Modern Optics, Prof. Ruiz, UNCA doctorphys.com
Chapter Y. Photons

Y0. Optics. As this class is the last of the semester, we will keep the math to an introductory physics level (the 200 level).

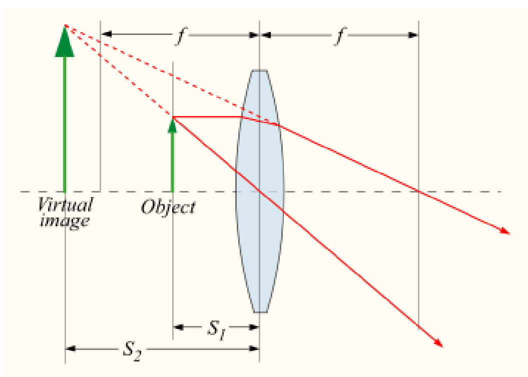
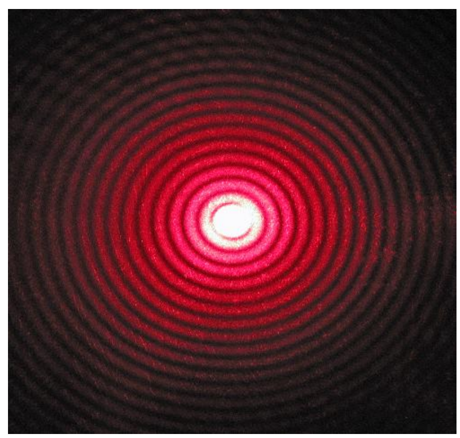
We return to our first class and list the three main areas that optics can be broken down into:

1. Geometrical Optics – light travels in straight lines,
2. Physical Optics – light as waves,
3. Quantum Optics – light as particles (quanta).

Our course description for *PHYS 323 Modern Optics* (3) in the UNCA Catalog is:

> A study of geometrical and physical optics. Prerequisite: PHYS 222. Even years Fall.

Our last class will focus on light as quanta. But remember that our discussions of electron transitions earlier with spectra and most recently with the laser included the particle idea of light.

	
Geometrical Optics	Physical Optics

Wikipedia: DrBob (left figure), Wisky (right figure). [Creative Commons](#)

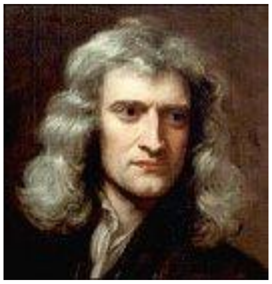
Basic Categorization of Light Properties

Realm of Geometrical Optics	Realm of Physical Optics
Reflection and Refraction	**Diffraction and Interference**

You can add polarization to the realm of physical optics.

Y1. Light: Wave or Particle?

Isaac Newton



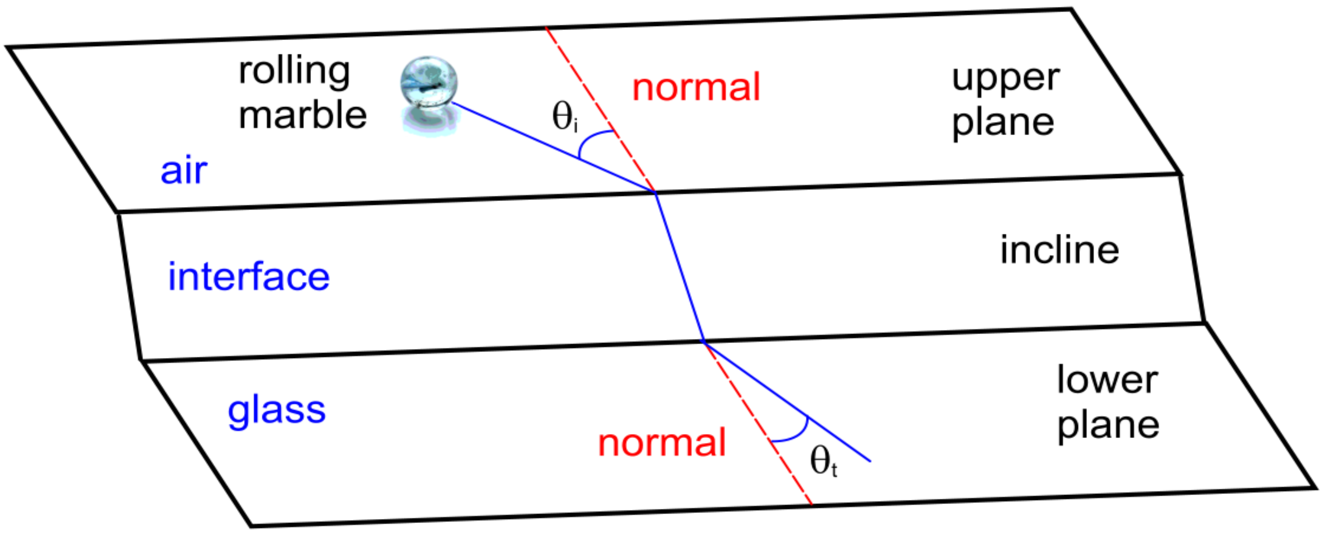
1642-1727

Albert Einstein

1879-1955

The greatest two physicists of all time are Newton and Einstein. Newton proposed in the 1600s that light consists of a stream of particles. This explained reflection since little balls bounce off walls (neglecting gravity) the way light reflects off mirrors.

A simplified way to think about Newton's particle model of refraction is to consider two planes at different heights and a ball rolling on the upper plane to an interface with a sliding board. Then light going from air to glass is like a particle traveling from a higher plane to a lower one, sliding down an incline in the process. But rolling marbles speed up as they deflect toward the normal, heading down an incline plane. When light bends toward the normal going from air to glass, the speed does the opposite - it decreases. But no one knew the speed of light in glass for even over a century later. So the particle model problems were not evident at the time.



Marble from Wikipedia: wjlonien. Creative Commons

Huygens was a contemporary of Newton and Huygens proposed a wave model. Remember the Huygens-Fresnel principle with the "baby" waves? Newton's stature though was much great and his fame helped to keep the particle model alive and well for over 100 years.

