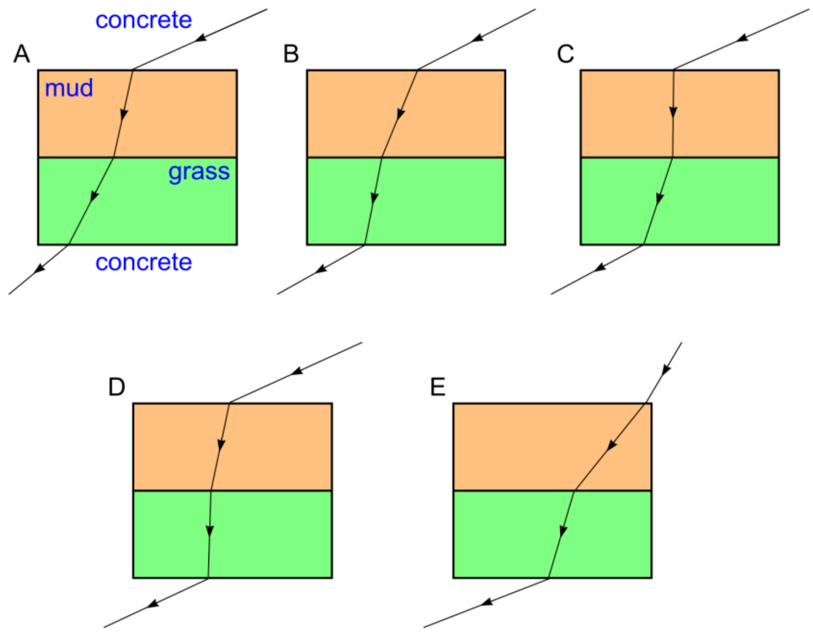
Modern Optics, Prof. Ruiz, UNCA SOLUTIONS Fall 2020

Exam 1 Closed Book, Closed Notes, Closed Everything, HONOR CODE
NOTE EXCEPTION: For this specific exam, a calculator is permitted.

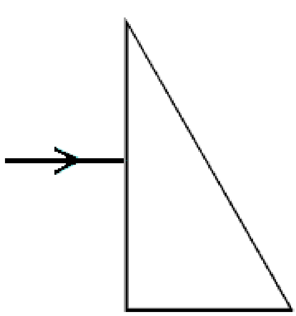
[30 pts] Multiple Choice GRE Style.

MC1. Which path below will take the least time if the

speed on concrete > speed on grass > speed on mud.



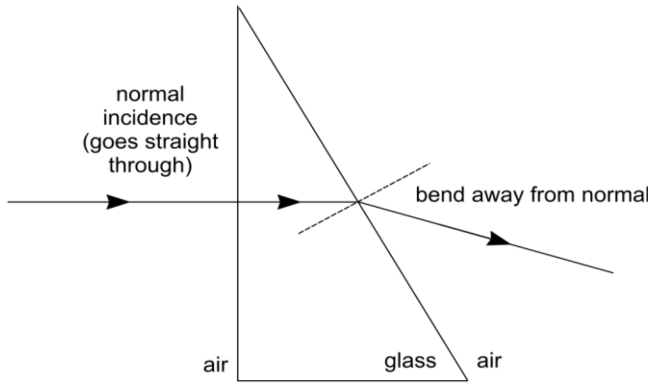
Answer MC1: Choice A. In class we learned that for the path of least time when there are media with different speeds, Snell's Law is satisfied at a boundary, i.e., the interface. Choice C is ruled out since the angle of refraction in the mud coming from the concrete is zero degrees. A zero angle of refraction would mean zero angle of incidence on the upper concrete side, which from the figure is clearly not the case. Choice D can be ruled out for a similar reason with its zero angle of refraction in the grass. When you go from a fast (concrete) to a slow medium (mud), you bend towards the normal, which rules out E. We are left with A and B. But going from slow (mud) to faster (grass) must bend away from the normal, as seen in A.



MC2. The ray of light shown entering the glass will ultimately leave the piece of glass traveling

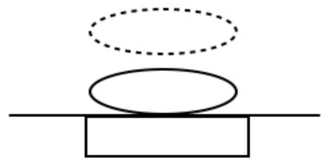
(A) somewhat downward
(B) in the same direction
(C) somewhat upward.

Answer MC2: Choice A.

