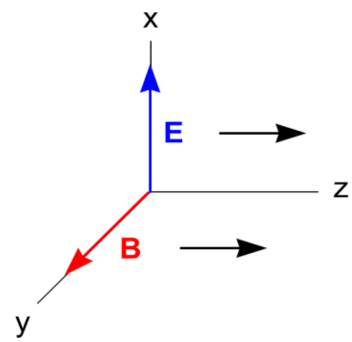
O1. Energy in EM Wave. We return to our EM wave traveling down the z-axis from a previous chapter.

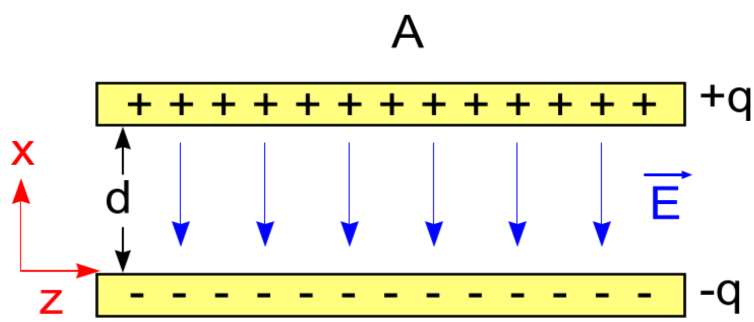


$$\vec{E} = E_o \sin(kz - \omega t)\hat{i} \text{ and } \vec{B} = B_o \sin(kz - \omega t)\hat{j} .$$

$$\frac{E_o}{B_o} = c = \frac{\omega}{k} \qquad c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

What is the energy transported due to the electric field? Well, we could pull out the energy density formula from your introductory physics course, but that would not be in our spirit and motto: “Derive Everything!” So let me give you a derivation tailored to our purposes and I will not even mention the word capacitor.

We look at two plates with charge since this arrangement is simple and we obtain a constant electric field inside the plates, neglecting edge effects. Stick close to the central region of the space between the plates.



We showed earlier that $E = \dfrac{q}{A\varepsilon_o}$ between two parallel plates.

The potential is $V = Ex$ since $-\dfrac{dV}{dx} = -E$ with $F = -q_{\text{test}}E$, i.e. a positive test charge placed between the plates will move downward. The potential for $x = d$ is

$$V = Ed .$$

It’s like gravity as the force downward is a constant between the plates.

The work needed to move an additional charge dq from the negative plate to the positive plate using an attached battery is

$$dW = V(q)dq .$$

The total energy to build up a total charge Q using the battery is

$$W = \int_0^Q V(q)dq .$$

Now it is time to plug in stuff and work out the integral. What do we have?

$$E = \frac{q}{A\varepsilon_o} \qquad \sigma = \frac{q}{A} \qquad V = Ed$$

Start with V and find $V = V(q)$ in order to do the integral.

$$V = Ed = \frac{q}{A\varepsilon_o} d = V(q)$$

Now we are ready for the integral.

$$W = \int_0^Q V(q)dq = \int_0^Q \frac{qd}{A\varepsilon_o} dq = \frac{d}{A\varepsilon_o} \int_0^Q qdq$$

$$W = \frac{d}{A\varepsilon_o} \frac{q^2}{2}\bigg|_0^Q = \frac{d}{A\varepsilon_o} \frac{Q^2}{2}$$

The next step is to get everything in terms of the electric field corresponding to the full charge.

Since $E = \dfrac{Q}{A\varepsilon_o}$ we have $Q = A\varepsilon_o E$.

Then $W = \dfrac{d}{A\varepsilon_o} \dfrac{Q^2}{2} = \dfrac{d}{A\varepsilon_o} \dfrac{1}{2} (A\varepsilon_o E)^2$.

$$W = \frac{d}{A\varepsilon_o}\frac{1}{2}A^2\varepsilon_o^2 E^2$$

$$W = d\frac{1}{2}A\varepsilon_o E^2$$

$$W = \frac{1}{2}Ad\varepsilon_o E^2$$

To adapt this physics to our wave we need the energy per unit volume.

The volume of the space between the plates is Ad .

The energy density for the EM wave due to the electric field is then

$$u_E = \frac{1}{2}\varepsilon_o E_o^2 .$$

For a substance between the plates like glass we have in general

$$u_E = \frac{1}{2}\varepsilon E_o^2 .$$

What about the magnetic field?

Well, I can pull out a formula from introductory physics dealing with inductors. Yikes!

I am going to do it instead by intuition.

I expect a $\frac{1}{2}$, μ , and B^2 .

What are the dimensions of these things?

A lesson in **Dimensional Analysis**!

The dimensions for $u_E = \frac{1}{2}\varepsilon_o E^2$ are energy per volume.

Energy is joules and best remembered from work as newton times meter. Then

$$[u_E] = \frac{J}{m^3} = \frac{N \cdot m}{m^3} = \frac{N}{m^2}.$$

We need to use μ, and B^2 to match it and then throw in the $\frac{1}{2}$.

W can obtain $[B]$ from $\vec{F} = q\vec{v} \times \vec{B}$:

$$[B] = \frac{N}{C \cdot m / s} = \frac{N \cdot s}{C \cdot m} \quad \text{and} \quad [B^2] = \frac{N^2 \cdot s^2}{C^2 \cdot m^2} = \frac{N}{m^2}\frac{Ns^2}{C^2}$$

We need to divide out that $\frac{Ns^2}{C^2}$ part to be left with $\frac{N}{m^2}$.

We proceed next to find the dimensions of μ_o.

We can find our answer from the current in a wire: $\oint \vec{B} \cdot \vec{dl} = \mu_0 i$, i.e., $B(2\pi r) = \mu_0 i$.

Using $[B] = \frac{N \cdot s}{C \cdot m}$ from above, $[\mu_0] = \frac{[B][r]}{[i]} = \frac{N \cdot s}{C \cdot m}\frac{[r]}{[i]}$.

Then $[\mu_0] = \frac{N \cdot s}{C \cdot m}\frac{m}{C / s} = \frac{N \cdot s^2}{C^2}$.

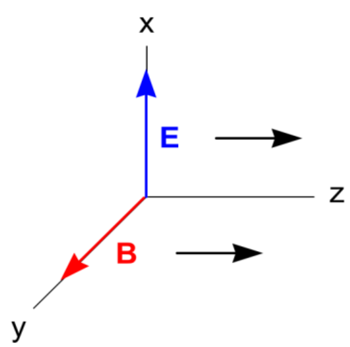
Comparing with $[B^2] = \frac{N}{m^2}\frac{Ns^2}{C^2}$, we want $\left[\frac{B^2}{\mu_o}\right]$.

Therefore, $u_B = \frac{1}{2}\frac{B^2}{\mu_o}$, and for the total energy density

$$u = u_E + u_B = \frac{1}{2}\varepsilon_o E_o^2 + \frac{1}{2}\frac{B_o^2}{\mu_o}$$ in vacuum.

In general $u = \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\frac{B^2}{\mu}$.

Let's go back to EM waves in the vacuum.



$$u = \frac{1}{2}\varepsilon_o E_o^2 + \frac{1}{2}\frac{B_o^2}{\mu_o}$$

$\vec{E} = E_o \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_o \sin(kz - \omega t)\hat{j}$.

$$\frac{E_o}{B_o} = c = \frac{\omega}{k} \qquad c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Substitute $E_o = cB_o$ in $u_E = \frac{1}{2}\varepsilon_o E_o^2$ and you get

$$u_E = \frac{1}{2}\varepsilon_o E_o^2 = \frac{1}{2}\varepsilon_o (cB_o)^2 = \frac{1}{2}\varepsilon_o c^2 B_o^2 = \frac{1}{2}\varepsilon_o \frac{1}{\varepsilon_o \mu_o} B_o^2 = \frac{1}{2}\frac{B_o^2}{\mu_o} = u_B \, .$$

The energy density due to the eclectic field is the same as that for the magnetic field.

Summary: $u_E = \frac{1}{2}\varepsilon E_o^2 \qquad u_B = \frac{1}{2}\frac{B_o^2}{\mu_o} \qquad u_E = u_B \qquad u = u_E + u_B$

Therefore, the total energy density for an EM wave is $u = \varepsilon_o E_o^2 = \frac{B_o^2}{\mu_o}$.

$$[u] = \frac{\text{Energy}}{\text{Volume}} = \frac{\text{J}}{\text{m}^3}$$

The energy travels at the speed o light. Therefore energy per time per area is

$$[uc] = \frac{\text{J}}{\text{m}^3}\frac{\text{m}}{\text{s}} = \frac{\text{J}}{s}\frac{1}{\text{m}^2} = \frac{\text{Watts}}{\text{m}^2} = \frac{\text{W}}{\text{m}^2}$$, the intensity.

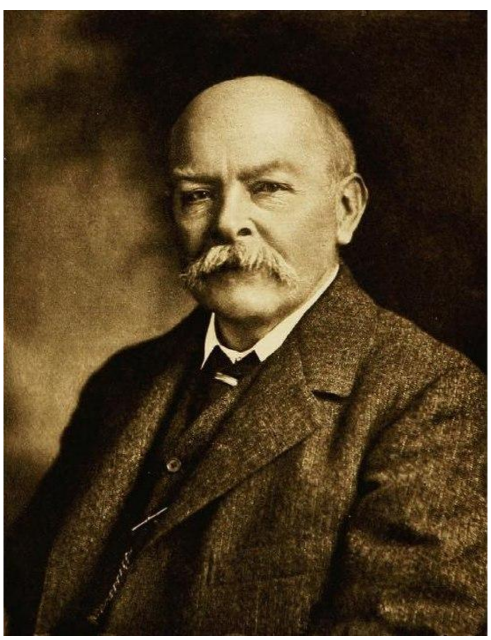
Note $\vec{E} \times \vec{B} \sim \hat{k}$, the propagation direction and $\left|\vec{E} \times \vec{B}\right| = E_o B_o$.