As we have noted earlier, around 1800, Young challenged Newton's idea and argued that light is a wave. His famous two-slit interference experiment cannot be explained by the particle model at all. Furthermore, Fresnel developed elegant wave equations which were substantiated by experiment. Remember Poisson's prediction of a crazy spot behind an obstacle using these equations in order to prove Fresnel wrong? But, as we discussed earlier in our course, the spot was found to everyone's surprise, and this phenomenon is now called the Poisson spot.

The scoreboard appears below comparing the wave and particle models in describing phenomena of light. A "1" means success, while a "0" is failure. You can easily see that the wave model wins hands down for the phenomena listed.

Wave and Particle Models of Light

Phenomenon	Wave	Particle
Reflection	1	1
Refraction	1	0
Diffraction	1	0
Interference	1	0
Polarization	1	0
Totals	**5**	**1**

A '1' indicates success, '0' means failure.

Y2. Light as Wave-Particle.

The story about light as particle in modern times beginning in 1900 brings us to Max Planck. The cornerstones of what we refer to as Modern Physics are quantum mechanics (1925) and relativity (1905). The early form of quantum mechanics took shape as the “quantum” concept was applied to energy levels. We think of energy as continuous in classical physics. As an example, we return to our familiar harmonic oscillator. The energy of a harmonic oscillator is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 ,$$

the sum of kinetic energy and potential energy. An object of mass m moves at speed v where the force on the mass satisfies Hooke’s Law: $F = -kx$. The parameter k is the spring constant and x is measured from the middle position, the equilibrium. When the mass is at the extreme position $x = A$ the energy is all potential. Since the energy is conserved, the total energy can be expressed as

$$E = \frac{1}{2}kA^2 .$$

Classically, we can imagine a continuous amount of possibilities for the energy. Until Max Planck came along and "quantized" the energy for the harmonic oscillator. In trying to understand radiation in situations similar to a kiln where the walls glow with different colors depending on the temperature, he was led to the quantization.

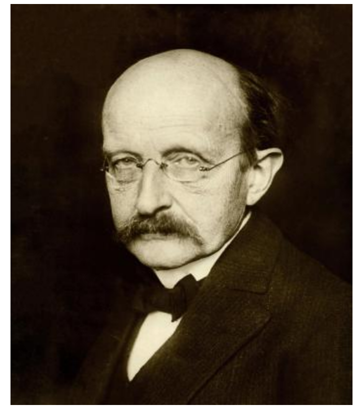
Max Planck (1858 – 1947)
German Theoretical Physicist

Left Photo 1878
Right Photo 1933

Energy Quanta

Birth of Quantum Physics

Nobel Prize in Physics 1918



The basic concept of Planck is that harmonic-oscillator energy comes in discrete units or chunks of energy where the fundamental unit is proportional to the oscillator frequency:

$$\Delta E = hf \text{ and } E_n = nhf \text{ with n = 0, 1, 2, 3, ….}$$

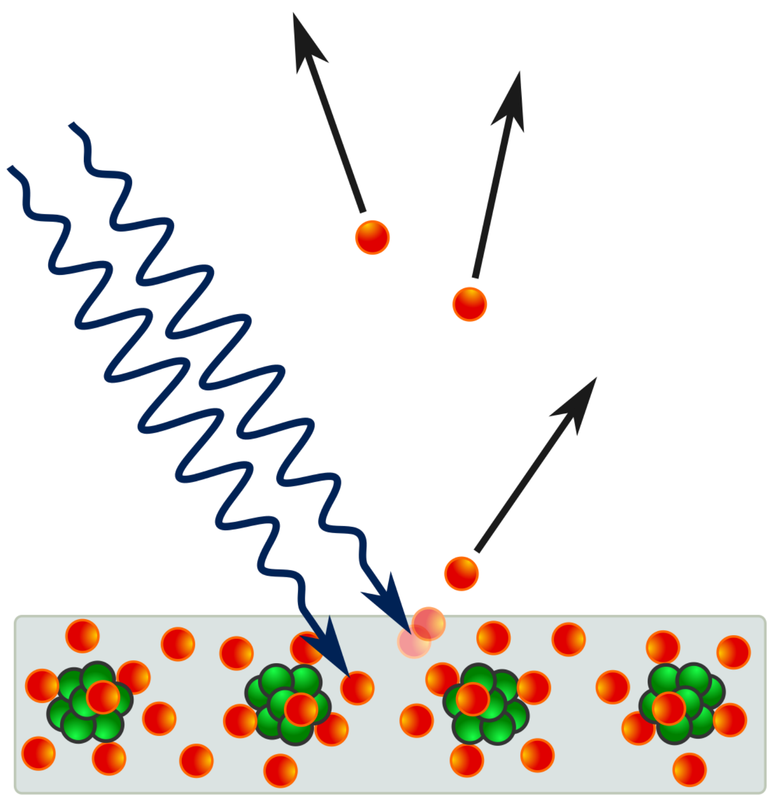
The constant $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$ is the Planck constant named in his honor. The unit of energy is so small that we never notice it in macroscopic oscillations like a car moving up and down on its shock absorbers. Note that the dimensions are joule · second since energy is joule and frequency is inverse second. The combination joule · second is called action.

Classically, we think of continuous energy since the amplitude A can be anything, a continuous variable. In Planck's scenario one can think of the amplitude as being restricted to jumping by discrete amounts. But this interpretation is wrong from later developments in quantum mechanics. The quantum mechanics of 1925 provides a framework of probabilities for the positions of the particle. The energy though is still quantized and with discrete steps given by Planck's formula. Another difference with the full quantum mechanics of 1925 is that you can't have zero energy for the lowest energy level. You find $E_n = (n + \frac{1}{2})hf$.

Einstein took the emission of light from a harmonic oscillator given by hf as actually a quantum of electromagnetic radiation. With Einstein, the quantum of light was born.

$$E_{\text{photon}} = hf$$

Einstein successfully employed his quanta to explain the photoelectric effect (1905), which is illustrated in the figure below. Incoming light in the form of quanta kick out electrons from a crystal. The reason why light as a wave does not work is that increasing the intensity of the wave does nothing! The effect occurs by considering light as quanta with energy $E_{\text{photon}} = hf$. When the frequency becomes high enough, there is sufficient kick even though the light intensity can be very low, i.e., very dim. One photon is all you need to do the trick. Zillions of photons without the necessary high frequency do nothing. Well, they can reflect, but no kicking out of electrons.



