac{1}{300}, \quad s_o = 300\text{ mm}.$$

The closest object has a distance of $s_o = 300 \text{ mm} = 30 \text{ cm} = 1 \text{ ft}$ from the camera lens

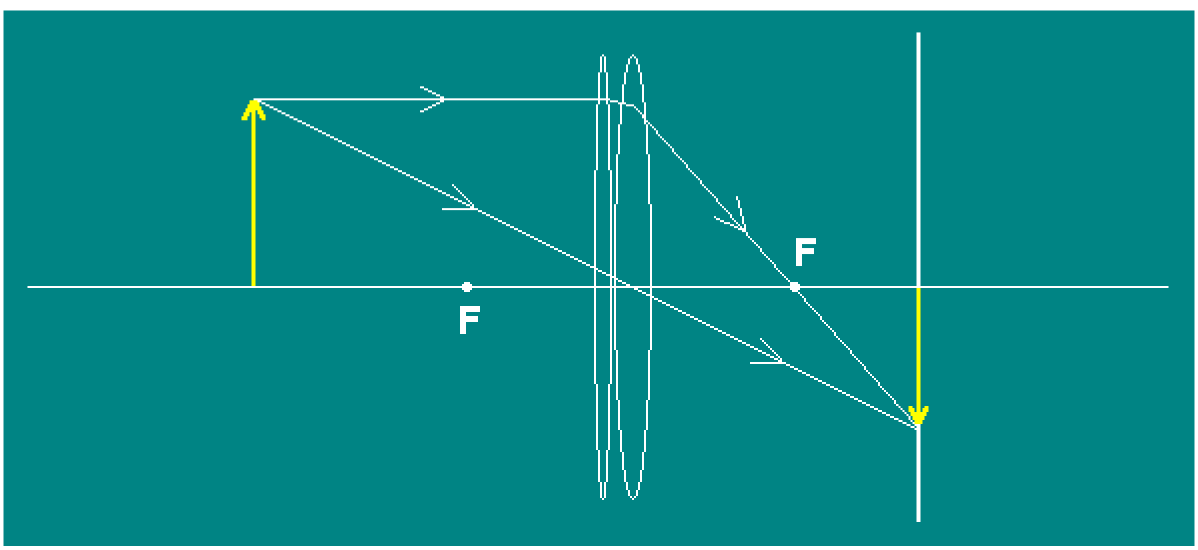
and the magnification is $M = -\dfrac{s_i}{s_o} = -\dfrac{60}{300} = -\dfrac{1}{5} = -0.2$.

The figure below illustrates how the image falls beyond the film when objects get too close. The film cannot get that far from the lens to be where we need it. The lens-film separation distance can only increase so much, i.e., by the 10 mm.

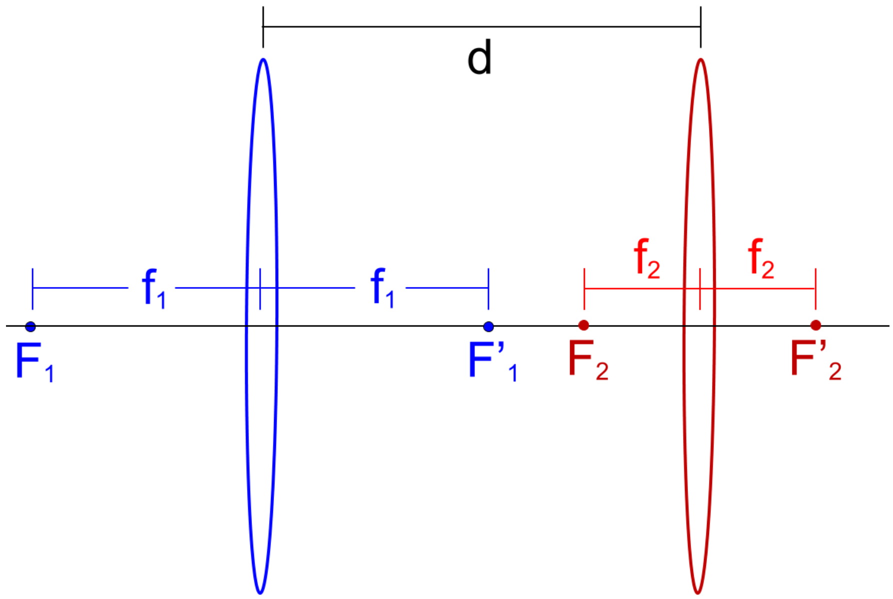


a) Close-Up Lens Attachments

By attaching another converging lens to the normal camera lens, additional focusing power is achieved.



What is the focal length of the combined system with a second lens attached to the main camera lens? We use a powerful formula for a two-lens system derived for a homework assignment, where lenses with focal lengths f_1 and f_2 are separated by distance d.



$$f = \frac{f_1 f_2}{f_1 + f_2 - d}\text{, or the form } \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}.$$

For attached lenses by threading one on to the other, $d = 0$, and we have

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

This formula is extremely convenient since the reciprocal of the focal length gives the strength of the lens. A lens with a very short focal length refracts light more dramatically and is therefore a strong lens. If the lens hardly refracts the light at all, the focal length will be very long. If you make the lens flat the light passes straight through. The focal length is infinite.

Eye doctors and opticians love the reciprocal since it tells them how strong the lens is. The reciprocal is called the power and the units are diopters when the focal length is expressed in meters.

$$P(\text{in diopters D}) = \frac{1}{f\,(\text{in meters})}$$

$$P(\text{in D}) = \frac{1}{f\,(\text{in m})} = \frac{100}{f\,(\text{in cm})} = \frac{1000}{f\,(\text{in mm})}$$

The big bonus is that we can replace

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{with} \quad P = P_1 + P_2\,.$$

THE DIOPTERS ADD!

You also have the related formulas

$$f\,(\text{in m}) = \frac{1}{P(\text{in D})} \qquad f\,(\text{in cm}) = \frac{100}{P(\text{in D})} \qquad f\,(\text{in mm}) = \frac{1000}{P(\text{in D})}$$

Close-up lens attachments usually come in a package of three with diopters 1D, 2D, and 4D.