For the intensity we need $uc = \varepsilon_o E_o^2 c = \dfrac{B_o^2}{\mu_o} c$.

But since $E_o = cB_o$, we can write uc where we have one E_o and one B_o .

$$uc = \varepsilon_o E_o^2 c = \varepsilon_o E_o E_o c = \varepsilon_o E_o (cB_o) c = \varepsilon_o c^2 E_o B_o = \varepsilon_o \frac{1}{\varepsilon_o \mu_o} E_o B_o$$

$$uc = \frac{E_o B_o}{\mu_o}$$



Since $\vec{E} \times \vec{B} \sim \hat{k}$ gives the direction of the EM wave, it is natural to define

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$, which is great since then $\left|\vec{S}\right| = uc$.

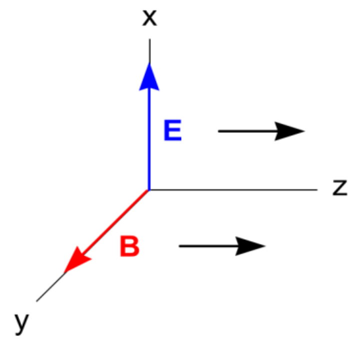
The vector $\vec{S} = \dfrac{1}{\mu_o} \vec{E} \times \vec{B}$ is called the Poynting vector after John Henry Poynting.

John Poynting (1852 – 1914)
English Physicist who Derived the Poynting Vector

POINTING VECTOR also – points in the travel direction.

O2. Irradiance.

$\vec{E} = E_o \sin(kz - \omega t)\hat{i}$ and $\vec{B} = B_o \sin(kz - \omega t)\hat{j}$.



$$\frac{E_o}{B_o} = c \qquad c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$u = \frac{1}{2}\varepsilon_o E_o^2 + \frac{1}{2}\frac{B_o^2}{\mu_o} \qquad \vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B}$$

Let's work out $\vec{S}$.

$$\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B} = \frac{1}{\mu_o}E_o \sin(kz - \omega t)\hat{i} \times B_o \sin(kz - \omega t)\hat{j}$$

$$\vec{S} = \frac{1}{\mu_o}E_o B_o \sin^2(kz - \omega t)\hat{i} \times \hat{j} = \frac{1}{\mu_o}E_o B_o \sin^2(kz - \omega t)\hat{k}$$

$$\vec{S} = \frac{1}{\mu_o}E_o B_o \sin^2(kz - \omega t)\hat{k}$$

Remember that this vector gives Watts per square meter.

Let's fix ourselves at z = 0 and note $[\sin(-\omega t)]^2 = [\sin(\omega t)]^2$.

Then $\vec{S} = \frac{1}{\mu_o}E_o B_o \sin^2(\omega t)\hat{k}$.

This vector rapidly varies in time. Recall that the frequency for visible light is 10^{15} Hz .

Measuring instruments will register the average intensity of these rapid fluctuations.

The average of the sinusoidal part over a cycle is

$$\frac{1}{T}\int_0^T \sin^2(\omega t)\,dt \equiv \langle \sin^2(\omega t) \rangle .$$

Let $\omega t = \theta$. Then $dt = \dfrac{d\theta}{\omega}$ and as $0 \to t \to T$, $0 \to \theta \to \omega T = 2\pi$.

$$< \sin^2(\omega t) >= \frac{1}{T}\int_0^T \sin^2(\omega t)\, dt = \frac{1}{T}\int_0^{2\pi} \sin^2(\theta) \frac{d\theta}{\omega}$$

$$< \sin^2(\omega t) >= \frac{1}{2\pi}\int_0^{2\pi} \sin^2(\theta)\, d\theta \; .$$

Since we are integrating over a full period, we have a shortcut taught to me by my advisor, Dr. Richard Houston (1935 - 2011), when I was an undergraduate. Dr. Houston taught at St. Joseph's College (which became St. Joseph's University in 1978) for 50 years!

Over a full period: $\int_0^{2\pi} \sin^2(\theta)\, d\theta = \int_0^{2\pi} \cos^2(\theta)\, d\theta$.

Then $\int_0^{2\pi} \sin^2(\theta)\, d\theta = \dfrac{1}{2}\left[\int_0^{2\pi} \cos^2(\theta)\, d\theta + \int_0^{2\pi} \sin^2(\theta)\, d\theta \right]$

$$\int_0^{2\pi} \sin^2(\theta)\, d\theta = \frac{1}{2}\left[\int_0^{2\pi} \cos^2(\theta) + \sin^2(\theta) \right] d\theta$$

$$\int_0^{2\pi} \sin^2(\theta)\, d\theta = \frac{1}{2}\int_0^{2\pi} 1\, d\theta = \frac{1}{2}\theta\Big|_0^{2\pi} = \frac{1}{2}(2\pi - 0) = \pi$$

Then $< \sin^2(\omega t) >= \dfrac{1}{2\pi}\int_0^{2\pi} \sin^2(\theta)\, d\theta = \dfrac{1}{2\pi}\pi = \dfrac{1}{2}$.

In summary, from $\vec{S} = \dfrac{1}{\mu_o} E_o B_o \sin^2(\omega t)\hat{k}$, the irradiance is

$$I = \frac{E_o B_o}{2\mu_o} = \frac{E_o E_o}{2c\mu_o} = \frac{E_o^2}{2}\frac{\sqrt{\varepsilon_o \mu_o}}{\mu_o} = \frac{E_o^2}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}} \; .$$

O3. Total Internal Reflection Surprise. Remember total internal reflection from earlier? Watch the Fresnel equations predict such as phenomena. I call this one another Fresnel surprise. The Maxwell equations are giving us everything about light.

We pull out the Fresnel equations and adapt them for going from glass or water to air. Let's start with the reflection coefficients for the p and s polarizations.

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \qquad r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

Going from glass or water to air, $n_1 = n$ and $n_2 = 1$, where the n stands for glass or water.

$$r_p = \frac{\cos\theta_1 - n \cos\theta_2}{n \cos\theta_2 + \cos\theta_1} \qquad r_s = \frac{n \cos\theta_1 - \cos\theta_2}{n \cos\theta_1 + \cos\theta_2}$$

Snell's law gives $n \sin\theta_1 = \sin\theta_2$.

We would like to get rid of $\cos\theta_2$ so we have everything in terms of $\cos\theta_1$.

Therefore, $\cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - n^2 \sin^2\theta_1}$.

All is well until $1 - n^2 \sin^2\theta_1$ goes negative. This situation will begin when the angle of incidence θ_1 gets large enough to reach a critical angle θ_c such that $1 - n^2 \sin^2\theta_c = 0$

$$1 - n^2 \sin^2\theta_c = 0 \quad \Rightarrow \quad 1 = n^2 \sin^2\theta_c \quad \Rightarrow \quad \sin^2\theta_c = \frac{1}{n^2}$$

$$\Rightarrow \quad \sin\theta_c = \frac{1}{n} \quad \Rightarrow \quad \theta_c = \sin^{-1}\frac{1}{n}.$$

For glass $\theta_c = \sin^{-1}\frac{1}{n} = \sin^{-1}\frac{1}{1.5} = 42^\circ$ and for water $\sin^{-1}\frac{1}{1.33} = 49^\circ$.

For $\theta_1 > \theta_c$ the square root factor goes imaginary:

$$\cos\theta_2 = \sqrt{1-n^2\sin^2\theta_1} = i\sqrt{n^2\sin^2\theta_1 - 1}.$$

When you take one of our reflection coefficients, say the s polarization one, you get

$$r_s = \frac{n\cos\theta_1 - \cos\theta_2}{n\cos\theta_1 + \cos\theta_2} = \frac{n\cos\theta_1 - i\sqrt{n^2\sin^2\theta_1 - 1}}{n\cos\theta_1 + i\sqrt{n^2\sin^2\theta_1 - 1}}.$$

Yikes!

What about R? If you square r, it will still have an imaginary part.

Don't square it and don't panic.

Remember what we did with phasors?

The mathematics leads us to do the same here: $R = rr*$, a more general version of r^2.

But at this point this math leads us to an astonishing result.

Consider a complex number of the form like we have,

$$z = \frac{a-ib}{a+ib}.$$

Then,

$$zz* = \frac{a-ib}{a+ib}\frac{a+ib}{a-ib} = 1.$$

What?

$$R_s = 1$$

Literally, total internal reflection. All the light reflects and $T_s = 1 - R_s = 0$.

For the other polarization, $r_p = \dfrac{\cos\theta_1 - n\cos\theta_2}{n\cos\theta_2 + \cos\theta_1}$ we have

$$r_p = \frac{\cos\theta_1 - ni\sqrt{n^2\sin^2\theta_1 - 1}}{ni\sqrt{n^2\sin^2\theta_1 - 1} + \cos\theta_1}$$

, which is also of the form

$$z = \frac{a - ib}{a + ib} .$$

Then again

$$zz^* = \frac{a - ib}{a + ib}\,\frac{a + ib}{a - ib} = 1 ,$$

with the same result $R_p = 1$ and $T_p = 1 - R_p = 0$.