Courtesy Wikipedia: Georg-Johann. Creative Commons License

Einstein received the Nobel Prize for his explanation of the photoelectric effect, for which you study the details in your Modern Physics course. Here are the Nobel prize descriptions for both Planck and Einstein.

Max Planck – 1918 Nobel Prize in Physics – “in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta.”

Albert Einstein – 1921 Nobel Prize in Physics – “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect.” He received the actual award in 1922.

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We now update our scoreboard with a devastating blow to the wave picture.

Wave and Particle Models of Light

Phenomenon	Wave	Particle
Reflection	1	1
Refraction	1	0
Diffraction	1	0
Interference	1	0
Polarization	1	0
Photoelectric	0	1
Totals	**5**	**2**

Physics is ruthless – one failure and the law or picture is destroyed! What do we do now? Well, if you are confused, so were physicists – and for about 20 years – until quantum mechanics came in 1925 and beyond. The picture we have today is one of a wave-particle duality. You could say, if you want to include our three areas of optics: geometrical, physical, and quantum, that light is a ray-wave-particle. But the conventional knowledge goes with wave-particle.

Y3. Leading to the Bohr Model. A very elegant and simple way to understand light as a photon is an area where physics and chemistry overlap: the atom. In particular, we look at the hydrogen atom. The Bohr model is semi-classical, appearing in 1913 and predating quantum mechanics by a decade. The model is due to Bohr and Rutherford, but often just called the Bohr model.

Here is the background leading up to the Bohr model of the hydrogen atom. **The Bohr model only works for hydrogen.** You need quantum mechanics to understand the other atoms.

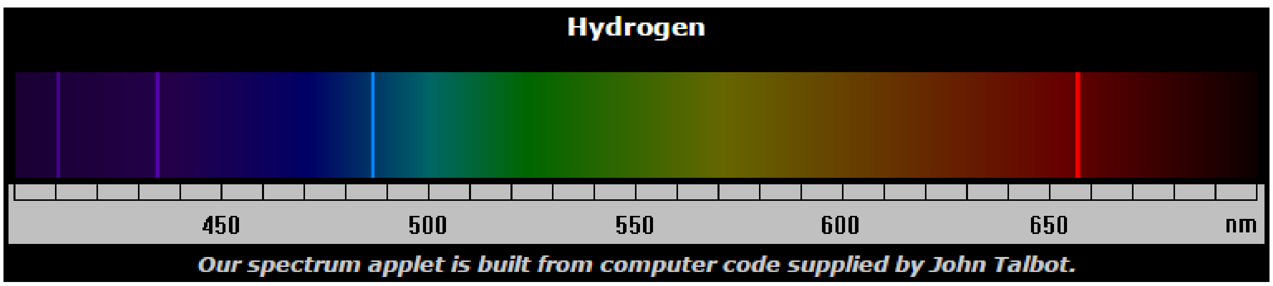
We first consider the experimental data that set the stage and need for a model of the atom.

Johann Jakob Balmer (1925 – 1898)
Swiss Mathematician

The story takes us back to a high school teacher, Dr. Balmer, at a school for young ladies in Basel, Switzerland (from 1859 – 1898). He also was a lecturer in mathematics at the University of Basel from (1865 to 1890).

Balmer cracked the code for the four visible hydrogen lines by finding an empirical formula in 1885:

$$\lambda = 364.56 \text{ nm}\left(\frac{m^2}{m^2-4}\right) \text{, where } m = 3, 4, 5\ldots.$$



The four visible lines are 410 nm, 434 nm, 486 nm, and 656 nm.

The Beautiful 656 nm Red (North American and Pelican Nebulae)

Let’s check the formula out. With m = 3,

$$\lambda = 364.56 \text{ nm}(\frac{3^2}{3^2-4}) = 364.56 \text{ nm}\frac{9}{5} = 656 \text{ nm}\ .$$

For m = 4,

$$\lambda = 364.56 \text{ nm}(\frac{4^2}{4^2-4}) = 364.56 \text{ nm}\frac{16}{12} = 486 \text{ nm}\ .$$

For m = 5,

$$\lambda = 364.56 \text{ nm}(\frac{5^2}{5^2-4}) = 364.56 \text{ nm}\frac{25}{21} = 434 \text{ nm}\ .$$

For m = 6,

$$\lambda = 364.56 \text{ nm}(\frac{6^2}{6^2-4}) = 364.56 \text{ nm}\frac{36}{32} = 410 \text{ nm}\ .$$

Awesome empirical formula. If we continue, we enter the ultraviolet (UV).

For m = 7,

$$\lambda = 364.56 \text{ nm}(\frac{7^2}{7^2-4}) = 364.56 \text{ nm}\frac{49}{45} = 397 \text{ nm}\ .$$

For m = 8,

$$\lambda = 364.56 \text{ nm}(\frac{8^2}{8^2-4}) = 364.56 \text{ nm}\frac{64}{60} = 389 \text{ nm}\ .$$

The lines are getting very close to each other. In fact for m = infinity,

$$\lim_{m\to\infty} \lambda = 364.56 \text{ nm} \lim_{n\to\infty}(\frac{m^2}{m^2-4}) = 364.56 \text{ nm} \cdot 1 = 364.6 \text{ nm}\ .$$

Johannes Rydberg (1854 – 1919)
Swedish Physicist

Known for the Rydberg Formula

Rydberg generalized Balmer's formula to describe more wavelengths in the hydrogen spectrum. His formula appeared in 1888.