Courtesy Andrew Magill via Flickr and Wikipedia

With the regular f = 50 mm lens without any attachments we found above that for $f = 50\text{ mm}$ and the max lens-to-film distance $s_i = f + 10 = 50 + 10 = 60\text{ mm}$, and we

the closest a subject can get is $s_o = 30 \text{ cm}$. Call this main camera lens 1 so that $f_1 = 50 \text{ mm}$. The power of this lens is $P_1 = \frac{1000 \text{ mm}}{f_1} = \frac{1000 \text{ mm}}{50 \text{ mm}} = 20\text{D}$.
Attaching the +1D lens to the main lens will let us get closer to the subject. The power of the combination is 21D with a corresponding $f = \frac{1000}{P} = \frac{1000}{21} = 47.62 \text{ mm}$. We can now solve for the new close object distance s_o.

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

$$\frac{21}{1000} = \frac{1}{s_0} + \frac{1}{60_i}$$

$$\frac{1}{s_0} = \frac{21}{1000} - \frac{1}{60}$$

$$s_0 = 231 \text{ mm}$$

$$M = -\frac{s_i}{s_o} = -\frac{60}{231} = -\frac{1}{5} = -0.26$$

Compared to the $M = -0.2$ without the attachment, magnification is larger. What about all three close-up lenses? Then, we need to add the diopter value for the 50-mm lens, which is 20D to + 1D + 2D + 4D. The total is 27D. Then, the closest we can get is found from

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$ where $f = \frac{1000}{27} \text{ mm}$ and $s_i = 60 \text{ mm}$.

$$\frac{27}{1000} = \frac{1}{s_0} + \frac{1}{60}$$

$s_0 = 97 \text{ mm}$ and $M = -\frac{s_i}{s_o} = -\frac{60}{97} = -0.62$

See the experimental results below.

Photos of License: Close-Up Lens Attachments

Photos are Dark Because No Compensation was Made for the Bright Background

Normal Lens Alone

With +1D Lens Attached

With +1 and +2 Lenses (+3D Attached)

With +1, +2, and +4 Lenses (+7D Attached)

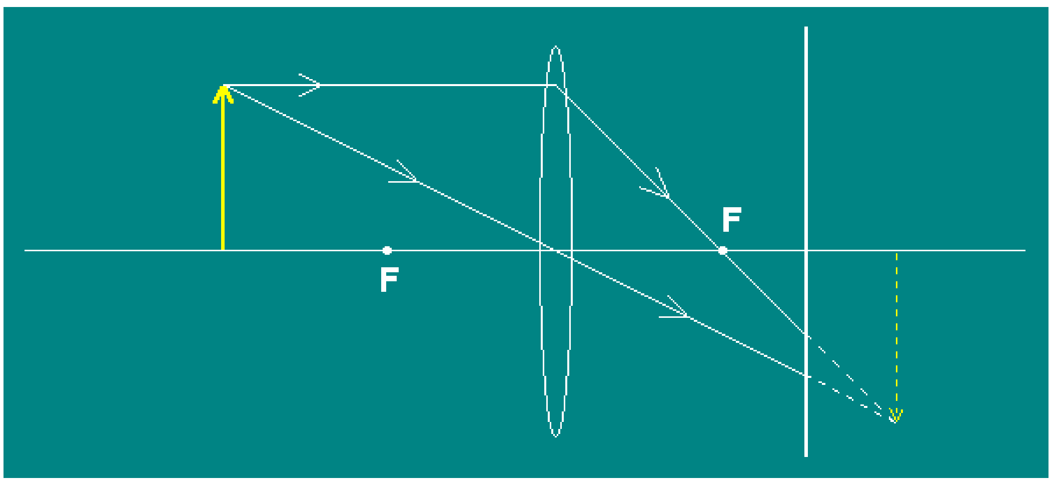
Photos by Prof. Ruiz (c. 1980)

The light meter was overwhelmed with the white background in that first photo, but adjusted well when the license took up the entire view.

b) Extension Tubes

Perhaps the easiest way to provide for shorter subject distances from the point of view of the physics is to simply extend the lens-to-film distance, i.e., the image distance. Cylindrical barrels that are inserted between the camera lens and camera body in order to ***extend*** the distance from the lens to the film plane are called ***extension tubes***. The usual set comes with three tubes of different lengths. For example, the tubes shown below have lengths 36 mm, 20 mm, and 12 mm. These can be used singly or in combination.

Courtesy Wikipedia: Fg2



Now we can move the film back to where the image is trying to form.

With all three extension tubes, the 36 mm, 20 mm, and 12 mm, added to the camera's maximum lens-to-film 60 mm, we can get a maximum

$$s_i = 60 + 36 + 20 + 12 = 128 \text{ mm}.$$

Then,

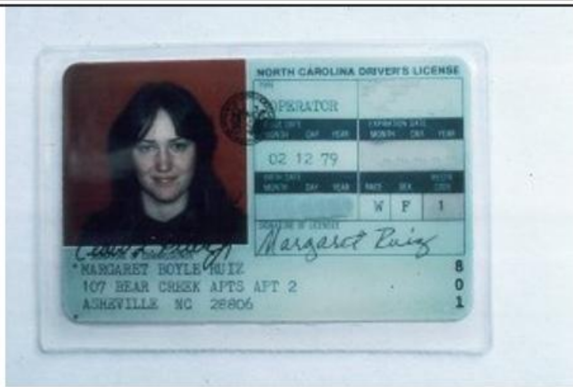
$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i} \quad \text{becomes} \quad \frac{1}{50} = \frac{1}{s_0} + \frac{1}{128}$$

$$\frac{1}{s_0} = \frac{1}{50} - \frac{1}{128}$$

$s_0 = 82 \text{ mm}$ with $M = -\dfrac{s_i}{s_o} = -\dfrac{128}{82} = -1.6$, which beats the close-up lenses big time!