O4. The Brewster Angle. Another Surprise!

We adapt the reflection coefficients for going from air to glass or water, etc. Then in

$$r_p = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_1\cos\theta_2 + n_2\cos\theta_1} \qquad r_s = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2}$$

we want to set $n_1 = 1$ and $n_2 = n$.

$$r_p = \frac{n\cos\theta_1 - \cos\theta_2}{\cos\theta_2 + n\cos\theta_1} \qquad r_s = \frac{\cos\theta_1 - n\cos\theta_2}{\cos\theta_1 + n\cos\theta_2}$$

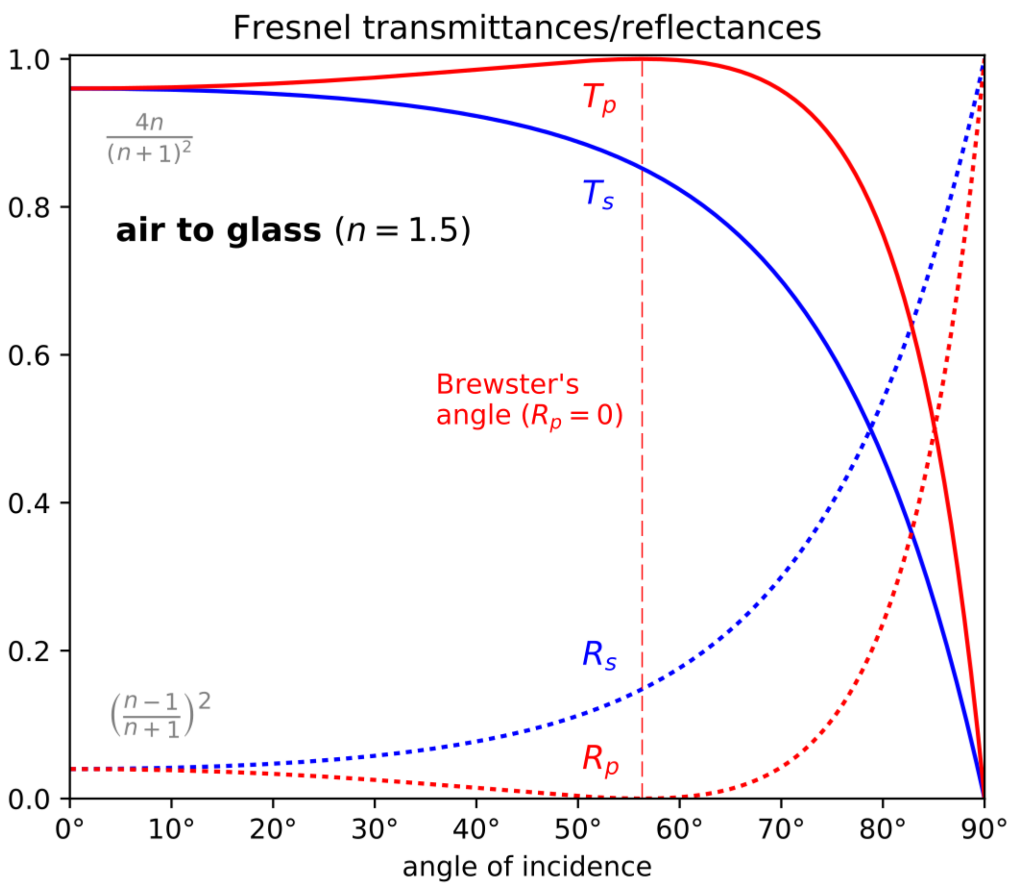
Going from a fast medium like air to glass or water, the light bends towards the normal.

Therefore, $\theta_2 < \theta_1$ provided you are not heading into the glass at normal incidence.

This inequality leads to $\sin\theta_2 < \sin\theta_1$ and $\cos\theta_2 > \cos\theta_1$.

Therefore, the numerator in r_p can go zero. Why not r_s ?

The result is that the p polarization gets zapped when this condition occurs. See where R_p dips to 0 in the figure. It looks like about 56°.



Wikipedia: Sbergjohansen. [Creative Commons License]

The condition is met when $n\cos\theta_1 - \cos\theta_2 = 0$, i.e., $\cos\theta_2 = n\cos\theta_1$

Bringing in Snell's law $\sin\theta_1 = n\sin\theta_2$.

Therefore $n = \dfrac{\sin\theta_1}{\sin\theta_2}$. Substituting into $\cos\theta_2 = n\cos\theta_1$. Leads to

$$\cos\theta_2 = \frac{\sin\theta_1}{\sin\theta_2}\cos\theta_1 ,$$

and the interesting combination $\sin\theta_1 \cos\theta_1 = \cos\theta_2 \sin\theta_2$.

1. Didn't we derive an identity involving $\sin\theta\cos\theta$?

2. Do you remember seeing this factor in the range for kicking a football in intro physics?

3. Oh heck. Just figure it out. Derive it!

$$\sin\theta\cos\theta = \frac{1}{2i}\left[e^{i\theta} - e^{i\theta}\right]\frac{1}{2}\left[e^{i\theta} + e^{i\theta}\right] = \frac{1}{4i}(e^{2i\theta} - e^{-2i\theta}) = \frac{1}{2}\sin(2\theta) .$$

You may forget trig identities but never forget Euler!

The equation $\sin\theta_1 \cos\theta_1 = \cos\theta_2 \sin\theta_2$ becomes $\sin 2\theta_1 = \sin 2\theta_2$.

Now we know from Snell's law that for an oblique angle that we can't have $\theta_1 = \theta_2$.

However, supplementary angles $2\theta_1 = \pi - 2\theta_2$ also work for $\sin 2\theta_1 = \sin 2\theta_2$.

Interestingly, $2\theta_1 = \pi - 2\theta_2$ means complementary angles for θ_1 and θ_2 since

equation $2\theta_1 = \pi - 2\theta_2$ brings us to $\theta_2 = \frac{\pi}{2} - \theta_1$.

Since we want to find the incident angle θ_1, Snell's law $\sin\theta_1 = n\sin\theta_2$ leads to

$$\sin\theta_1 = n\sin(\frac{\pi}{2} - \theta_1)$$
$$\sin\theta_1 = n\cos(\theta_1)$$

$$\tan\theta_1 = n .$$

Do you remember from intro physics where the coefficient of static friction of a quarter on a book can be measured by slanting the book until the quarter starts sliding at an angle θ ?
Then, $\tan\theta = \mu$.

This special angle where the p polarization gets zapped is called the Brewster angle θ_B :

$$\tan\theta_B = n \quad \text{and} \quad \theta_B = \tan^{-1} n\ .$$

For glass, $\theta_B = \tan^{-1} 1.5 = 56°$ and for water $\theta_B = \tan^{-1} 1.33 = 53°$.

If you look back to $r_p = \dfrac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$ in general where $n_1 < n_2$,

$$n_2 \cos\theta_1 - n_1 \cos\theta_2 = 0 \text{ gives } \cos\theta_2 = \frac{n_2}{n_1}\cos\theta_1 ,$$

and repeating the same steps leads to

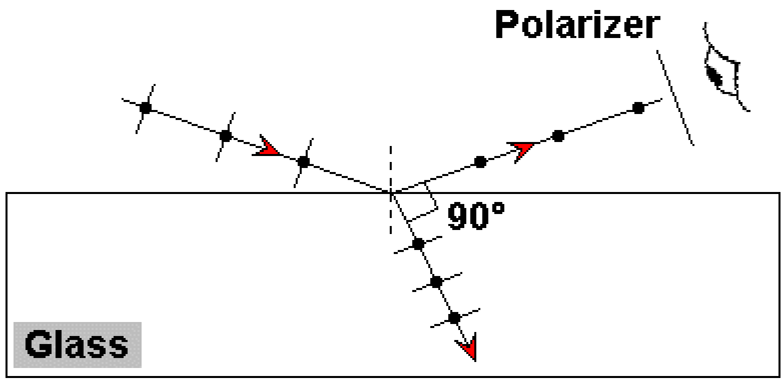
$$\tan\theta_B = \frac{n_2}{n_1} \quad \text{and} \quad \theta_B = \tan^{-1}\frac{n_2}{n_1}\ .$$

Here's how you can explain the general idea without math to general students.

Sir David Brewster (1781-1868) of the famous angle, the ***Brewster's angle*** or the ***Brewster angle*****.** Brewster derived a law that reflected light becomes linearly polarized at a special angle, the Brewster angle. The diagram below illustrates why. We designate the two basic polarizations of the incoming light with a little line (p polarization) and dot (s polarization). The line represents vibrations of the electric field in the plane of your computer monitor, while the dot indicates a vibrating electric field moving out of your computer monitor and then behind your monitor, and so on.

Actually, the incident light has mixed polarization with all possible slants as long as all of these slanted lines are perpendicular to the direction of propagation, preserving light's transverse character. It helps to think of two mutually perpendicular polarization states since any slanted mode can be expressed as a mixture of these two basic states to varying amounts. For example, on a map you can focus on north-south and east-west. Any slanted direction can be expressed as a combination, e.g., moving at a 45° slant is moving north and east (NE) at the same time, then south and west (SW).