Here is a way to see how his formula can be generalized from Balmer's formula. Write Balmer

$$\lambda = 364.56 \text{ nm}(\frac{m^2}{m^2-4})$$ as

$$\frac{1}{\lambda} = \frac{1}{364.56 \text{ nm}} (\frac{m^2-4}{m^2}).$$

Then continue as follows.

$$\frac{1}{\lambda} = \frac{1}{364.56 \text{ nm}} (1 - \frac{4}{m^2}) \quad => \quad \frac{1}{\lambda} = \frac{1}{364.56 \text{ nm}} (\frac{4}{4} - \frac{4}{m^2})$$

$$\frac{1}{\lambda} = \frac{4}{364.56 \text{ nm}} (\frac{1}{4} - \frac{1}{m^2})$$

$$\frac{1}{\lambda} = \frac{4}{364.56 \text{ nm}} (\frac{1}{2^2} - \frac{1}{m^2})$$

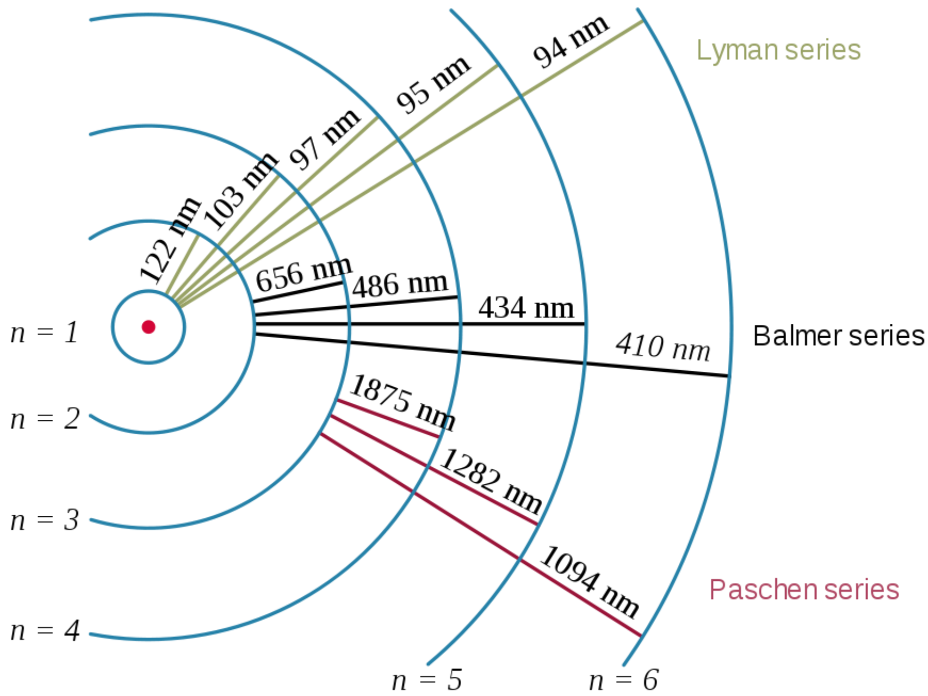
Rydberg's generalization is

$$\frac{1}{\lambda} = \frac{4}{364.56 \text{ nm}} (\frac{1}{n^2} - \frac{1}{m^2}),$$

where $n = 1, 2, 3...$ and $m = n+1, n+2, n+3...$

Each value of n consists of a series. The n = 2 case is the Balmer series.

Electron Transitions and Series (NOT TO SCALE)



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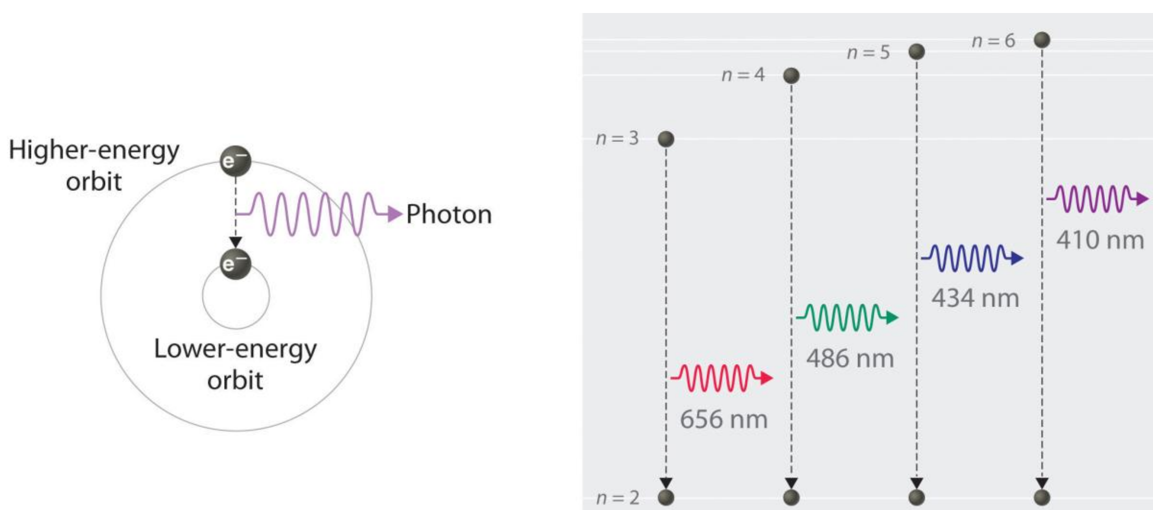
The Balmer series has the four visible lines. Each visible spectral line is shown with its electron transition. Energy levels below are more to scale. We will derive the equation for the radii.



