**Modern Optics, Prof. Ruiz, UNCA** **doctorphys.com**
**Chapter H Solutions. Camera Lenses**

## HW H1. Close-Up Photography

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| Courtesy Andrew Magill via Flickr and Wikipedia | Courtesy Wikipedia: Fg2 |

You have there close-up lens attachments $+1D$, $+2D$**, and** $+4D$ along with three extension tubes of lengths $12\text{ mm}$, $20\text{ mm}$, and $36\text{ mm}$. You have an SLR camera with its standard $50\text{ mm}$ focal length lens. The most you can turn the focusing mechanism to focus close up gets you a maximum lens-to-film distance of $s_i = 60\text{ mm}$, which allows you to bring a subject as close as $s_o = 300\text{ mm}$ for a magnification of $M = -\frac{s_i}{s_o} = -\frac{60}{300} = -\frac{1}{5} = -0.2$. Calculate the closest you can bring an object now for photographing and determine the magnification. Report your answer to two significant figures. Comment on your magnification comparing it to life size.

With the three extension tubes $s_i = 60\text{ mm} + 12\text{ mm} + 20\text{ mm} + 36\text{ mm} = 128\text{ mm}$.

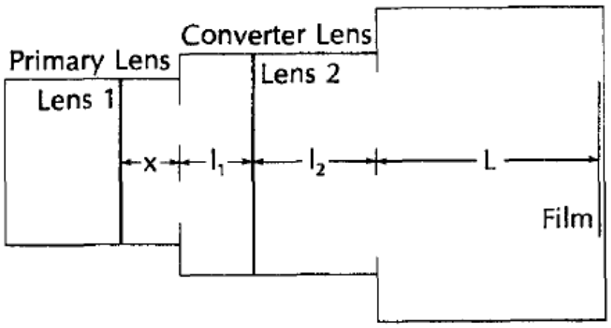
We will also add the close-up lenses: $\frac{1}{f} = \frac{1}{f_{camera}} + 1 + 2 + 4$. Note $\frac{1}{f_{camera}} = \frac{1000}{50} = 20D$.

$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$ leads to $20 + 1 + 2 + 4 = \frac{1}{s_0} + \frac{1}{0.128}$, using diopters and meters.

$\frac{1}{s_0} = 27 - \frac{1}{0.128} = 19.19$ => $s_0 = \frac{1}{19.19}$ => $s_0 = 0.052\text{ m} = 52\text{ mm}$

$M = -\frac{s_i}{s_o} = -\frac{128\text{ mm}}{52\text{ mm}} = -2.5$. Magnification is two and a half times life size!!!

**HW H2. The Teleconverter: Formula for Diverging Insert**




A primary camera lens with focal length $f_{\text{camera}} = f_1$ can be converted into a telephoto lens by inserting a diverging lens, the converter secondary lens with focal length $f_2$, between the primary lens and camera body.

Derive the following formula for the focal length of the diverging lens.

$$f_2 = -\frac{(L+l_2)(L-l_1)}{(l_1+l_2)}$$

Hints: For telephoto lens analysis all subject distances are very far away. In your analysis you should justify why $x + L = f_1$.

Start with $f_b = \dfrac{f_2(f_1 - d)}{f_1 + f_2 - d}$.

When the regular camera lens is only attached to the camera body and you focus on objects far away, then $s_o = 50\ \text{mm}$. But from the figure with the converter lens removed $s_o = x + L$. Therefore $f_1 = x + L$. The separation distance between the lenses is $d = x + l_1$ from the figure. Finally, the back focal length $f_b = l_2 + L$ from the figure.

Summary: From the figure $f_1 = x + L$, $d = x + l_1$, and $f_b = l_2 + L$.

Then $f_b = \dfrac{f_2(f_1 - d)}{f_1 + f_2 - d}$ becomes $l_2 + L = \dfrac{f_2\left[(x+L)-(x+l_1)\right]}{(x+L)+f_2-(x+l_1)}$

$$l_2 + L = \frac{f_2(L-l_1)}{L+f_2-l_1}$$

$$L + l_2 = \frac{f_2(L-l_1)}{f_2+L-l_1} \quad \Rightarrow \quad (f_2 + L - l_1)(L + l_2) = f_2(L - l_1)$$

$$f_2(L+l_2) + (L-l_1)(L+l_2) = f_2(L-l_1)$$

$f_2 L + f_2 l_2 + (L-l_1)(L+l_2) = f_2 L - f_2 l_1$ and the $f_2 L$ will cancel on each side