The Brewster Angle and Polarized Reflection



When the refracted beam and reflected beam are 90° apart from each other, the little lines can no longer travel along the reflected beam since this would give the reflection a vibrating contribution parallel to the direction of propagation, something forbidden by the Maxwell equations. Light is a transverse wave, not a longitudinal wave. Therefore, this component vanishes and the reflected light is polarized as described by the dots. If the observer uses a polarizer to view the beam, the beam can be made to vanish completely if the polarizer axis is aligned properly. The s polarization is untouched, as we mentioned earlier, it is the p polarization equation in the Fresnel equation that gets effected by a possibly vanishing numerator.

Back to Physics Majors.

We can use the above figure to derive the Brewster angle in a couple of lines.

And remember, it is always good to derive things in more than one way.

Why can we immediately write $\theta_B + 90° + \theta_2 = 180°$?

$$\theta_2 = 90° - \theta_B$$

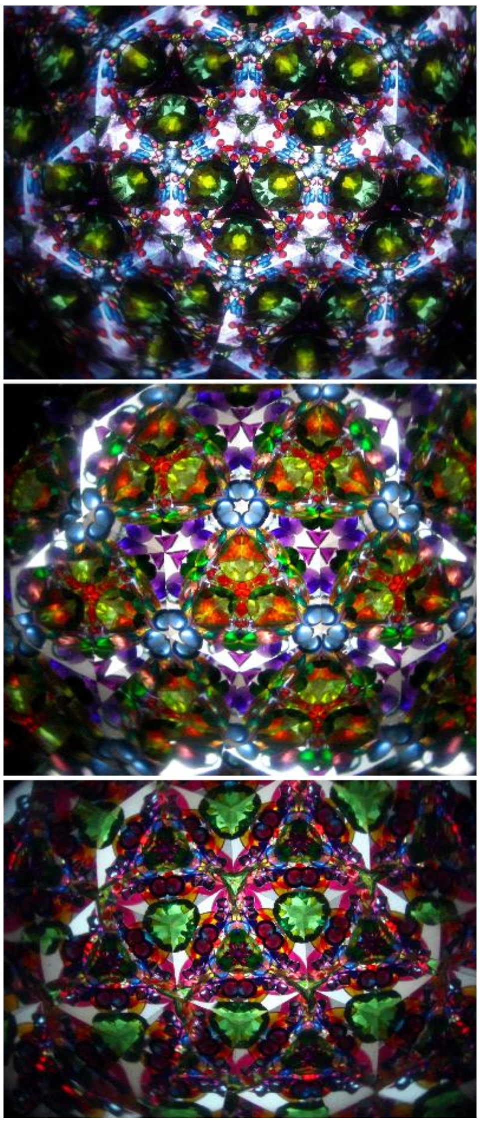
$$\sin\theta_B = n\sin\theta_2$$

$$\sin\theta_B = n\sin(90° - \theta_B) = n\cos\theta_B$$

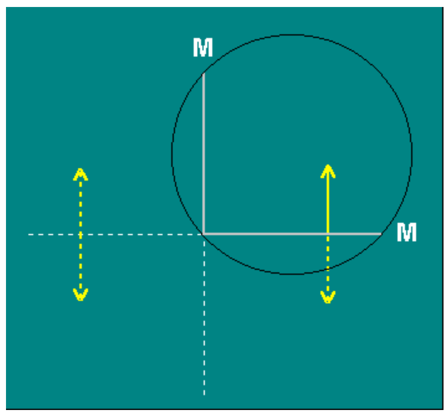
$$\tan\theta_B = n$$

O5. Brewster and the Kaleidoscope. Brewster's work in optics led him to invent a kaleidoscope possibly as early as 1814, which principle was known historically before him. He patented his kaleidoscope in 1817. But the patent didn't stop manufacturers from ripping off his idea. Word had gotten out about the design before the patent. He should have kept super quiet about the invention until the patent was official. The result was that Brewster did not become rich when hundreds of thousands started selling.

He called the invention of joining two mirrors at an angle with colorful bits of material to be reflected in multiple ways **kaleidoscope** from the Greek *kalos* (beautiful), *eidos* (form), and English scope (observing instrument). You might translate kaleidoscope as an instrument to observe beautiful forms.



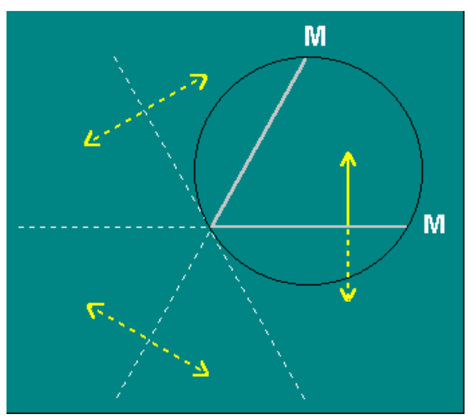
Kaleidoscope Images.
Wikimedia: αvცოթ€ɲ. Creative Commons License



Mirrors at 90° produce three images. If you count the object, you have 4 things.

Note that 360°/90° = 4.

Note the double reflection for the lower image at the far left.



Mirrors at 60° produce five images. If you count the object, you have 6 things.

Note that 360°/60° = 6.

Can you find the image formed by three reflections?

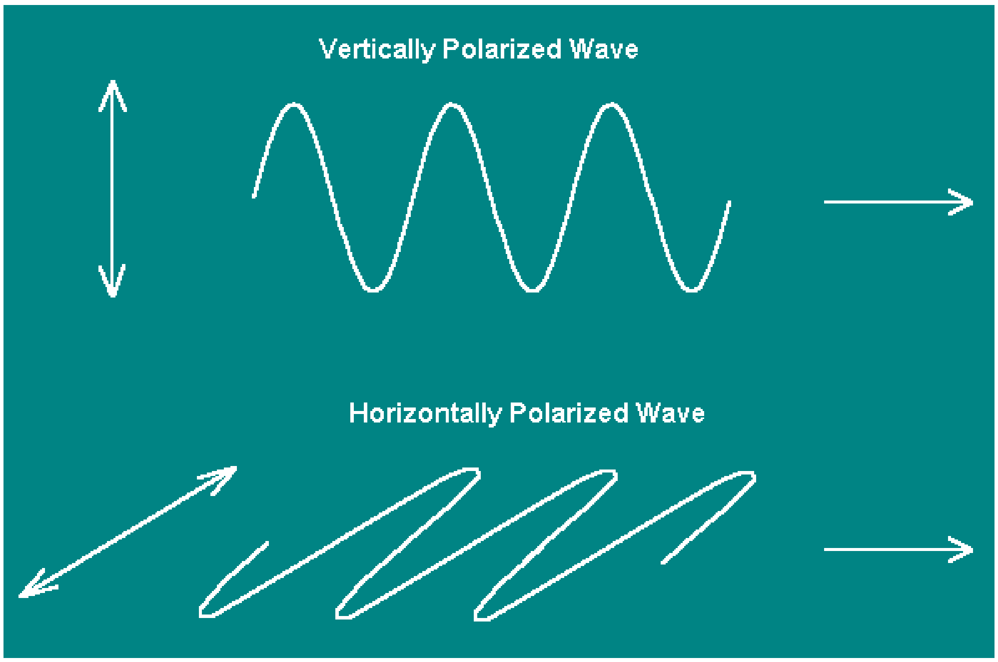
The formula for the number of things seen when the angles are chosen to get symmetric overlapping images is

$$n = \frac{360^\circ}{\theta}.$$

O6. The Polarizer and Edwin Land. Let's delve into polarization more. Light is transverse and sound is longitudinal wave. Light does not need a medium to travel in. Note than a longitudinal wave does not have polarization. Look at the two ***transverse waves*** below. If these represent rope waves, you produce the upper one by shaking your hand up and down, i.e., parallel to a vertical wall. If they represent the electric field of a light wave, you make them by shaking a charged particle up and. Don't worry about the magnetic field (which is not shown).

To produce the lower one, you shake back and forth parallel to the floor. The ***polarization*** of a transverse wave is determined by the orientation of the crests and troughs. It is important that the wave be transverse because longitudinal waves do not have transverse polarization. In the top case of the figure, the crests go up and troughs go down along an imagined vertical wall. We say that the wave is ***polarized vertically*** and we have ***vertical polarization***. For the lower wave, the crests and troughs are parallel to the floor. In this latter case, the wave is ***polarized horizontally*** or has ***horizontal polarization***.

Polarization



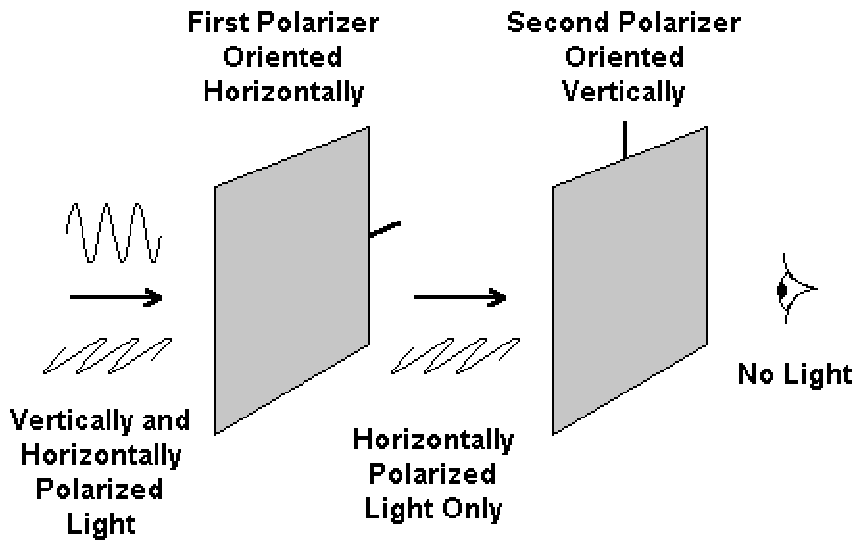
A way to quickly check the polarization is to hold your hand flat and place it against the wave so that the crest and troughs brush against your hand. Your hand needs to be oriented vertically like you are going to place your hand against a wall for the upper wave. For the lower case, your hand needs to be oriented horizontally as if you are going to place your hand on the floor. You can shake at slanted angles and thereby obtain many different types of polarization. It is customary to designate vertical polarization as 0° and horizontal polarization as 90°. Polarizations with angles between 0° and 90° have slanted polarization.