Photos of License: Extension Tubes

Darkness is Eliminated as the Bright Surrounding Region is Excluded



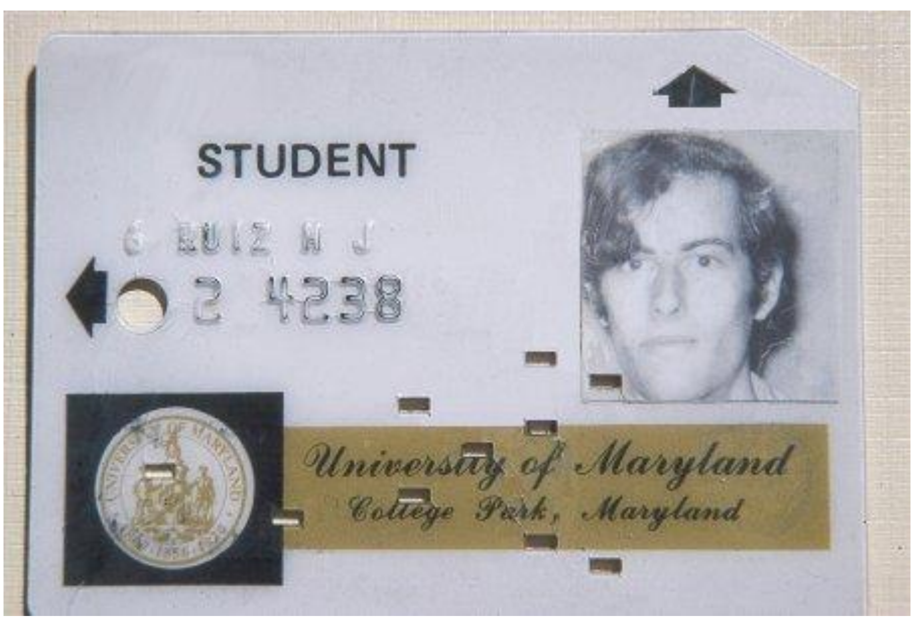
With the Smallest Extension

With the Middle Extension

With the Longest Extension

All Three Extensions

Photos by Prof. Ruiz (c. 1980)



H2. The Telephoto Lens.

Remember the shots of the Biltmore House in the previous chapter? Many were telephoto photos. Think telescope. You can take a picture of something far away. Telephoto lenses are very useful in wildlife photography as they can magnify distant subjects compared to the usual 50-mm lens. This is consistent with their narrow angles of view.

Telephoto Photo of Bear at UNCA (Use a Telephoto Lens – You do not want to get close.)

Photo of Bear by Doc March 19, 2019, Students in Class at Whitesides Hall in Background

For $s_o \to \text{large}$ in

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}, \text{ we have } s_i \to f \text{ and } M = -\frac{s_i}{s_o} \to -\frac{f}{\text{large}}.$$

Let $M_{50\text{mm}} = -\dfrac{f}{\text{large}} = -\dfrac{50}{\text{large}}$. Then the magnification for some arbitrary f is $M_f = -\dfrac{f}{\text{large}}$. The relative magnification compared to the f = 50 mm case is then

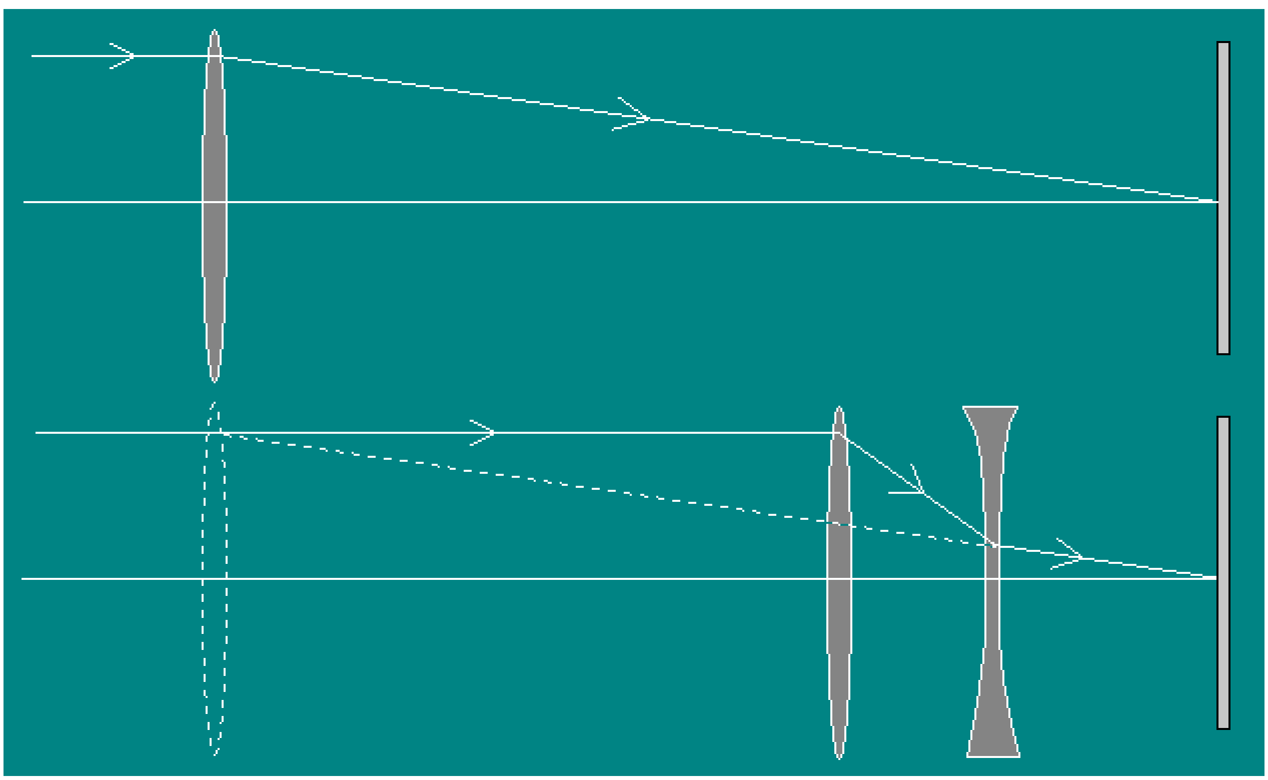
$$M_{\text{relative}} = \frac{-\dfrac{f}{\text{large}}}{-\dfrac{50}{\text{large}}} = \frac{f}{50}.$$

For a 100 mm telephoto you get 2x magnification relative to the normal lens.