(a) Electronic emission transition **(b) Balmer series transitions**

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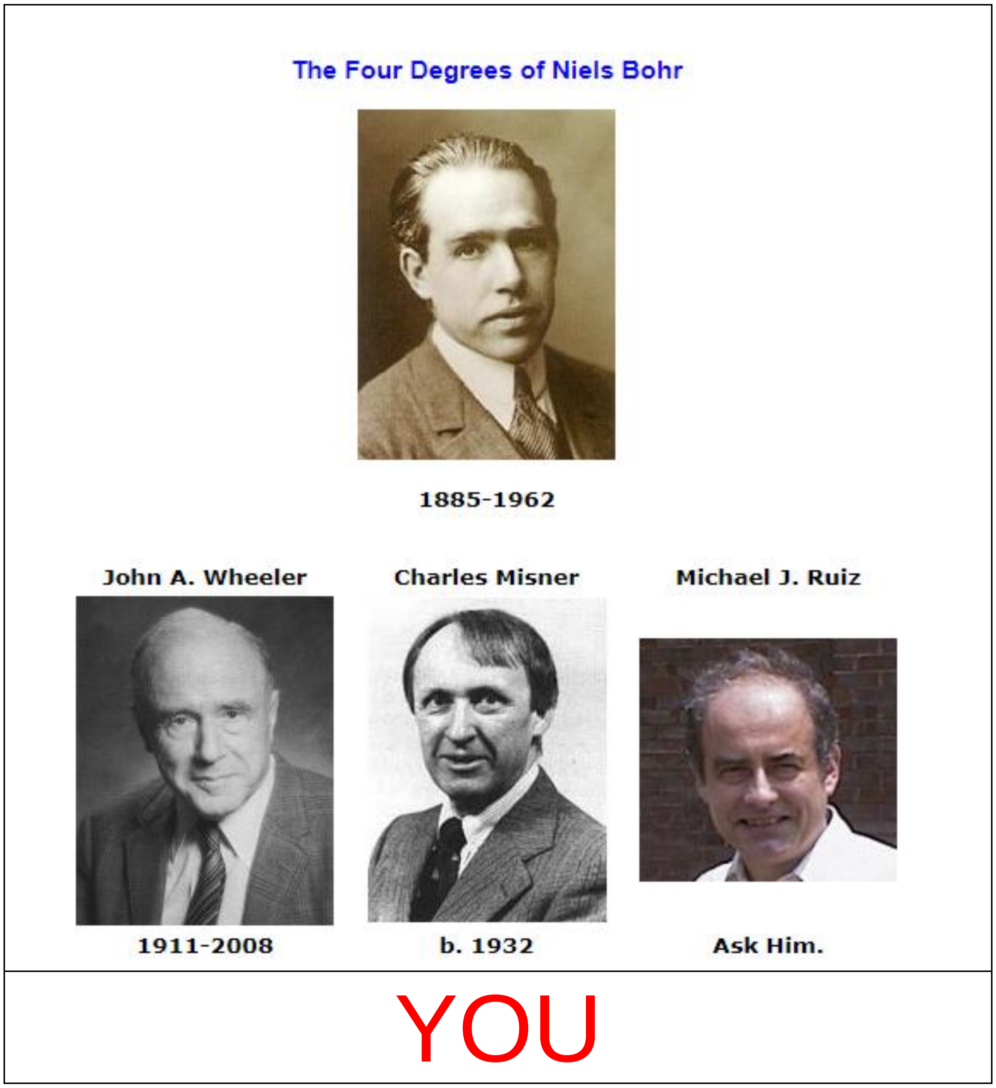
Y4. The Bohr Model. Bohr is special to us since we can are connected to him by teachers.



Similar to the game ***The Six Degrees of Kevin Bacon***, invented by US college students in the mid 1990s, the analysis goes as follows.

Bohr Student Number 0. Niels Bohr himself.

Bohr Number 1. John Wheeler studied under John Archibald Wheeler in the 1930s. Wheeler coined the word ***black hole*** in 1968.

Bohr No. 2. Charles Misner (while at ***Princeton***) studied under John Wheeler 1950s. Later, Wheeler (Princeton) and Misner (then teaching at the ***University of Maryland***), along with Kip Thorne (***Caltech***) authored what came to be known as the "bible" of Einstein's theory of general relativity. This nearly 1300-page text is called ***Gravitation*** and was published in 1971.

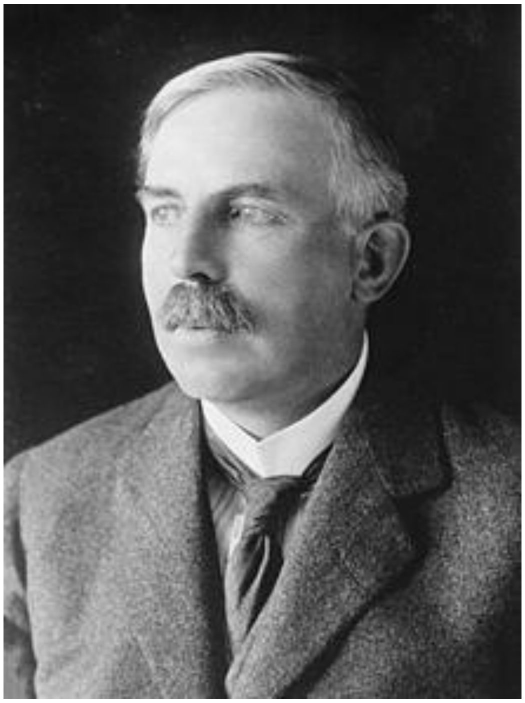
Bohr No. 3. Michael J. Ruiz took a course on Einstein's theory of general relativity at the ***University of Maryland*** c. 1975, using ***Gravitation*** AKA MTW (Misner-Thorne-Wheeler).

Bohr No. 4. YOU are studying under Michael **J. Ruiz**, known as "Doc" or "doctorphys." Therefore, you have a ***Bohr Student Number*** of **4**, representing four degrees of Niels Bohr.

Kevin Bacon after whom "The Six Degrees of Kevin Bacon" is named. Can you connect any actor to Kevin Bacon by Six Degrees of Movies?

American Actor, b. 1958 in Philadelphia

Wikipedia: Gage Skidmore. Creative Commons License



Ernest Rutherford (1871 – 1937)
British Physicist

Known as the Father of Nuclear Physics

Known for the Bohr-Rutherford Model
(Bohr Model for Short)

Known for the Modern Atomic Model
a nucleus at the center of the atom

Credited with the Discovery of the Proton (1911)

Known for Rutherford Scattering in nuclear physics

Nobel Prize in Chemistry (1908)

Nobel Committee: “for his investigations into the disintegration of the elements, and the chemistry of radioactive substances.”

Niels Bohr (1885 – 1962)
Danish Physicist

Nobel Prize in Physics (1922)

Nobel Committee: "for his services in the investigation of the structure of atoms and of the radiation emanating from them."