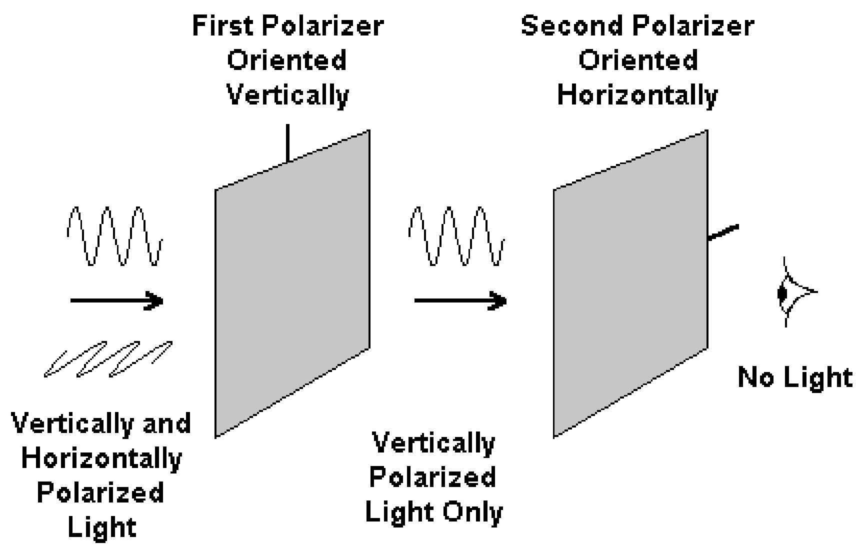
Next we introduce Land, a great inventor. He was a hero to many, including the late Steve Jobs. Edwin Land, Steve Jobs, and Bill Gates all dropped out of college to start up companies.

Edwin Land (1906 – 1991)
Photo origin. Using a thumb size as fair educational use.

At the dawn of a new century in 1900 it was known that certain crystals polarize light. Normally, light has mixed polarization as charges shake randomly in sources like light bulbs or the sun.

In 1926, a 17-year-old freshman physics major at Harvard came up with the idea that perhaps aligned tiny crystals embedded in plastic might make it possible to manufacture inexpensive sheets that would make light polarized after the light passed through it. The young physics major left Harvard to pursue his idea and became one of the famous American inventors of the 20th century.





Land called his inexpensive polarizer sheet a ***polaroid***. Afterwards he founded a company by the same name. Land would give out free samples of his invention to encourage people to come up with applications. He found the most popular application in what became known as ***polaroid sunglasses*** - to reduce glare. Allowing only the components of light vibrating along the axis of the polaroid, the effect is that 50% of the light is transmitted and this 50% is polarized.

Reducing glare is related to Brewster since on reflection light can become polarized.

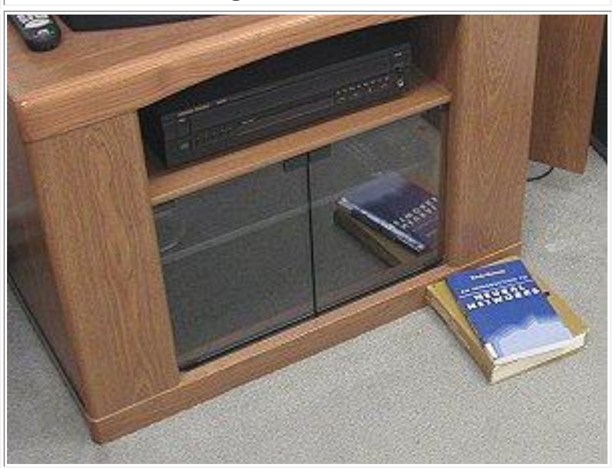
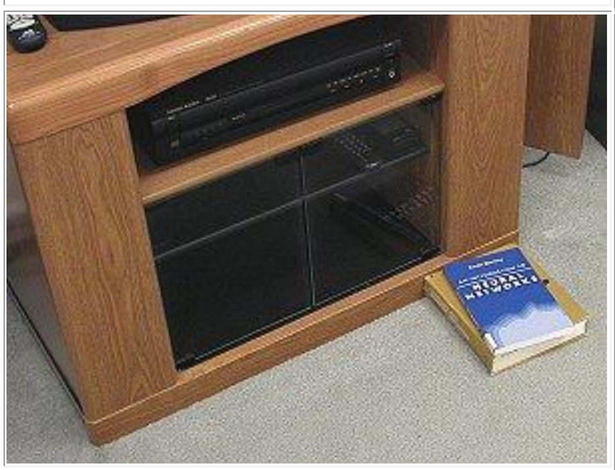
Photos Near the Brewster Angle (What's That "Poison" on the Floor?)

Polarizer Aligned to See Reflection	**Polarizer Turned 90°**
	

Photos by Doc at His Home, March 5, 2002

See the reflection from the glass doors vanish in the right photo below.

Polarization by Reflection

Polarizer Aligned to See Reflection	**Polarizer Turned 90°**
	

Photos by Doc at His Home, March 4, 2002

Below is Prof. Tracy Brown from UNCA's Department of Psychology at a desk with a reflecting glass top. I was on the tenure committee with him at the time. I had finished reading the student evaluations of the candidates for tenure and came by while he was reading them. UNCA takes teaching seriously. The tenure committee does read the student evals.

Polarization by Reflection

Polarizer Aligned to See Reflection	**Polarizer Turned 90°**
	

Photos by Doc at Phillips Hall, UNCA, March 5, 2002

The next photo shows the polarization of light due to reflections from a piano..

Piano and Polarized Reflected Light

Polarizer Aligned to See Reflected Light	Polarizer Turned 90°
	

Photos by Doc of His Steinway B, March 5, 2002

Do you ever notice especially dark reflections from floors, tables, or lakes when you wear polaroid sunglasses? If so, that is because the reflected light is polarized and has much trouble getting through your sunglasses unless the polaroid glasses happen to be aligned for transmission. The Brewster angle for the air-glass interface is 56°, while for air-water it is 53°. Remember that the Brewster angle is the angle of incidence that gives a completely polarized reflection. Of course, the angle of reflection is equal to the angle of incidence as always.

Edwin H. Land and Daughter Jennifer.
Source Polaroid Corporation.
Thumbnail use due to uncertainty in copyright.
Educational Use.

"The history of the instant photograph began with a legendary question. During a holiday in Santa Fe at Christmas 1943 Jennifer, the daughter of Polaroid founder Edwin H. Land, wanted to know: 'Daddy, why can't I see the picture now?'

"In the following hours Land thought about it and developed the whole concept of Instant Photography. From there it took him three more years to develop one of the biggest inventions: the Instant Photograph. It was finally presented in February 1947." ***Polaroid Corporation***

Edwin H. Land (1909-1991), inventor of the polarizer sheet he called the polaroid, patented over 500 inventions during his lifetime. Land and other inventors like Thomas Edison (1847-1931) and Nicola Tesla (1856-1943), are among the "hall-of-fame" of inventors. The unit for the magnetic field **B** is the tesla, named after the inventor.

Edison's famous statement about invention was that invention is 1% inspiration and 99% perspiration. Land once commented about invention: "You always start with a fantasy. Part of the fantasy technique is to visualize something as perfect. Then with the experiments you work back from the fantasy to reality, hacking away at the components." (Reference Edwin Herbert Land, May 7, 1909 - March 1, 1991 by Victor K. McElheny, National Academy of Sciences).

Land's statement is a good philosophy to apply to any goal in life. The full realization of Land's dream took 30 years: color development at the push of a button, where the photograph pops out and develops right before your eyes in full daylight and without the need to discard any chemicals. The breakthroughs leading up to this sensational camera, called the SX-70 One Step, are given in the table below.

Date	Description
1943	Goal: "Daddy, why can't I see the picture now?" Jennifer Land
1947	Land's Polaroid Black and White Instant Photography (60 sec)
1963	Polaroid Color Instant Photography (60 sec)
1972	Polaroid Color Instant Photography (No Chemical Refuse)

Today the smart phone reigns as the main instrument for photography in the general population.

O6. Malus's Law. When unpolarized light goes through a linear polarizer sheet or filter, only polarization along the aligned axis is allowed. Consider an incoming EM wave with polarization along the x-axis

$\vec{E} = E_x \hat{i}$, traveling down the z-direction with its associated $\vec{B} = B \hat{j}$ field.