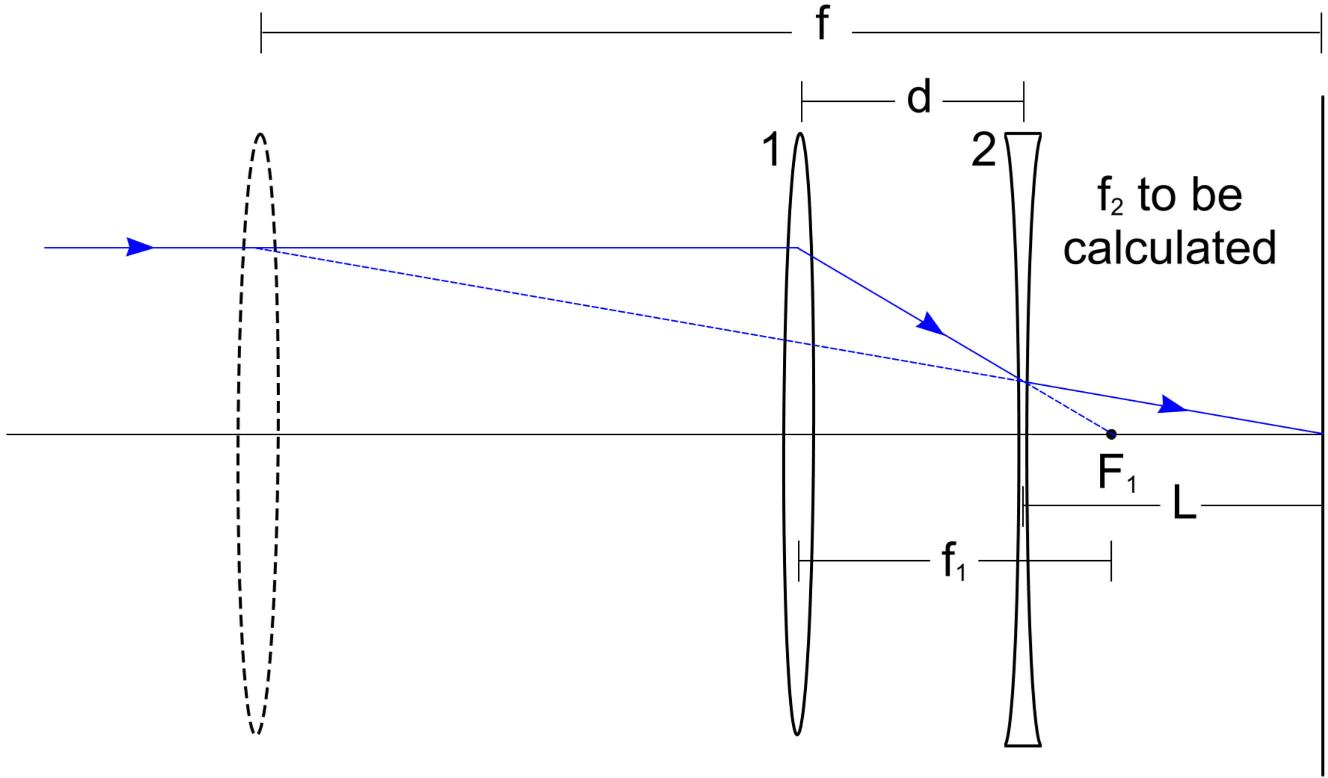
For a 400 mm telephoto, you get 4x.

The Problem: The long focal length will stick out too far the camera hard to handle.

We need to design a telephoto lens that is compact. The secret is to use two lenses. In reality, it will be two lens groups to minimize for the aberrations. But the basic idea is a compound system with one converging lens and one diverging lens. The ray diagram illustrates the invention. The effective or equivalent focal length in the two-lens arrangement is the same as the single lens shown above it.



Let's design one.



We will design one with $f_1 = 50\text{ mm}$ so we can manufacture the corresponding diverging lens in a cheap version as an insertion where the user can pop off their $f_1 = 50\text{ mm}$ lens and insert the diverging lens between it and the camera body. Such a diverging lens insert is called a teleconverter. But the unit we sell with our own converging lens as a two-lens system will be the expensive version since aberrations will be minimized for the telephoto system. The regular f = 50 mm camera lens is optimized for the standard use of the camera. Popping off the f = 50 mm lens and adding the teleconverter will not result in an ideal match for minimizing aberrations. But it is cheaper to construct your own telephoto by adding the teleconverter. It converts your regular lens to a telephoto.

The backward focal length for a two-lens system, derived in homework is

$$f_b = \frac{f_2(d - f_1)}{d - f_1 - f_2} = \frac{f_2(f_1 - d)}{f_1 + f_2 - d}\text{, and the effective focal length is } f = \frac{f_1 f_2}{f_1 + f_2 - d}.$$

We will see what happens taking $L = 40\text{ mm}$ and $d = 30\text{ mm}$.

Note from the diagram that f_1 must be greater than d, and it is for our choices.

We can now solve for f_2 since from the figure we see that $f_b = \dfrac{f_2(f_1 - d)}{f_1 + f_2 - d} = L$.

$$f_b = \frac{f_2(50-30)}{50 + f_2 - 30} = 40$$

$$\frac{f_2(20)}{20 + f_2} = 40$$

$$f_2(20) = 800 + 40 f_2$$

$$-20 f_2 = 800$$

The focal length f_2 is going to be negative as expected, a diverging lens.

$$f_2 = -40 \text{ mm}$$

Now let's see what our design leads to for the effective focal length.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{50 \cdot (-40)}{50 + (-40) - 30} = \frac{-2000}{50 - 70} = \frac{-2000}{-20} = 100 \text{ mm}$$

We have a telephoto, one with $f = 100 \text{ mm}$.

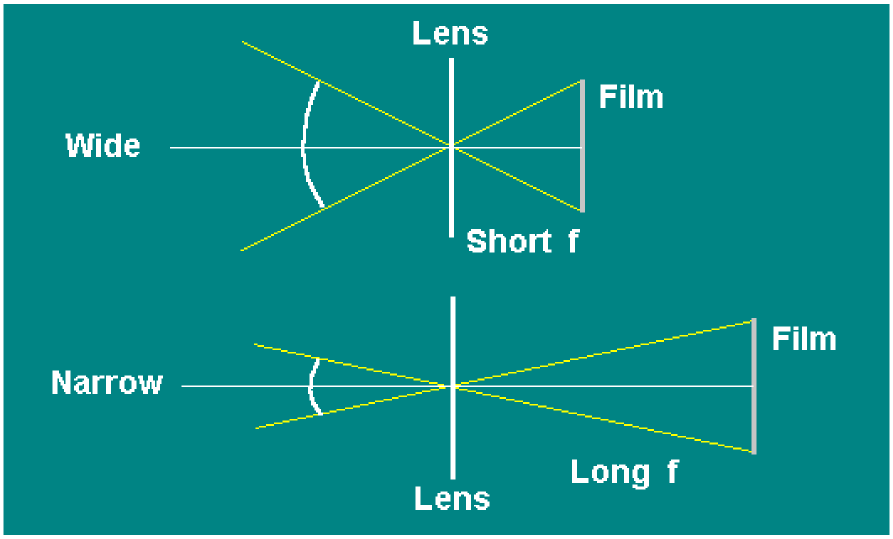
We market the diverging lens separately labeling it as a 2x converter.