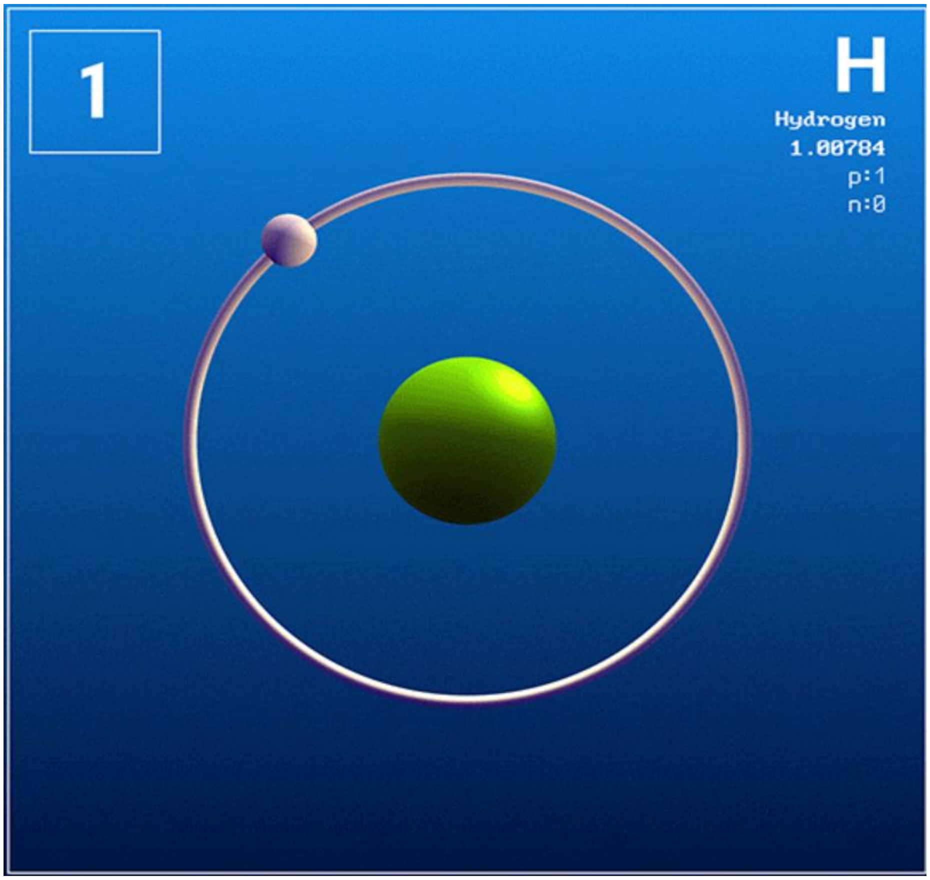
Known for the Bohr Model which he developed with Ernest Rutherford (1913).

Known for his Principle of Complementarity in Quantum Mechanics – applies to the Wave-Particle aspects of light and matter.

Bohr’s son, the nuclear physicist Aage Bohr (1922 – 2009), born when Dad got the Nobel, shared the Nobel Prize in Physics (1975) with Ben Mottelson and James Rainwater over 50 years later.

A Mother-Daughter Nobel is Marie Curie (Physics 1903 with husband Pierre Curie, Chemistry 1911) and later her daughter Irène Joliot-Curie shared the prize in Chemistry (1935) with her husband Frédéric. A family of Nobel laureates. A fifth Nobel involved a Peace Prize to UNICEF (United Nations Children’s Fund), accepted by its director, a son-in-law of Marie Curie.

Now on to the Bohr Model. The hydrogen atom consists of an electron in orbit around a proton in the classical way of thinking – the planetary model (like a planet orbiting the Sun).



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The Coulombic force between the electron and proton a distance r apart is

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_p q_e}{r^2},$$

where the charge of the proton is $q_p = +e > 0$ and

the charge on the electron is $q_e = -e$.

These substitutions give the attractive force

$$F = -\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}.$$

Often the electric constant is defined: $k_e \equiv \dfrac{1}{4\pi\varepsilon_o}$. Therefore, we can write

$$F = -k_e \frac{e^2}{r^2} \, .$$

In the simple model we consider the electron traveling in a circular orbit. Since we know the mass of the proton is 1836 times the mass of the electron, we consider the proton mass infinite and unmovable at the origin. Newton's Second Law is

$F = ma = -m_e \dfrac{v^2}{r}$, where the acceleration is inward to match the force.

We include a subscript to emphasize that the mass is that of the electron.

Combining Newton's Second Law with Coulomb's Law,

$$\boxed{m_e \frac{v^2}{r} = k_e \frac{e^2}{r^2}}$$

Conservation of energy gives us another equation:

$$E = \frac{1}{2} m v^2 + U \, .$$

But what is the potential energy? We know how to get it since $F = -k_e \dfrac{e^2}{r^2} = -\dfrac{dU}{dr}$.

Therefore $U = -k_e \dfrac{e^2}{r}$.

Our energy equation is given below.

$$\boxed{E = \frac{1}{2} m v^2 - k_e \frac{e^2}{r}}$$

Before delving into the Rutherford-Bohr model, let's derive one more classical equation from our two equations

$$E = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r} \quad \text{and} \quad m_e \frac{v^2}{r} = k_e \frac{e^2}{r^2} .$$

The second equation can be written as $m_e v^2 = k_e \frac{e^2}{r}$.

Then $\frac{1}{2} m_e v^2 = \frac{1}{2} k_e \frac{e^2}{r}$ and

$E = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$ becomes $E = \frac{1}{2} k_e \frac{e^2}{r} - k_e \frac{e^2}{r}$.

$$\boxed{E = -\frac{1}{2} k_e \frac{e^2}{r}}$$

You will encounter this kind of relation in advanced classes when you study the Virial Theorem. Now on to the Bohr Model and experience the profound insight of Bohr and Rutherford. They thought out of the box.

First off, we have a serious classical problem. An electron traveling in a circle around a proton is accelerating and will thus emit electromagnetic radiation. As the system gives up energy, the electron will lose energy and spiral down to the proton – the hydrogen atom will cease to exist. That means atoms will not exist and therefore you will not exist. But since you have Bohr Number 4, we know you exist.

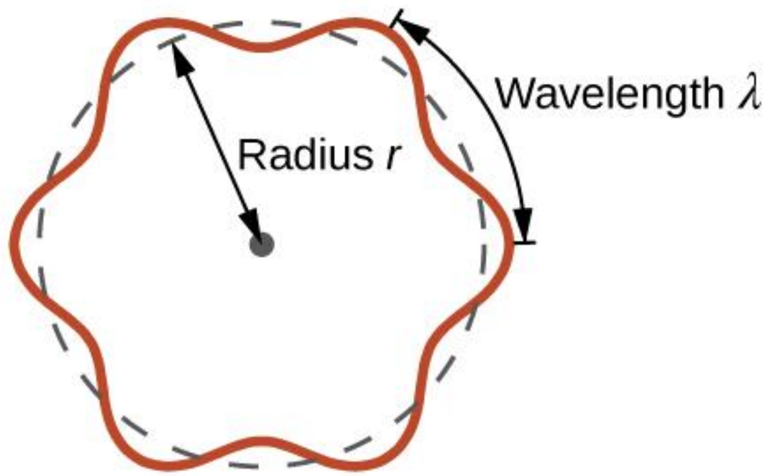
So here come the postulates of the model. They are stated as new laws of physics, laws superseding classical laws of electromagnetic radiation. The general postulates are given below. Details and the math will follow.

