## Modern Optics, Prof. Ruiz, UNCA Chapter O. Polarization – Homework

**HW O1. Elliptical Polarization.** Consider the electric wave traveling down the z-axis with the most general polarization state

$$\vec{E} = E_x \hat{i} + E_y \hat{j} = E_{ox} \hat{i} \cos(kz - \omega t) + E_{oy} \hat{j} \cos(kz - \omega t + \phi).$$

Choosing different values for  $\phi$  leads to a rich variety of polarizations. Choosing

 $\phi = 0$  leads to linearly-polarized light and taking  $E_{ox} = E_{oy}$  with  $\phi = \frac{\pi}{2}$  leads to

circularly polarized light. Here you will take  $\phi = \frac{\pi}{2}$  and  $E_{ox} > E_{oy}$ . Set z = 0 and

let  $\theta = -\omega t$  like we did in class and show that the electric field vector traces out an ellipse. Give the equation for the ellipse in terms of the variables  $E_x$ ,  $E_y$ , and constants  $E_{ox}$ ,  $E_{oy}$ . Finally give the eccentricity in terms of the constants.

HW O2. Quarter Wave Plate Design. A birefringent material allows for double refraction affecting perpendicular polarization states differently. The different wave speeds for each polarization state can allow us to retard one polarization state by  $\pi/2$  and produce elliptically polarized light. Consider normal incidence and quartz, where for 590 nm light the index of refraction  $n_e = 1.553$  for the extraordinary wave and the index of refraction for the ordinary wave  $n_o = 1.544$ . By the way, calcite is too brittle for a wave plate.

(a) Show that the relative phase shift between the light for the 2 polarization directions is

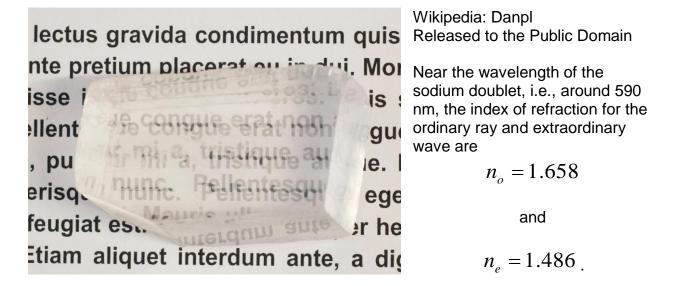
$$\Delta \phi = (2\pi / \lambda) d (n_e - n_o),$$

where  $\lambda$  is the wavelength of the incoming light in vacuum or air, d is the thickness of the birefringent plate, and  $n_o$ ,  $n_e$  are the indexes of refraction for the ordinary and extraordinary rays. If d  $(n_e - n_o) = \lambda / 4$ , then  $\Delta \varphi = \pi / 2$ . But d  $(n_e - n_o) = (4m + 1) \lambda / 4$  also works where m = 1, 2, 3 ..., i.e. an integral number of wavelengths plus  $\lambda / 4$ .

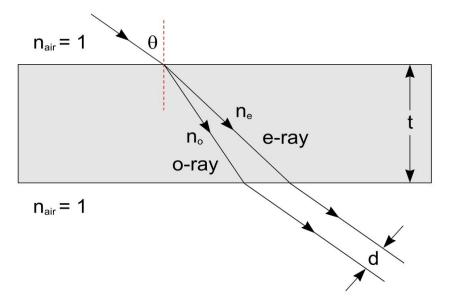
(b) Find the minimum thickness d, i.e. m = 0, tailored to 590 nm for quartz.

(c) Give the thickness for a quarter-wave quartz plate tailored to 590 nm where m = 200.

HW O3. Birefringence in Rocks.



Pick up the calcite so that there is a layer of air between the print and the calcite.



Find d in terms of  $n_{o}$  ,  $n_{e}$  , the incident angle  $\theta$  , and thickness t .

Find d to 3 sig figs for calcite where  $\theta = 30.0^{\circ}$ , and t = 7.00 cm.