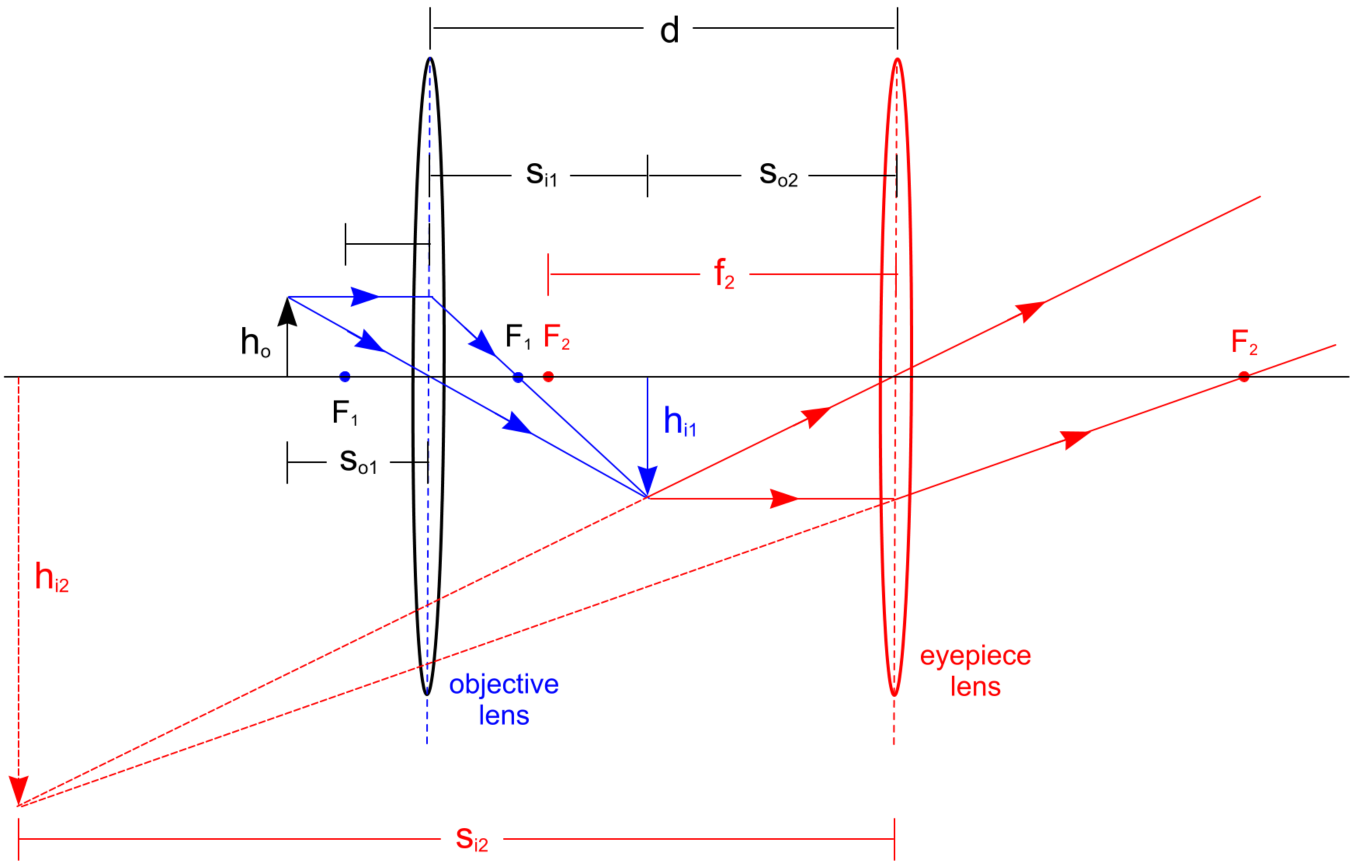
HW-J1. The Microscope.

is a detailed drawing. **Important:** Why must $f_1 < s_{o1} < 2f_1$ and $s_{o2} < f_2$?