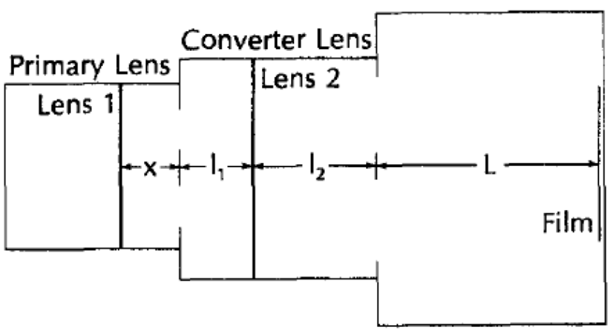
$$f_2 l_2 + (L - l_1)(L + l_2) = -f_2 l_1$$

$$f_2(l_1 + l_2) + (L - l_1)(L + l_2) = 0$$

$$f_2(l_1 + l_2) = -(L - l_1)(L + l_2)$$

$$\boxed{f_2 = -\frac{(L + l_2)(L - l_1)}{(l_1 + l_2)}}$$

**HW H3. The Teleconverter: Multiplication Factor**




A primary camera lens with focal length $f_{\text{camera}} = f_1$ can be converted into a telephoto lens by inserting a diverging lens, the converter secondary lens with focal length $f_2$, between the primary lens and camera body.

The multiplication factor $\alpha$ is defined as the ratio of the effective focal length for the system compared to $f_{\text{camera}}$. As an example, if with your inserted teleconverter your new focal length for the lens system is now $f = 2 f_{\text{camera}}$, then you have $\alpha = 2$ we say you have a 2x converter.

Find the simplest form for multiplication factor $\alpha$ in terms of the following parameters you find in the schematic: $l_1$, $l_2$, and $L$. The function $\alpha = \alpha(l_1, l_2, L)$ will not be a function of $x$.

Start with the effective focal length for a two-lens system: $f = \dfrac{f_1 f_2}{f_1 + f_2 - d}$.

$$\alpha \equiv \frac{f}{f_1} = \frac{f_2}{f_1 + f_2 - d}$$

Also from HW H2: $f_1 = x + L$, $d = x + l_1$, and $f_2 = -\dfrac{(L + l_2)(L - l_1)}{(l_1 + l_2)}$.

Use substitution first with $f_1 = x + L$ and $d = x + l_1$.

$$\alpha = \frac{f_2}{(x + L) + f_2 - (x + l_1)}$$

$$\alpha = \frac{f_2}{L + f_2 - l_1}$$

$$\alpha = \frac{f_2}{f_2 + (L - l_1)}$$

Now we pull in $f_2 = -\dfrac{(L + l_2)(L - l_1)}{(l_1 + l_2)}$ from HW H2.

$$\alpha = \frac{-\dfrac{(L + l_2)(L - l_1)}{(l_1 + l_2)}}{-\dfrac{(L + l_2)(L - l_1)}{(l_1 + l_2)} + (L - l_1)}$$

$$\alpha = \frac{-(L + l_2)(L - l_1)}{-(L + l_2)(L - l_1) + (l_1 + l_2)(L - l_1)}$$

The $(L - l_1)$ factor cancels out everywhere.

$$\alpha = \frac{-(L + l_2)}{-(L + l_2) + (l_1 + l_2)}$$

$$\alpha = \frac{-(L + l_2)}{-L - l_2 + l_1 + l_2}$$

$$\alpha = \frac{-(L + l_2)}{-L + l_1}$$

$$\boxed{\alpha = \frac{L + l_2}{L - l_1}}$$

**HW H4. Depth of Field.** In the previous chapter we found that the depth of field can be approximated as $DoF = \frac{2s_o^2 c}{fd}$. Explain in words why each parameter affects the DoF the way it does.

1) Object Distance $s_o$. Formula says increase $s_o$ => increase DoF. When you focus on something far away, your image is extremely small and the images are extremely close to the focal point. The small images will result in circles of confusion smaller than the allowed maximum c. Therefore, such super small images will allow for a greater depth of focus, with its associated greater depth of field.

2) Circle of Confusion c. Formula says increase c => increase DoF. Since c is your allowed circle of confusion, if c increases, your toleration increases. If you can tolerate a greater circle of confusion, then your depth of field will increase until you meet the new toleration criterion.

3) Focal Length f. Formula says increase f => decrease DoF. If you increase f, you increase the sizes of images and the associated circles of confusion will increase beyond the allowed maximum circle of confusion c. Therefore your DoF will have to decrease to stay within the allowed maximum circle of confusion.

4) Aperture diameter d. Formula says increase d => decrease DoF. If you increase d, there are more light rays entering away from the optic axis, which off-axis rays increase circles of confusion beyond the allowed circle of confusion c. So you need to decrease DoF to bring the circles of confusion back to within the allowed maximum value for c. Or, you can think about decreasing d. If you decrease d, you approach the pinhole camera, which has an infinite depth of field. Therefore, smaller d means increased DoF and vice versa.