It meets up with a polarizer that allows polarization along the slanted axis

$$\hat{n} = \cos\theta \hat{i} + \sin\theta \hat{j} .$$

Only the component projected along the $\hat{n}$ axis can pass.

$$\vec{E} \cdot \hat{n} = E_x \cos\theta .$$

The transmitted wave is

$$\vec{E}_{\mathrm{t}} = E_x \cos\theta \hat{n}$$

The intensity of the transmitted beam, now polarized along the $\hat{n}$ axis is

$$I \sim (E_x \cos\theta)^2 .$$

The maximum transmission will occur when you align the polarizer along the x-axis.

For $\hat{n} = \hat{i}$, $I \sim E_x^2$, and we can write

$$\boxed{I = I_{\max} \cos^2 \theta}$$

This relation is Malus's law.

Notice how we cleverly avoid the Poynting vector by using proportional statements.

O7. Circular Polarization.

Linear polarization along x-axis: $\vec{E} = E_o \cos(kz - \omega t)\hat{i}$

Linear polarization along y-axis: $\vec{E} = E_o \cos(kz - \omega t)\hat{j}$

Linear along slanted-axis: $\vec{E} = (E_{ox}\hat{i} + E_{oy}\hat{j})\cos(kz - \omega t)$

Now let $E_{ox} = E_{oy} \equiv A$ and observe from z = 0.

$$\vec{E} = A(\hat{i} + \hat{j})\cos(-\omega t)$$

Now introduce a phase shift of $\frac{\pi}{2}$ for the y-component.

$$\vec{E} = A[\hat{i}\cos(-\omega t) + \hat{j}\cos(-\omega t + \frac{\pi}{2})]$$

Let $\theta = \omega t$.

$$\vec{E} = A[\hat{i}\cos(-\theta) + \hat{j}\cos(-\theta + \frac{\pi}{2})]$$

$$\vec{E} = A[\hat{i}\cos(-\theta) + \hat{j}\sin(\theta)]$$

$$\vec{E} = A[\hat{i}\cos\theta + \hat{j}\sin\theta]$$

$$\vec{E} = A[\hat{i}\cos(\omega t) + \hat{j}\sin(\omega t)]$$

Counterclockwise circular motion!

Curve fingers of your right hand counterclockwise and your thumb points along z-axis.
Remember that the z-axis is the direction of propagation.

This light is therefore, said to be **right-circularly polarized**.

If your rotate clockwise, you need your left hand to have the thumb point along the z-axis.
That light is therefore, said to be **left-circularly polarized**.

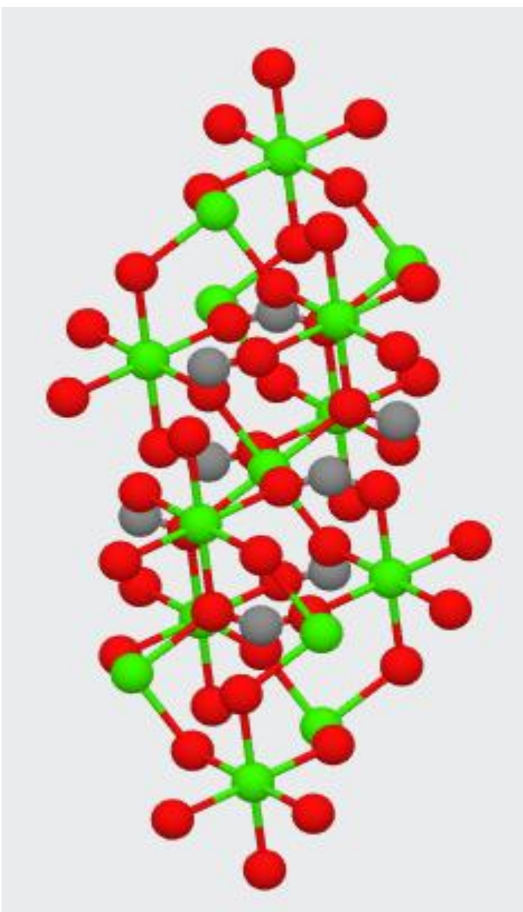
For $E_{ox} \neq E_{oy}$, you get **elliptically-polarized light**.

O8. Birefringence.

Rasmus (Erasmus) Bartholin (1625 - 1698)

The Danish medical professor Erasmus Bartholin (1625-1698), also known as Batholinus, discovered the phenomenon called **double refraction**. At that time, calcite crystals had been recently discovered in Iceland. These became known as ***Icelandic Spar***, which is ***calcite***, the formal chemical name being calcium carbonate. The chemical formula is $CaCO_3$, i.e., its molecule consists of one calcium atom (Ca), one carbon atom (C), and three oxygen atoms (O).

When a beam of light is split into two refractive beams we have ***double refraction***. This bi-refraction is also called ***birefringence***. In studying crystals, the geometrical arrangement of the molecules is also important. The larger configuration of a crystal is based on the shape of the basic unit. For example, common table salt, sodium chloride (NaCl), consists of sodium (Na) and chlorine (Cl) atoms, where the NaCl molecules are packed together in a cubic geometry.



Calcite $CaCO_3$ Crystal Structure
materialsproject.org

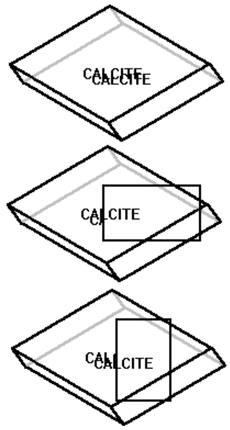
Green, Calcium (Ca) atoms
Gray, Carbon (C) atoms
Red, Oxygen (O) atoms

See the calcite crystal structure at the left. It is not symmetric in all dimensions. When light enters the calcite crystal, there are two speeds for the electromagnetic wave.

Two speeds imply two indexes of refraction since n = c/v, where v is the speed of light in the medium. Thus we get double refraction and the light is polarized for each case!

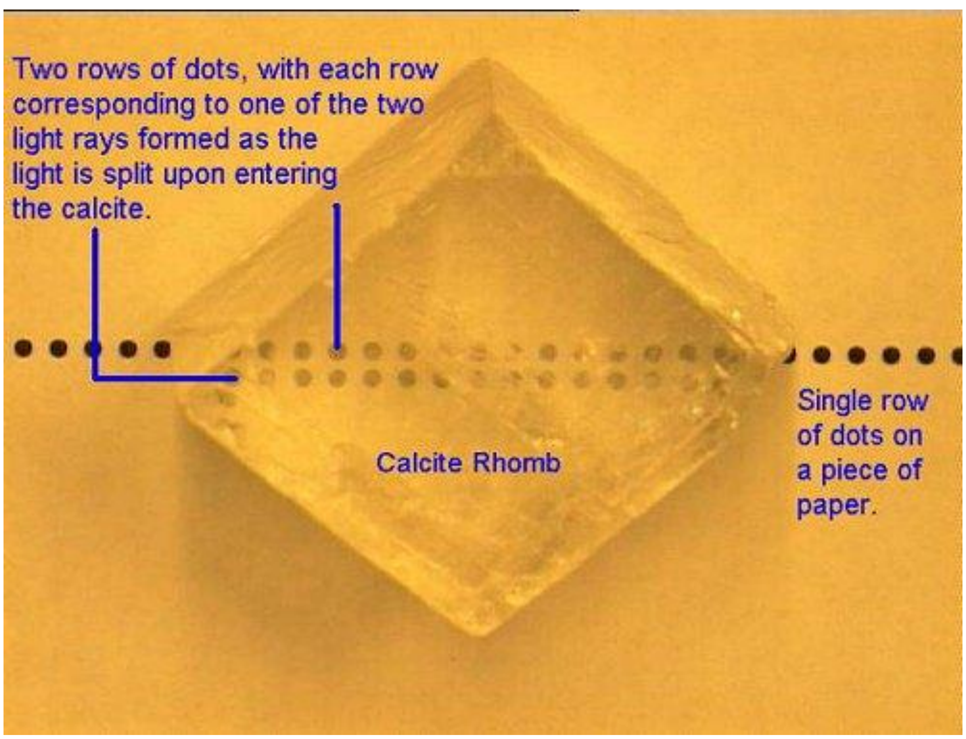
Below is a schematic showing the double image due to double refraction. The lower two images demonstrate that the light from the letters is polarized in each case.

Sketch of Double Images Due to Birefringence and Polarizer



Below is a photo of calcite over a piece of paper with a single row of dots. The "double-vision" effect is due to the birefringence. Bartholin called the ray of light that behaves as expected the ***ordinary ray***. He referred to the surprise ray as the ***extraordinary ray***.

Photo of Double Images Due to Birefringence



Courtesy Department of Earth Sciences, Brock University

If you rotate the above calcite crystal the extraordinary image shifts around the ordinary image. Bartholin, who also discovered polarization along with birefringence, considered the double image wonderful. He was also the father-in-law of a scientist we discussed concerning the speed of light, Roemer.