When you pop off your 50-mm lens and insert the 2x converter, you get 2 x 50 = 100 mm.

Of course we have to manufacture several teleconverters to specs for different cameras since the threads or mounting connectors differ for different camera companies.

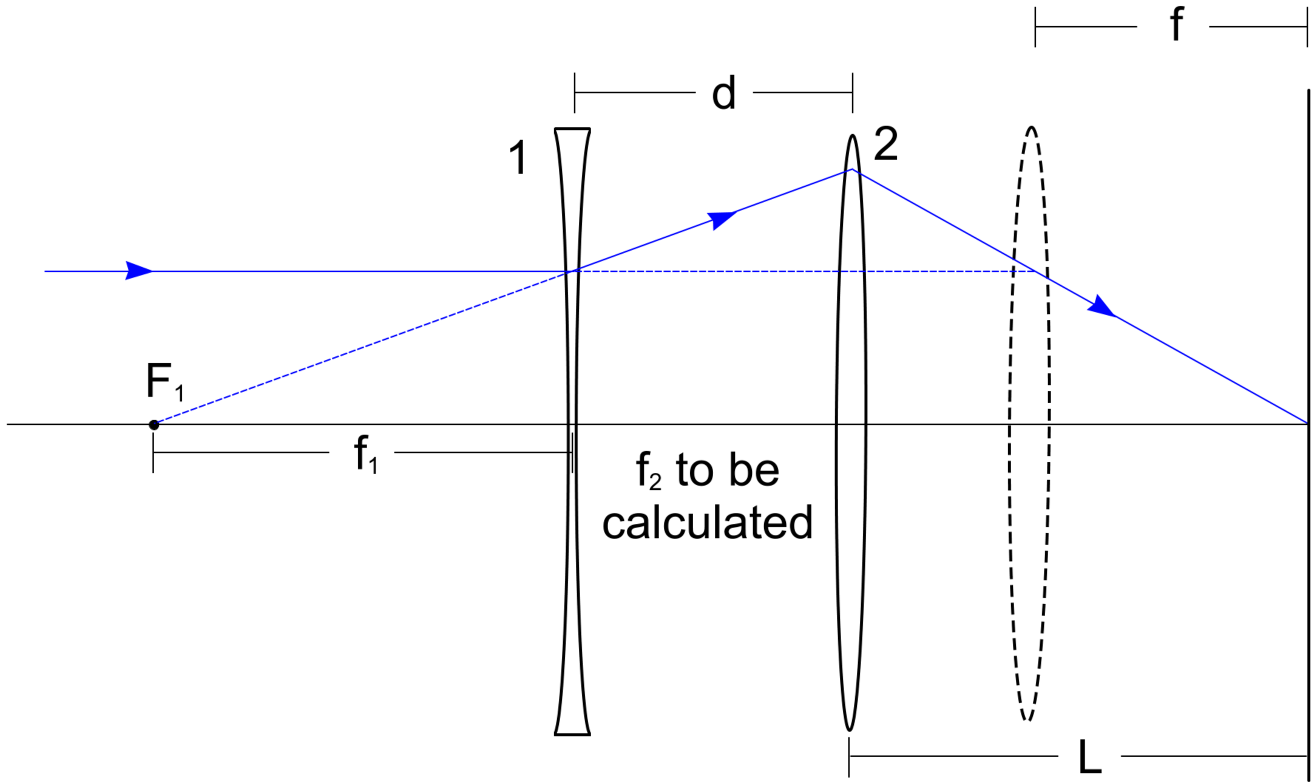
H3. The Wide-Angle Lens.

Did you ever want to take a picture of a large group and fit them all in the photo? For that, you want a wide angle less, one with a focal length f < 50 mm. Two historical standards for wide-angle lenses are the classic 24 mm and 35 mm lenses.



Super Wide-Angle Shot – the Fisheye Lens where f = 8 mm

The secret here is to turn around the telephoto design. Have you ever looked backwards into a pair of binoculars? Your binoculars are a pair of telephotos and looking in the wrong way everything looks small. You see a wide angle of view.



To give an example of this design with specific numbers, let's use

$$f_1 = -30 \text{ mm}, \qquad d = 20 \text{ mm}, \qquad L = 50 \text{ mm}.$$

We will need the general formulas $f_b = \dfrac{f_2(f_1 - d)}{f_1 + f_2 - d} = L$ and $f = \dfrac{f_1 f_2}{f_1 + f_2 - d}$.

Start with the back focal length:

$$f_b = \frac{f_2(-30-20)}{-30+f_2-20} = L = 50 \qquad \frac{-f_2 50}{f_2 - 50} = 50 \qquad \frac{-f_2}{f_2-50} = 1$$