Microscope. Inspired by Maier A, Steidl S, Christlein V, et al., editors. Medical Imaging Systems, Cham (CH): Springer; **2018.** Creative Commons License

$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \;\; => \;\; f_1 = \frac{s_{o1}s_{i1}}{s_{o1}+s_{i1}} \;\; => \;\; (s_{o1}+s_{i1})f_1 = s_{o1}s_{i1} \;\; => \;\; s_{o1}f_1 + s_{i1}f_1 = s_{o1}s_{i1}$$

$$s_{o1}f_1 = s_{o1}s_{i1} - s_{i1}f_1 \;\; => \;\; s_{o1}f_1 = (s_{o1}-f_1)s_{i1} \;\; => \;\; s_{i1} = \frac{s_{o1}f_1}{s_{o1}-f_1}$$

$$M_1 = -\frac{s_{i1}}{s_{o1}} = -\frac{s_{o1}f_1}{s_{o1}-f_1}\frac{1}{s_{o1}} = -\frac{f_1}{s_{o1}-f_1}$$

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \quad \text{and} \quad s_{i2} = \frac{s_{o2} f_2}{s_{o2} - f_2} \quad \text{where} \quad s_{o2} = d - s_{i1}\,.$$

$$M_2 = -\frac{s_{i2}}{s_{o2}} = -\frac{s_{o2} f_2}{s_{o2} - f_2}\frac{1}{s_{o2}} = -\frac{f_2}{s_{o2} - f_2} = -\frac{f_2}{d - s_{i1} - f_2}$$

$$M_2 = -\frac{f_2}{d - \dfrac{s_{o1} f_1}{s_{o1} - f_1} - f_2}$$

$$M = M_1 M_2 = \left(-\frac{f_1}{s_{o1} - f_1}\right)\left(-\frac{f_2}{d - \dfrac{s_{o1} f_1}{s_{o1} - f_1} - f_2}\right)$$

$$M = M_1 M_2 = \frac{f_1 f_2}{(s_{o1} - f_1)(d - f_2) - s_{o1} f_1}$$

$$M = M_1 M_2 = \frac{f_1 f_2}{s_{o1} d - s_{o1} f_2 - f_1 d + f_1 f_2 - s_{o1} f_1}$$

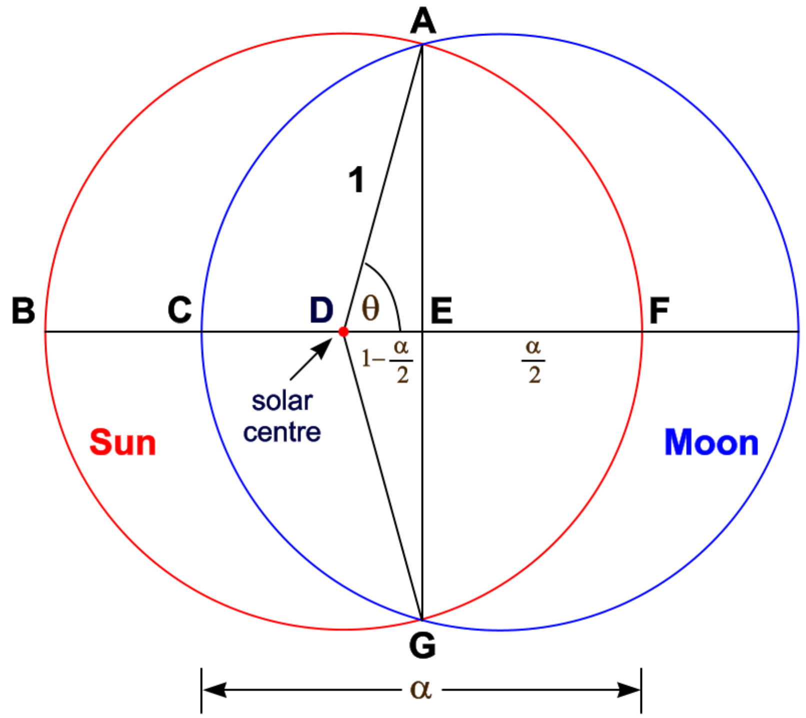
$$M = M_1 M_2 = \frac{f_1 f_2}{s_{o1}(d - f_1 - f_2) - f_1 d + f_1 f_2}$$

Check with case done in text.

