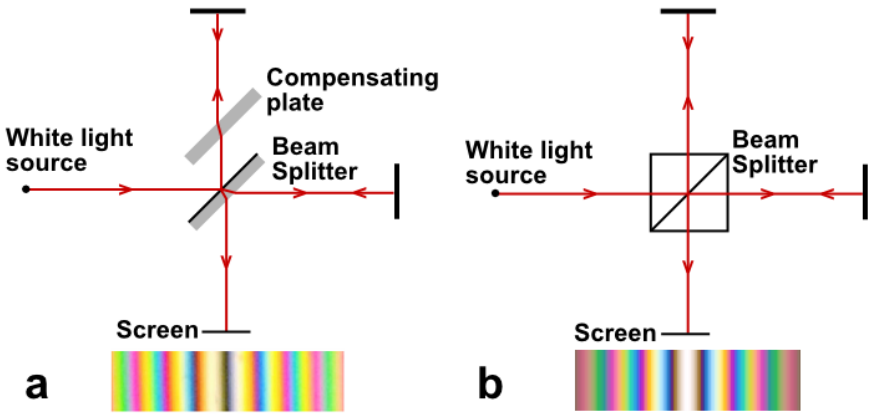
HX1. Michelson Interferometer.



"White light fringes in a Michelson interferometer. A Michelson interferometer using white light. (a) Michelson interferometer with a mirror beam splitter. A compensator plate is required to equalize the two paths. The two beams differ in the number of phase inversions. The central fringe representing equal path length is dark. (b) Michelson interferometer with a cube beam splitter, which provides equal phase shifts for the longitudinal and transverse beams. The central fringe representing equal path length is bright." Wikipedia: Stigmatella aurantiaca

Monochromatic light enters a Michelson interferometer. One mirror is carefully moved 25 microns and the observer notices 100 bright-dark pairs of fringes transpire during the move. In other words, as the mirror is moved with precision the observer sees bright, dark, bright, dark, etc. as the interference pattern shifts from constructive interference to destructive interfere over and over again. What is the wavelength of light in nm to the nearest nm? Be careful, when the mirror is moved a distance d, remember that the light has to travel to the mirror and back.

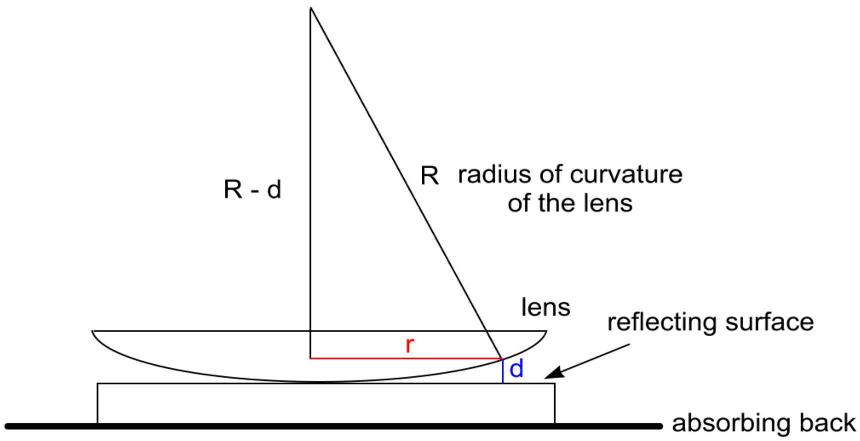
SOLUTION

When a mirror is moved a distance d, the path difference between the two beams changes by $2d$. For a bright fringe of constructive interference to pass though darkness and back to bright again means that $2d = m\lambda$, where $m = 1, 2, 3...$. Therefore,

$$d = m\frac{\lambda}{2} \text{ and the wavelength is } \lambda = \frac{2d}{m}$$

$$\lambda = \frac{2d}{m} = \frac{2 \cdot 25\ \mu\text{m}}{100} = \frac{50\ \mu\text{m}}{100} = 0.5\mu\text{m} = 500\text{ nm}$$

HX2. Newton's Rings.

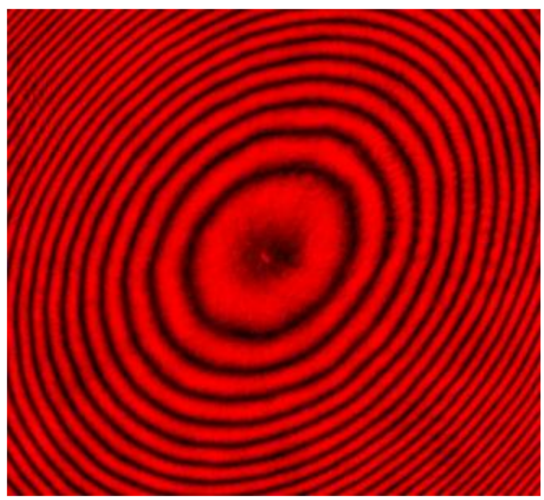


A lens sits on a flat reflecting surface. Take R to be very far away. Then, light traveling downward reflects partially at the back curved surface of the lens while the some light passes on and reflects off the optical flat. We will consider these two as the main beams.

The light hitting optical flat undergoes a 180° phase change since it is a hard reflection (air to glass).

Note that the extra optical path taken by the light passing a distance d beyond the lens and back is 2d. Normally for constructive interference we want 2d = mλ, where m = 1, 2, 3, … But because of the hard reflection, this condition is destructive interference. But that's okay, we will work with dark fringes.

Show that for R >> d, that radii r for the dark fringes are given by the formula $r = \sqrt{mR\lambda}$.



"Newton's rings interference pattern created by a plano-convex lens illuminated by 650 nm red laser light, photographed using a low-light microscope. The illumination is from above, leading to a dark central region." Wikipedia: Robert D. Anderson. Creative Commons

SOLUTION

From the triangle in the figure $(R-d)^2 + r^2 = R^2$.

$$R^2 - 2Rd + d^2 + r^2 = R^2$$

$$-2Rd + d^2 + r^2 = 0$$

$$r^2 = 2Rd - d^2$$

$$r^2 = Rd(2 - \frac{d}{R})$$

$$d << R \quad \Rightarrow \quad r^2 = 2Rd$$

Now introduce

$2d = m\lambda$ (destructive interference).

Then, $r^2 = 2Rd = 2R(m\frac{\lambda}{2}) = Rm\lambda$.

$$r^2 = mR\lambda$$

$$\boxed{r = \sqrt{mR\lambda}}$$

Dark fringe radii.