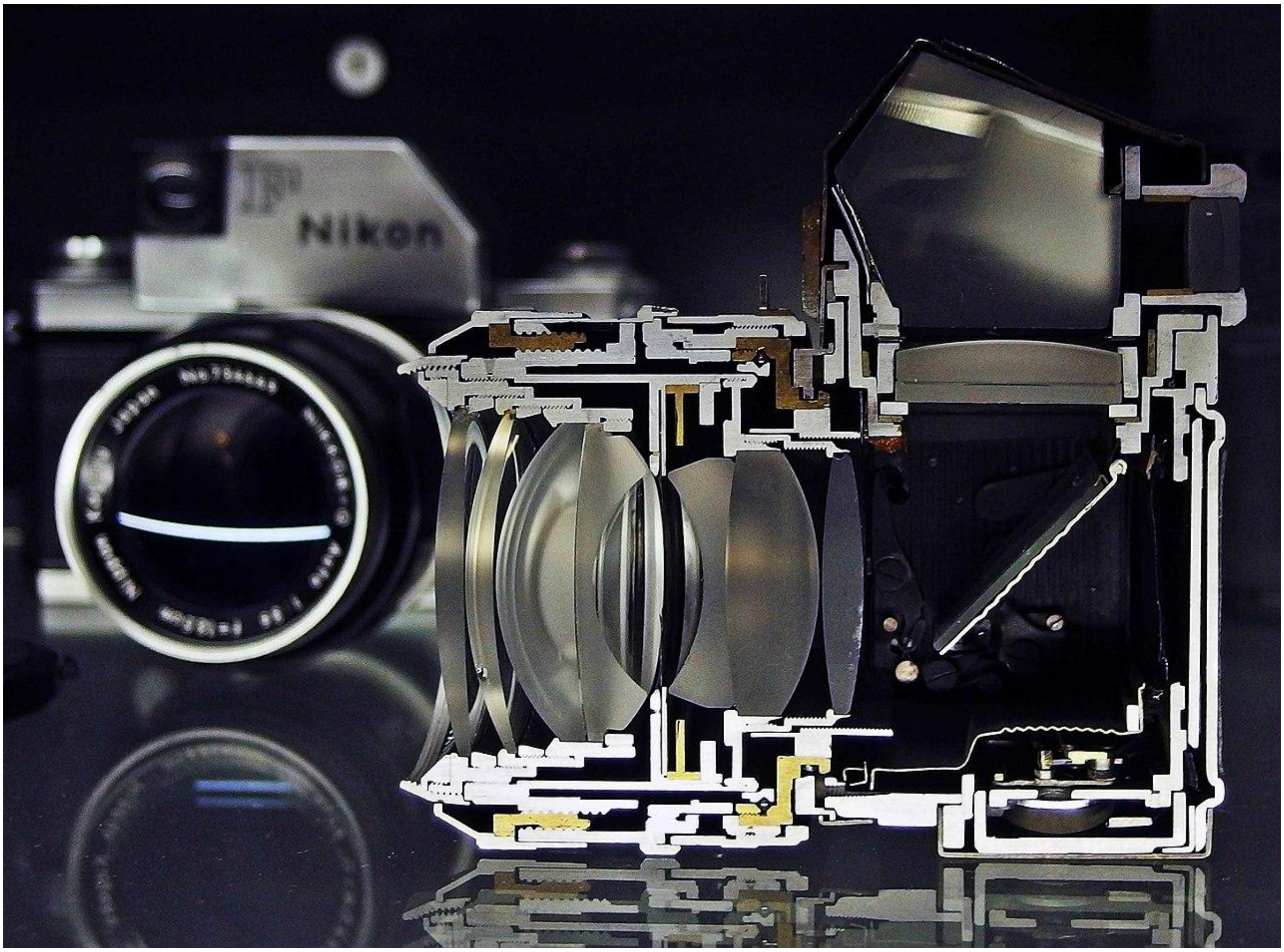
$$-f_2 = f_2 - 50 \qquad 2f_2 = 50 \qquad f_2 = 25 \text{ mm}$$

$f = \dfrac{-30 \cdot 25}{-30+25-20} = \dfrac{-750}{-25} = 30 \text{ mm}$, a wide-angle focal length since $f < 50 \text{ mm}$.

But wait! Since $f_2 = 25\text{ mm} < 50\text{ mm}$ is wide angle, why not just this lens?

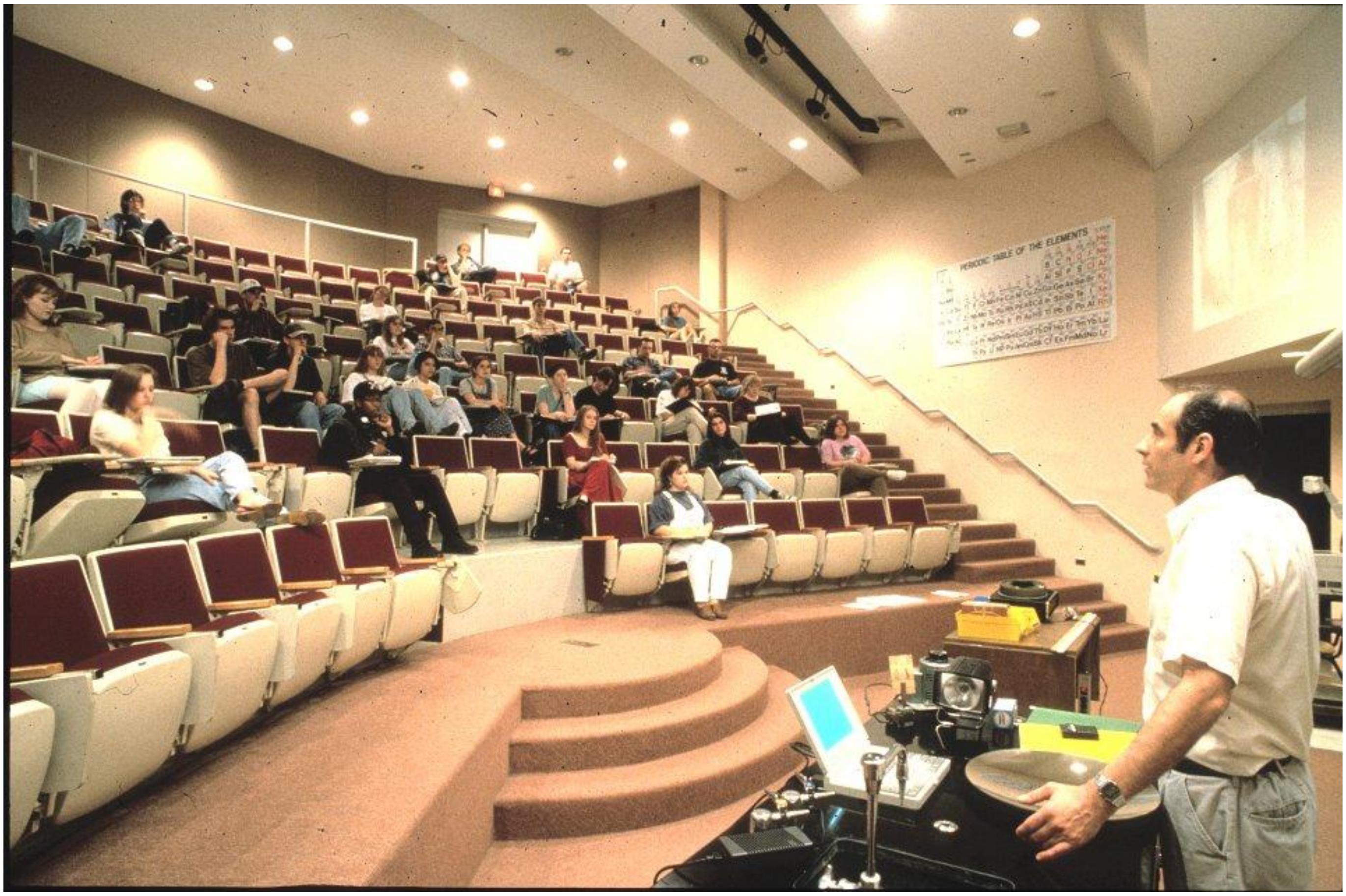
You can't!