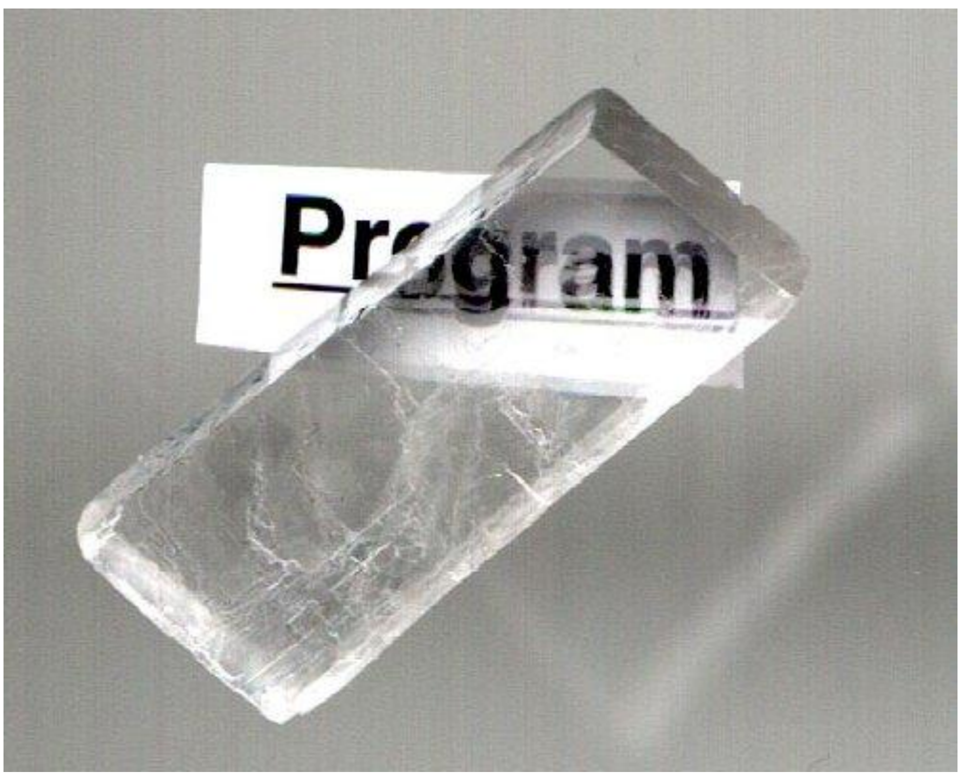
"As my investigation of this crystal proceeded there showed itself a wonderful and extraordinary phenomenon: objects which are looked at through the crystal do." ***Erasmus Bartholin***. Reference: ***Optics (3rd ed.) by Eugene Hecht (Addison-Wesley, Reading, MA, 1998).***

A calcite crystal is placed over a word below. The double image you see depends on the orientation of the crystal over the word and the viewing direction.

We can add Geology to our list of interdisciplinary topics!

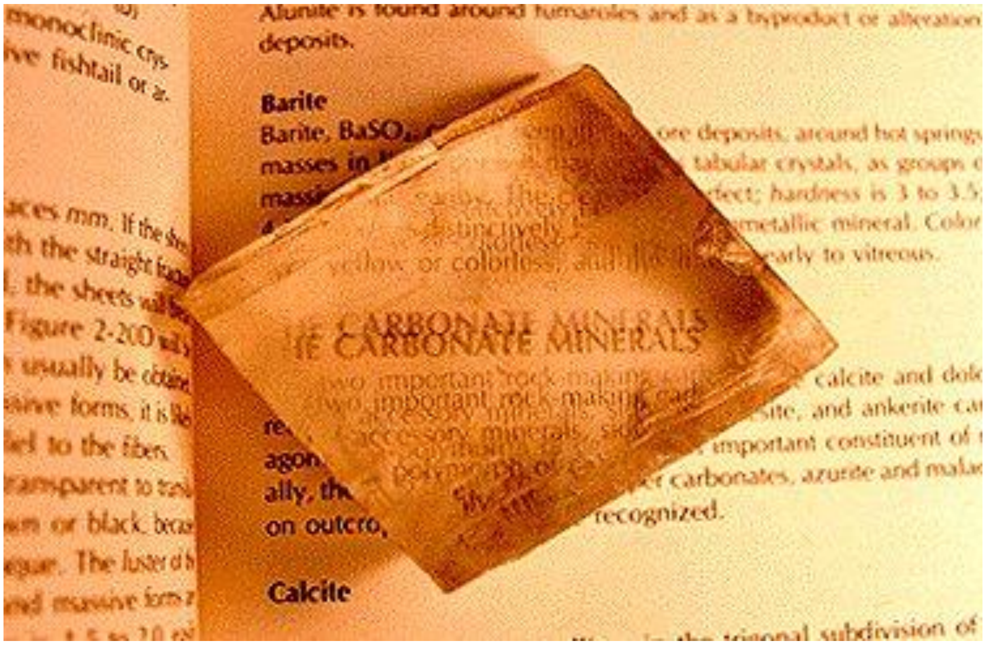
- **GEOL, Geology – birefringence in gems (earlier we discussed diamond)**

Birefringence and the Word Program



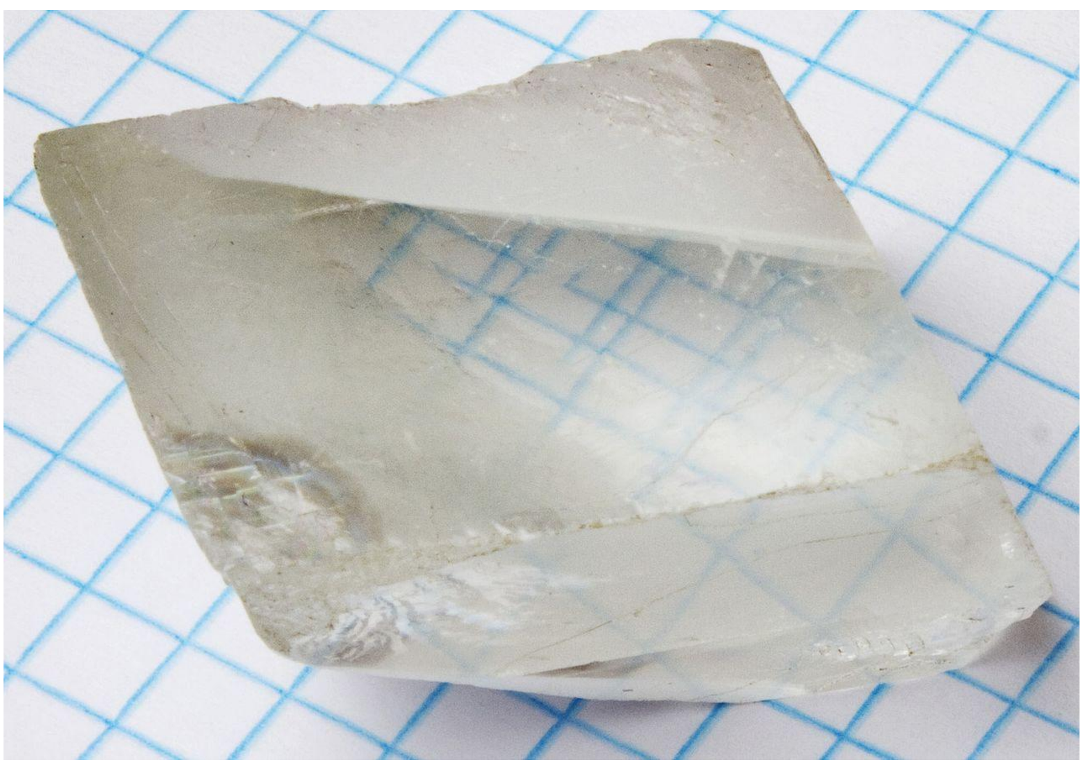
Courtesy Göran Axelsson, Scandinavian Mineral Gallery

Birefringence and Book



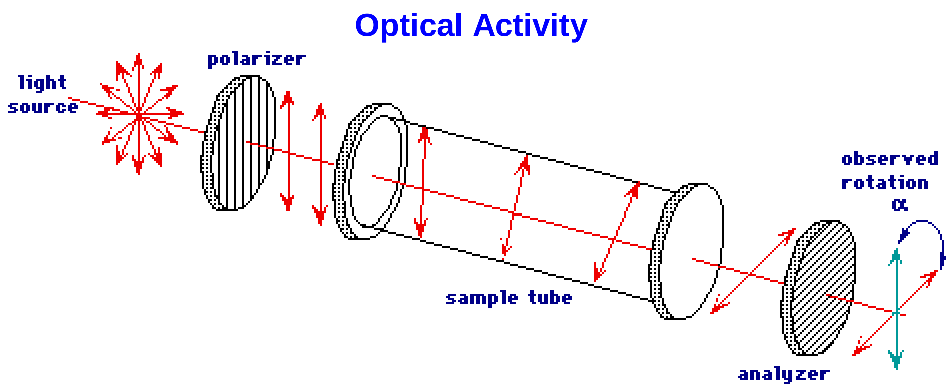
Courtesy 1996 Geology 150 class mineral project, Hartwick College

Birefringence: Calcite over Graph Paper



Wikimedia: APN MJM. [Creative Commons License]

O9. Optical Activity. Many molecular structures rotate linearly-polarized light as it passes through. This phenomenon is called ***optical activity*** and the structures are said to be ***optically active***. Below is a diagram to illustrate optical activity. Light from a source has mixed polarization. This means that the zillions of charges wiggling at the source send out electromagnetic waves where the electric field wiggles in all possible directions perpendicular to the direction of propagation. Remember that light being a transverse wave dictates this. The transverse character of light follows from the Maxwell Equations.



