doctorphys.com

Modern Optics, Prof. Ruiz, UNCA Chapter P. Why is the Sky Blue?

HW P1. Dispersion Revisited. Show that $n^2 = 1 + \frac{n_e e^2}{\varepsilon_o m(\omega_o^2 - \omega^2)}$ for

 $\lambda >> \lambda_o = \frac{c}{f_o} = \frac{2\pi c}{\omega_o} \text{ can be put in the form } n^2(\lambda) = \alpha + \frac{\beta}{\lambda^2}. \text{ Give the}$

constants lpha and eta in their simplest forms in terms of n_e , e , ${\cal E}_o$, m , c , and λ_o .

HW P2. Scattering. From class:
$$E_{\theta} = -\frac{ae\sin\theta}{4\pi\varepsilon_o c^2 r}$$
 and $x = -\frac{eE_o}{m(\omega_o^2 - \omega^2)}e^{i\omega t}$
Show that the amplitude of the scattered wave is $\left|E_{\theta}\right| = \frac{e^2\omega^2 E_o\sin\theta}{4\pi\varepsilon_o c^2 rm(\omega_o^2 - \omega^2)}$.

Then show that the irradiance is
$$I = \frac{1}{2} \sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{e^4 \omega^4 E_o^2 \sin^2 \theta}{(4\pi\varepsilon_o)^2 c^4 r^2 m^2 (\omega_o^2 - \omega^2)^2}.$$

Show that for a ribbon area $dA = 2\pi r^2 \sin \theta d\theta$, the power radiated is given by

$$dP = \frac{2\pi I_o e^4 \omega^4 \sin^3 \theta}{(4\pi\varepsilon_o)^2 c^4 m^2 (\omega_o^2 - \omega^2)^2} d\theta \text{ where the initial incident irradiance}$$

 $I_o = \frac{1}{2} \sqrt{\frac{\varepsilon_o}{\mu_o} E_o^2}$. The power P is the reradiated power, the scattered power.

 $r \sin \theta r d\theta$

Integrate over the angle θ from $\theta = 0$ to $\theta = \pi$ and show that the total scattered power is

$$P = \frac{8\pi}{3} \left(\frac{e^{2}}{4\pi\varepsilon_{o}mc^{2}}\right)^{2} \frac{\omega^{4}}{\left(\omega_{o}^{2} - \omega^{2}\right)^{2}} I_{o}$$