You can't put a lens that close to the film. The flipping mirror mechanism is there.



Courtesy Wikipedia: Martin Vorel

There needs to be at least 40 mm clearance. So our design is needed. Our lenses are located 50 mm and more away from the film, but the two-lens system has an effective focal length only 30 mm from the film!



Wide-angle shot by Asheville Professional Photographer John Warner
Robinson Hall 125 (c. 1995)

Super Wide Angle Shot of Biltmore House from the Gazebo (Doc Ruiz, c. 1980)

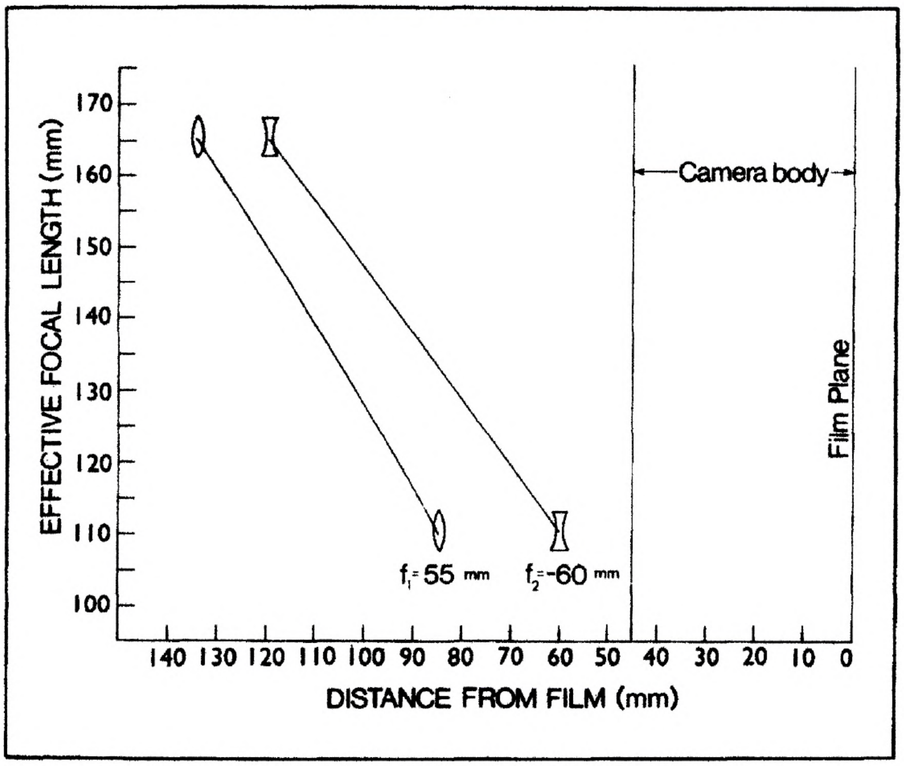
H4. The Zoom Lens.

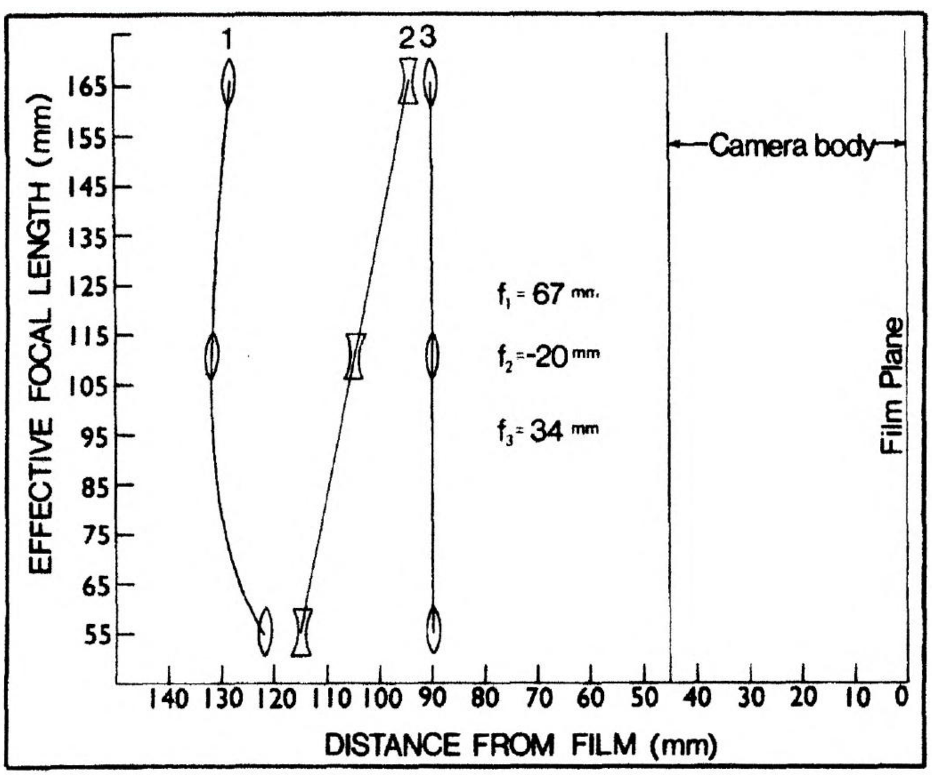
Zoom lenses have variable focal lengths. This invention makes it hard to correct for aberrations for all the different focal lengths. But a zoom is convenient. So there is a trade off. If you want the best optics as a professional photographer you have to fork out the cash to buy several fixed-focal length lens, all optimized for their specific focal lengths. See figure below.