Postulate 1. An electron in orbit around a proton DOES NOT RADIATE.
Postulate 2. The electron angular momentum is quantized – giving quantized orbits.
Postulate 3. A photon is emitted when the electron drops to a lower orbit.

The prescription for the quantization of angular momentum is given below.

$$\boxed{L = m_e v r = n\hbar}$$

Here is a cool way to derive this result using the de Broglie relation that came later. You wrap a wave around the circumference so that $2\pi r = n\lambda$, i.e., an integral number of wavelengths. Then you use de Broglie. Watch below.



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Fast forwarding to 1924 and the de Broglie relation.

$$2\pi r = n\lambda$$

$$\lambda = \frac{h}{p}$$

$$2\pi r = n\frac{h}{p}$$

$$pr = n\frac{h}{2\pi}$$

$$mvr = n\hbar$$

$$L = n\hbar$$

We will find the expression for the radius of the n^th^ electron orbit.

Start with $\boxed{L = m_e v r = n\hbar}$ and $\boxed{m_e \frac{v^2}{r} = k_e \frac{e^2}{r^2}}$.

$$m_e \frac{v^2}{r} = k_e \frac{e^2}{r^2} \quad \Rightarrow \quad m_e v^2 = k_e \frac{e^2}{r}$$

We want to eliminate v.

$$L = m_e v r = n\hbar \quad \Rightarrow \quad v = \frac{n\hbar}{m_e r} = \frac{nh}{2\pi m_e r}$$

$$m_e v^2 = k_e \frac{e^2}{r} \quad \Rightarrow \quad m_e \left(\frac{nh}{2\pi m_e r}\right)^2 = k_e \frac{e^2}{r}$$

$$m_e \left(\frac{n^2 h^2}{4\pi^2 m_e^2 r^2}\right) = k_e \frac{e^2}{r}$$

$$\frac{n^2 h^2}{4\pi^2 m_e r^2} = k_e \frac{e^2}{r} \quad \Rightarrow \quad \frac{n^2 h^2}{4\pi^2 m_e} = k_e e^2 r$$

$$r_n = \frac{n^2 h^2}{4\pi^2 m_e k_e e^2}$$

$$k_e = \frac{1}{4\pi\varepsilon_o} \quad \Rightarrow \quad r_n = \frac{4\pi\varepsilon_o n^2 h^2}{4\pi^2 m_e e^2} \quad \Rightarrow \quad r_n = \frac{\varepsilon_o n^2 h^2}{\pi m_e e^2}$$

$$\boxed{r_n = \frac{\varepsilon_o n^2 h^2}{\pi m_e e^2}}$$

The increasing radii of the circular orbits goes as n-squared.

The Bohr radius is the radius for the ground state.

$$r_n = \frac{\varepsilon_o n^2 h^2}{\pi m_e e^2} \quad => \quad a_o \equiv r_1 = \left.\frac{\varepsilon_o n^2 h^2}{\pi m_e e^2}\right|_{n=1} = \frac{\varepsilon_o h^2}{\pi m_e e^2}$$

$$a_o = \frac{(8.85418781 \times 10^{-12})(6.62607015 \times 10^{-34})^2}{\pi (9.10938370 \times 10^{-31})(1.602176634 \times 10^{-19})^2}$$

$$a_o = 5.29 \times 10^{-11} \text{ m}$$

$$a_o = 0.529 \times 10^{-10} \text{ m}$$

$$a_o = 0.529 \ \mathring{\text{A}}$$

Energy Levels are Next.

$$\boxed{E = -\frac{1}{2} k_e \frac{e^2}{r}} \quad => \quad E_n = -\frac{1}{2} k_e \frac{e^2}{r_n} \text{ with } r_n = \frac{\varepsilon_o n^2 h^2}{\pi m_e e^2}.$$

$$E_n = -\frac{1}{2} k_e e^2 \frac{1}{r_n} = -\frac{1}{2} \frac{e^2}{4\pi\varepsilon_o} \frac{\pi m_e e^2}{\varepsilon_o n^2 h^2}$$

$$E_n = -\frac{1}{8} \frac{e^2}{\varepsilon_o} \frac{m_e e^2}{\varepsilon_o n^2 h^2} \quad => \quad E_n = -\frac{1}{8} \frac{m_e e^4}{\varepsilon_o^2 n^2 h^2}$$

$$\boxed{E_n = -\frac{1}{8} \frac{m_e e^4}{\varepsilon_o^2 n^2 h^2}}$$

Energy levels goes as 1 over n-squared. They get closer to each other.

Ground State Energy is $E_1 = -\frac{1}{8}\frac{m_e e^4}{\varepsilon_o^2 h^2}$. Remember, the zero reference is at infinity.

$$E_1 = -\frac{1}{8}\frac{(9.10938370\times10^{-31})(1.602176634\times10^{-19})^4}{(8.85418781\times10^{-12})^2(6.62607015\times10^{-34})^2}$$

$$E_1 = -2.1799\times10^{-18}\ \mathrm{J}$$

$$E_1 = -2.1799\times10^{-18}\ \mathrm{J}\,\frac{6.242\times10^{18}\ \mathrm{eV}}{1\ \mathrm{J}} = -13.6\ \mathrm{eV}$$

You need 13.6 eV of energy to kick a ground-state electron out of a hydrogen atom.