$s_{o1} = 5.2\text{ mm} \qquad f_1 = 5\text{ mm} \qquad f_2 = 22\text{ mm} \qquad d = 150\text{ mm} \qquad M = -275$

$$M = \frac{5 \cdot 22}{5.2(150 - 5 - 22) - 5 \cdot 150 + 5 \cdot 22} = \frac{110}{-0.4} = -275$$

HW-J2. Solar Eclipse Model.



The Moon is moving across the Sun from right to left. In this ideal scenario, the Moon blocks the Sun perfectly during the eclipse for an instant and the total eclipse will last for only that instant. Find the area ACGFA of the covered Sun. Then, subtract this area from the area $\pi \cdot 1^2$ of the Sun in our units where the radius of the Sun is 1.

Work with angles in radians for this entire problem.

(a) Show that the exposed area of the Sun (the bright part ABGC) is given by

$$A_{\exp osed} = \pi - 2\theta + 2\cos\theta \sin\theta \,.$$

The area ACGFA of the covered Sun is twice the area of the chord section AEGFA. This chord section is equal to the angular pie section ADGFA minus the triangular region ADGEA. The area of the angular pie section ADGFA is

$$A_{ADGFA} = \pi 1^2 \frac{2\theta}{2\pi} = \theta \,,$$

where θ is measured in radians. The area of the triangular section ADGEA is two times the area of the triangle ADEA.

$$A_{ADGEA} = 2 \cdot \frac{1}{2} \cos\theta \sin\theta = \cos\theta \sin\theta$$

As noted above, the chord section AEGFA = ADGFA - ADGEA,

$$A_{AEGFA} = A_{ADGFA} - A_{ADGEA} = \theta - \cos\theta \sin\theta \,.$$

The area of the covered Sun is twice the area given in equation.

$$A_{\text{covered}} = A_{ACGFA} = 2A_{AEGFA} = 2\theta - 2\cos\theta \sin\theta$$

Finally, the area of the exposed Sun ($A_{\exp osed}$) is $\pi \cdot 1^2 - A_{\text{covered}}$,

$$A_{\exp osed} = \pi - 2\theta + 2\cos\theta \sin\theta \,.$$

(b) Then express this area in terms of the parameter α in the figure. When $\alpha = 0$ the Moon begins its journey across the Sun. When $\alpha = 1$ the Moon is completely covering the Sun.

The parameter α is related to the relevant trig functions as follows.

$$\cos\theta = 1 - \frac{\alpha}{2} = \frac{2-\alpha}{2}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \frac{\sqrt{4\alpha - \alpha^2}}{2}$$

With these trig relations,

$$A_{\exp osed} = \pi - 2\cos^{-1}\left[\frac{2-\alpha}{2}\right] + \left[\frac{2-\alpha}{2}\right]\sqrt{4\alpha - \alpha^2} \,.$$

(c) What percent of the Sun is exposed when $\alpha = \dfrac{1}{2}$? Don't Look at the Sun!

For $\alpha = \dfrac{1}{2}$, $A_{\exp osed} = \pi - 2\cos^{-1}\left[\dfrac{2-\alpha}{2}\right] + \left[\dfrac{2-\alpha}{2}\right]\sqrt{4\alpha - \alpha^2}$ becomes

$$A_{\exp osed} = \pi - 2\cos^{-1}\left[\frac{2-0.5}{2}\right] + \left[\frac{2-0.5}{2}\right]\sqrt{4(0.5)-(0.5)^2}$$

$$A_{\exp osed} = \pi - 2\cos^{-1}\left[\frac{3}{4}\right] + \left[\frac{3}{4}\right]\sqrt{\frac{7}{4}} = 3.15926 - 1.4455 + 0.9922 = 2.6986$$

Percent: $A_{\exp osed} = \dfrac{2.688}{\pi} \times 100\% = 85.6\%$