Photo: Rainer Knäpper, Free Art License (http://artlibre.org/licence/lal/en/), https://commons.wikimedia.org/wiki/File:Minolta_Rokkor_Teleobjektive.jpg

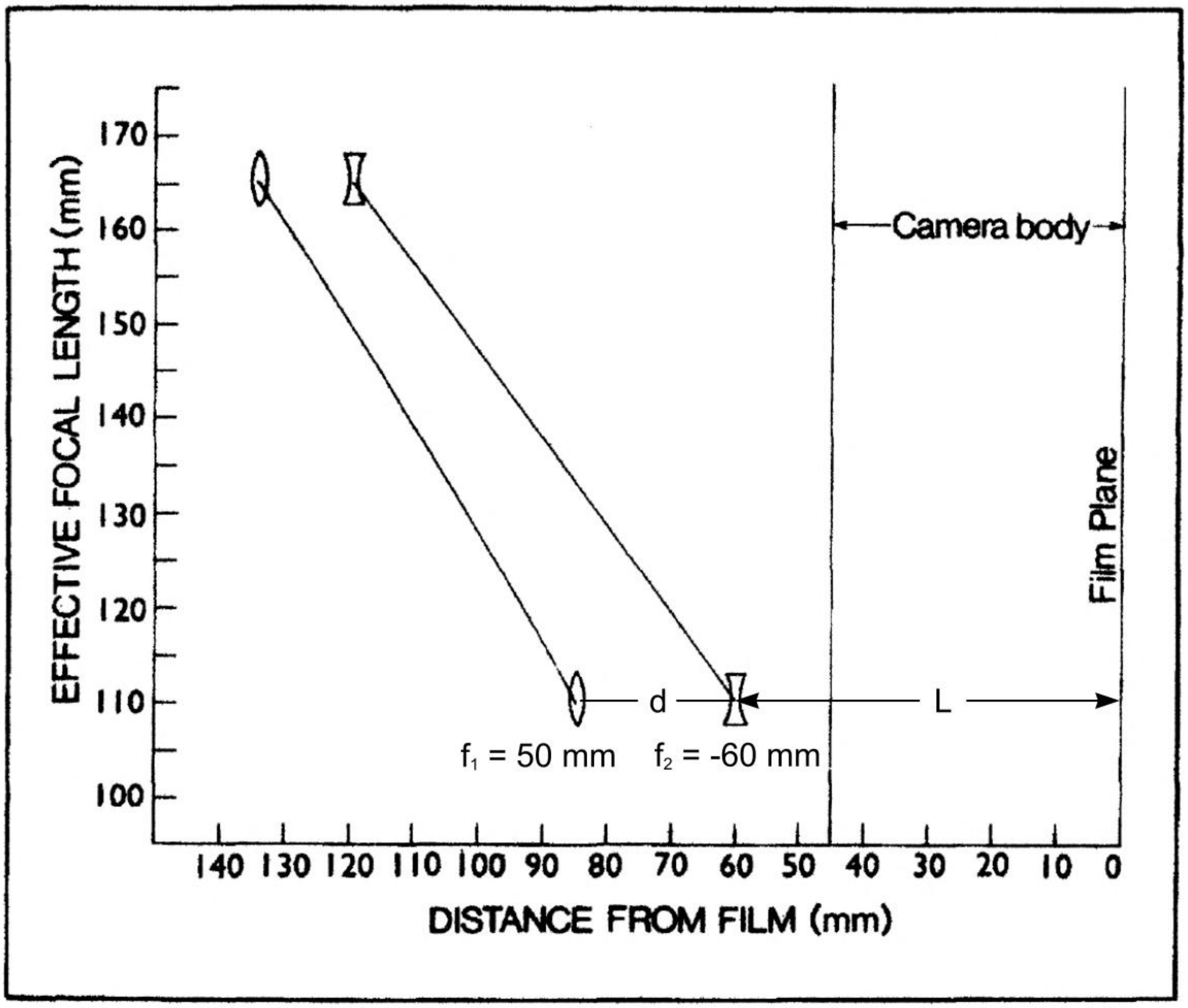
Below are two simple zoom lens designs.

M. J. Ruiz, "Camera Optics," *The Physics Teacher* **20**, 372 (September 1982), an invited article and **cover** article. **pdf**





You can see from the figures that the component lenses have to move with respect to each other as you zoom in and out. This movement keeps the focus as the effective focal length changes. The above figures are accurate to scale. Let's change the lenses and derive some formulas for the parameters d and L that change during the zoom operation.



We need as our stating point the following.

$$f_1 = 50 \text{ mm} \qquad f_2 = -60 \text{ mm} \qquad f_b = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} = L \qquad f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

We can easily get d as a function of L and f as a function of d from the following.

$$f_b = \frac{-60(50-d)}{50-60-d} = L \qquad f = \frac{50(-60)}{50-60-d}$$

$$\frac{-3000+60d}{-10-d}=L \qquad f=\frac{-3000}{-10-d}$$

$$-3000+60d=-10L-Ld \qquad f=\frac{3000}{10+d}$$

$$60d+Ld=3000-10L \qquad f=\frac{3000}{10+d}$$

$$d(60+L)=3000-10L \qquad f=\frac{3000}{10+d}$$

$$d=\frac{3000-10L}{60+L} \qquad f=\frac{3000}{10+d}$$

Let's say for our mechanism, we can make it so the L can go from 60 mm to 120 mm.

For the L = 60 mm extreme position:

$$d=\frac{3000-10\cdot 60}{60+60}=\frac{2400}{120}=20\text{ mm}$$

$$f=\frac{3000}{10+d}=\frac{3000}{10+20}=\frac{3000}{30}=100\text{ mm}$$

A nice telephoto focal length.

For the L = 120 mm extreme position:

$$d=\frac{3000-10\cdot 120}{60+120}=\frac{1800}{180}=10\text{ mm}$$

$$f=\frac{3000}{10+d}=\frac{3000}{10+10}=\frac{3000}{20}=150\text{ mm}$$

Our zoom has a focal length range of 100 mm to 150 mm.
A 2x to 3x Zoom relative to the normal 50 mm lens.

I bought a 100 mm to 300 mm zoom one day – very nice.

I took this photo of a tiger across a moat at f = 300 mm, Washington, DC Zoo.