Referring to $E_n = -\frac{1}{8}\frac{m_e e^4}{\varepsilon_o^2 n^2 h^2}$,

a transition from m to n where m > n gives off a photon with energy

$$\Delta E = E_m - E_n = -\frac{1}{8}\frac{m_e e^4}{\varepsilon_o^2 n^2 h^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = hf .$$

$$\Delta E = -\frac{1}{8}\frac{m_e e^4}{\varepsilon_o^2 n^2 h^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = h\frac{c}{\lambda}$$

$$\Delta E = -\frac{1}{8}\frac{m_e e^4}{\varepsilon_o^2 n^2 h^3 c}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = \frac{1}{\lambda}$$

$$\frac{1}{\lambda} = \frac{m_e e^4}{8\varepsilon_o^2 h^3 c}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

The Rydberg constant is

$$R_H = \frac{m_e e^4}{8\varepsilon_o^2 h^3 c}\ .$$

$$\frac{1}{\lambda} = R_H \left[\frac{1}{n^2} - \frac{1}{m^2} \right]$$

$$R_H = \frac{(9.10938370 \times 10^{-31})(1.602176634 \times 10^{-19})^4}{8(8.85418781 \cdot 10^{-12})^2 (6.62607015 \times 10^{-34})^3 (2.99792458 \times 10^8)}$$

$$R_H = 1.0973735 \times 10^7 \text{ m}^{-1}$$

$$R_H = \frac{1}{0.9112667 \times 10^{-7} \text{ m}} = \frac{1}{91.12667 \times 10^{-9} \text{ m}} = \frac{1}{91.12667 \text{ nm}}$$

$$R_H = \frac{4}{4} \cdot \frac{1}{91.12667 \text{ nm}} = \frac{4}{364.5 \text{ nm}}$$

This is our Balmer result. We did take the proton mass to be infinite, we expect some deviation. But we agree with the Balmer result to three significant figures.

Y5. Cool Derivation Orbit Radii. Start with $E = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$ and $L = m_e v r$.

Replace speed in the first equation with $v = \frac{L}{m_e r}$ to get

$$E = \frac{1}{2} m_e \left(\frac{L}{m_e r}\right)^2 - k_e \frac{e^2}{r}\ .$$

$$E = \frac{1}{2}\frac{L^2}{m_e}\frac{1}{r^2} - k_e e^2 \frac{1}{r}$$

Now note that the angular momentum L is a constant.

Minimize the energy for a given angular momentum constant.

$$\frac{dE}{dr} = \frac{1}{2}\frac{L^2}{m_e}(-2\frac{1}{r^3}) - k_e e^2(-\frac{1}{r^2}) = 0$$

$$\frac{1}{2}\frac{L^2}{m_e}(-2\frac{1}{r^3}) + k_e e^2(\frac{1}{r^2}) = 0$$

$$\frac{1}{2}\frac{L^2}{m_e}(2\frac{1}{r^3}) = k_e e^2(\frac{1}{r^2})$$

$$\frac{L^2}{m_e}(\frac{1}{r^3}) = k_e e^2(\frac{1}{r^2})$$

$$\frac{L^2}{m_e} = k_e e^2 r$$

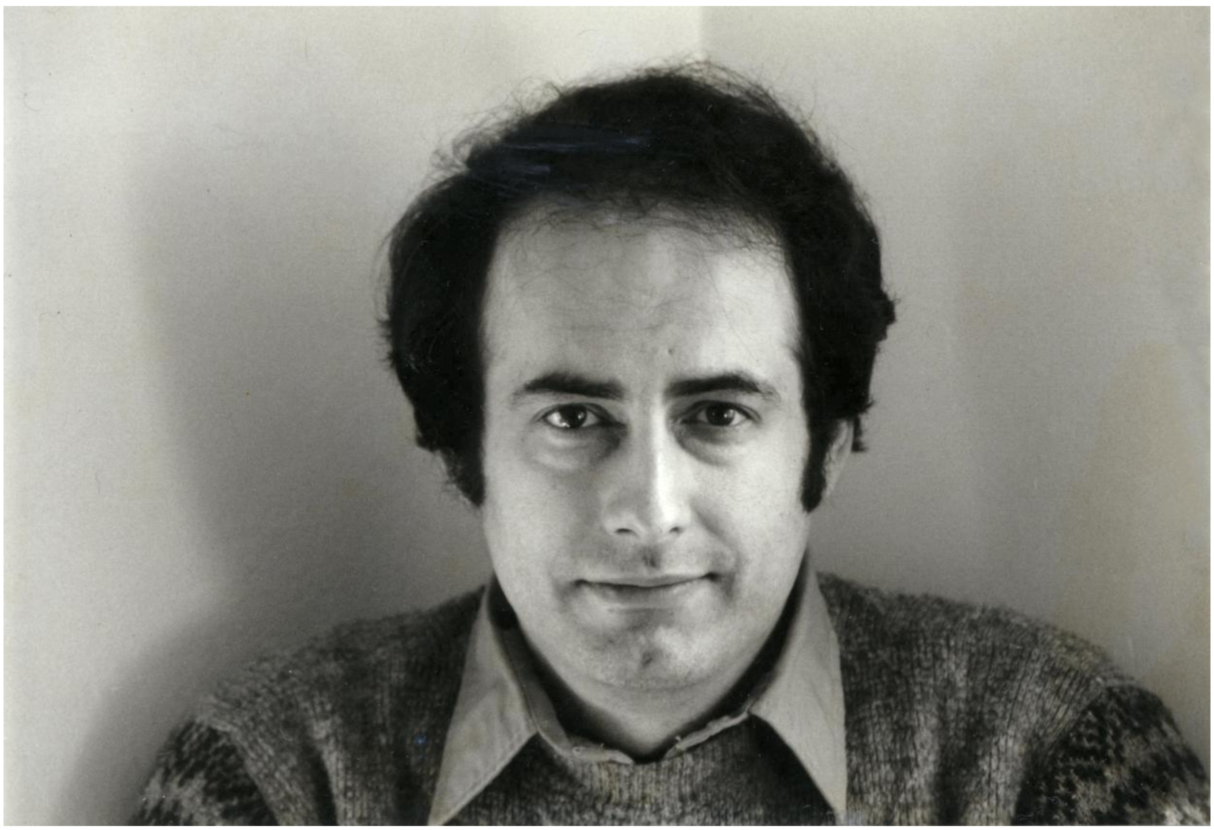
$$r = \frac{L^2}{m_e k_e e^2} \quad \Rightarrow \quad r = \frac{4\pi\varepsilon_o L^2}{m_e e^2}$$

With $L = m_e v r = n\hbar$ you get

$$r_n = \frac{4\pi\varepsilon_o n^2\hbar^2}{m_e e^2} = \frac{4\pi\varepsilon_o n^2}{m_e e^2}\frac{h^2}{4\pi^2} \quad \Rightarrow \quad \boxed{r_n = \frac{\varepsilon_o n^2 h^2}{\pi m_e e^2}}$$

THE END

When I last taught Optics I looked like this.



Have a Great Break.