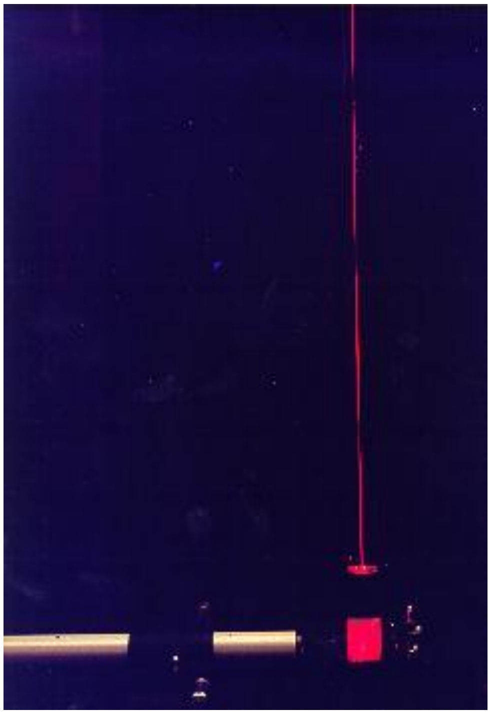
Courtesy Paul R. Young, Department of Chemistry, Univ. of Illinois at Chicago

A polarizer such as a polaroid sheet is placed in the path of the light to produce linearly-polarized light, indicated in the figure as a grid with vertical lines since the polarizer is oriented this way. After passing through, the electric field now wiggles up and down. As the light passes through the optically active material, the polarization begins to rotate. Another polarizer is placed in the path of the light emerging from the optically active region. This polarizer, called the ***analyzer*** is rotated in order to determine the polarization of the final beam. When the analyzer is rotated such that maximum intensity is transmitted, then we have determined the polarization. The angle a indicates the amount of rotation, which in this case is about 90°. Look at the schematic grooves on the analyzer, which match the polarization direction of the final beam of light.

Next is an actual experiment performed at the University of Tennessee, similar to the arrangement described above.

"Sugars such as sucrose, fructose, and maltose are familiar examples of materials which exhibit optical activity. Corn syrup is a mixture of various sugars and has a reasonably large specific rotation which makes it an ideal material to use as a demonstration of optical activity. A HeNe laser was used as the coherent, linearly polarized light source. To ensure maximum linear polarization of the laser beam, a polarizer was placed between the laser and the tube. The beam is angled 90° with a turning prism (this is the bright red square in the photograph) and enters a tube mounted vertically with a ring stand. The tube was simply a 1-m glass tube (open at the top) filled with Karo syrup. As the beam propagates through the corn syrup, the scattered light is observed at right angles with respect to the direction of propagation. As the beam is transmitted up the solution, the plane of polarization is rotated as indicated by the alternating bright and dark pattern of the light." ***Robert Compton, Department of Physics and Astronomy, University of Tennessee, Knoxville, TN.***

Optical Activity in Karo Syrup



Courtesy Robert Compton, Department of Physics and Astronomy, Univ. of Tennessee, Knoxville

Regions of darkness appear since light cannot scatter to your eye if the polarization is along your line of site. Changing electric fields must always be transverse to the direction of propagation. You do not need a second polarizer to observe this effect since the scattering of the light selects the polarization that heads your way. You see changes in intensity as the polarization rotates along the tube.

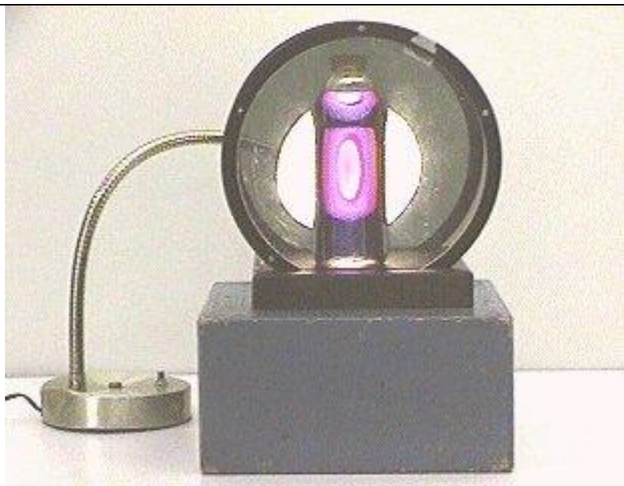
To convince yourself fully how this works, take your left finger and hold it in front of you so you can see the entire length of your finger. If your finger represents a wiggling electric field, this field can move toward your eye. Everything is fine since the finger points transverse to your line of sight. Now turn your finger so it points to your eye. The electric fields now DO NOT radiate toward your eye. However, any observer that can see the length of your finger, gets the light. If you finger is partially turned, then you see some light but not the full amount. A very clever demonstration!

Red laser light is used in this experiment. Such red light from a helium-neon laser is monochromatic with one wavelength, namely 633 nm. If you shine white light through a bottle of karo syrup, different wavelengths rotate to different degrees.

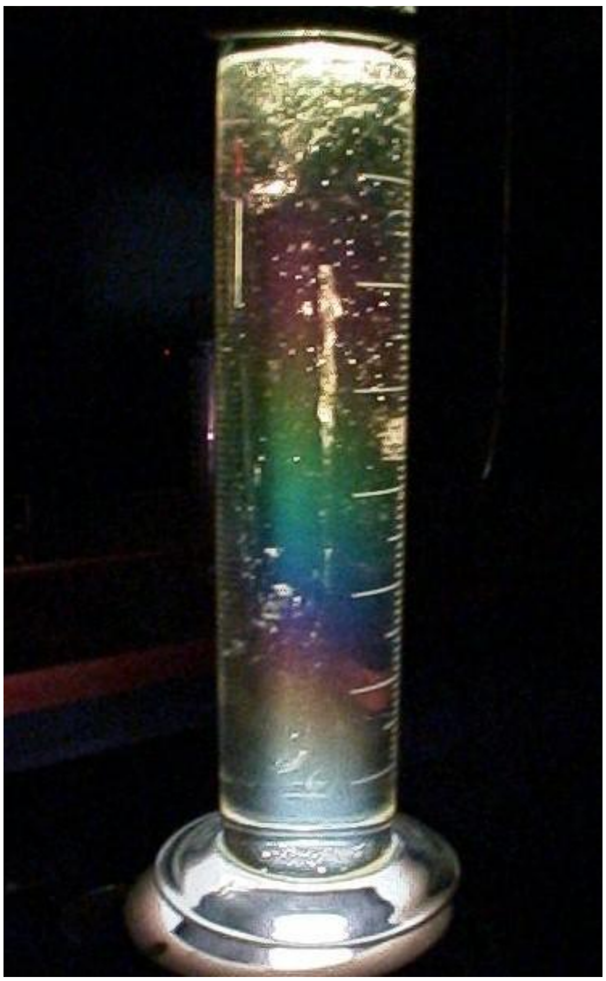
When viewed through a polarizer, different mixtures of color appear, depending on the orientation of the polarizer. A polarizer is held up to light passing through karo syrup in the photos below. The polarizer lets through 100% of the color if the polarization is correct, i.e., coordinated with the polarizer orientation.

The polarizer lets through progressively less and less amounts of the wavelengths with polarizations differing more and more from the polarizer orientation. For the wavelength that has polarization perpendicular to the axis of the polarizer, nothing gets through. The result is a nice mixture of color that is complementary to the mixture obtained when the polarizer is rotated 90°

Optical Activity with White Light and Karo Syrup

Color with Polarizer at First Orientation	Complementary Color: Polarizer Rotated 90°
	

Courtesy Richard E. Berg, Lecture Demonstration Facility, University of Maryland



Physics at the University of Virginia

"Optical activity refers to the property where polarized light is rotated when passing through some solid or liquid materials.

"Take a cardboard sheet with a 1 inch hole in the center and place it on the overhead. Place a polarizer over the hole and the cylinder full of corn syrup on the polarizer. As the polarized light passes through the corn syrup, the polarization direction is rotated. The light is also scattered, when viewed at an angle of 90 degrees to the direction of propagation (from the side), only light that is polarized perpendicular to the scattering plane will be seen. Since the rate of rotation of the polarization is frequency dependent, different colors are rotated by different amounts as the light travels up the tube and a rainbow of colors is seen emanating from the cylinder." U of VA

"Hints: Turn out the light!" U of VA

O10. Polage Art.

In 1967, Austine Wood Comarow invented an art form that utilizes stress birefringence, optical activity, and polarizers. She calls the new art ***Polage***, which is a trademark of Austine Studios. We will let the artist herself explain both the art and the physics of the Polage™.

"I began developing Polage (polarized light collage) in 1967. It consists of layers of clear cellulose sandwiched between two polarizing filters. Polarizing filter is used in Polaroid sunglasses, polarizing filters for camera lenses and in liquid crystal displays. Very few people are aware of the phenomenon which occurs when certain 'birefringent' materials such as cellulose, mica, stressed plastics and sugar solutions are viewed between two polarizing filters. White light, as it passes through, becomes separated by wavelength (color). I am able to control the color in my Polages by the thickness and orientation of all the tiny little pieces of cellulose which make up the image.

"I work with a razor knife at a light table, cutting out the hundreds of small shapes of clear cellulose which make up the *painting*. I wear polarizing sun glasses in order to see the image which is colorless until completed. Then it is framed in a light box containing a rotating polarizing filter. The finished Polage is *alive*. It glows and the colors appear to flow as it cycles through it's multiple views. It is moving, changing *stained glass*. Polage is a medium for the 21st Century: stained glass meets the video screen."

Comarow calls her studio "Austine Studios" and goes by her first name. Austine's work can be found in homes, offices, and museums. One of her Polages was made for The National Headquarters of the Materials Research Society in Warrendale, PA, near Pittsburgh.