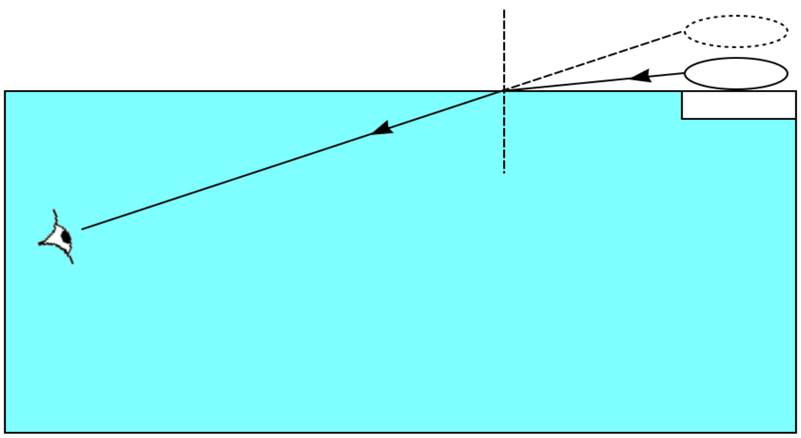


MC3. A bar of soap is sitting on a soap stand at one end of a bath tub, where the water level just covers the rectangular soap stand. You are laying in the bathtub of water with your head at the opposite end. Sketch to the right (not to turn in) what you see if you slip down under the water (with goggles so you can see better). Before you drown, you will see the soap

(A) shifted upward above the surface.
(B) appearing to be in its actual location.
(C) shifted downward below the surface.

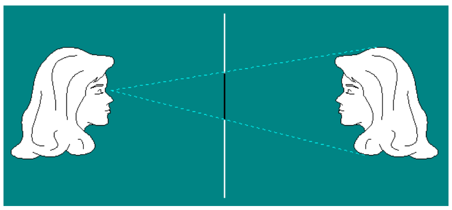


Answer MC3: Choice A.



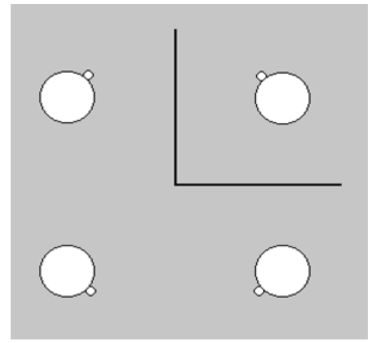
MC4. You pull out a pocket 4 cm x 4 cm plane mirror and look at your face. You will be able to see at most

(A) 1 cm x 1 cm of your face.
(B) 2 cm x 2 cm of your face.
(C) 4 cm x 4 cm of your face.
(D) 8 cm x 8 cm of your face.
(E) 16 cm x 16 cm of your face.



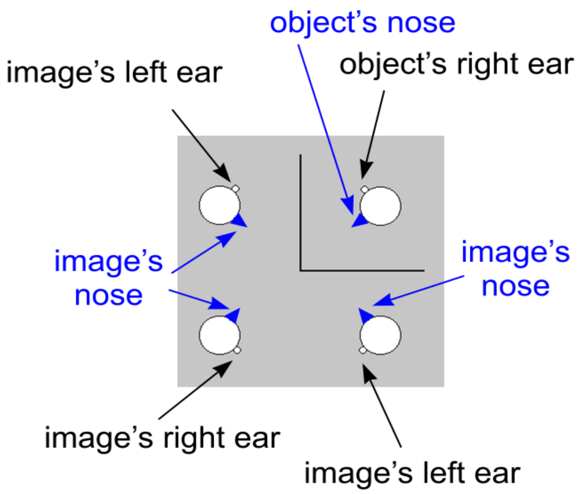
Answer MC4: Choice D. See figure from text reproduced here showing mirror region half the linear dimension of the face.

MC5. You are visiting a hair stylist. Two walls have mirrors joined at a corner. You see three reflected images of yourself. You look toward the corner of the corner-mirror to observe the central image. You then touch your right cheek with your right hand.

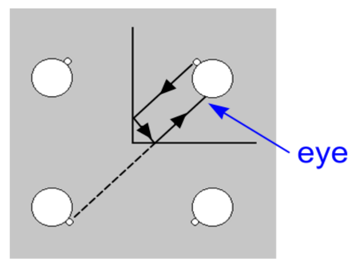


(A) The central image's left hand touches the image's left check.
(B) The central image's left hand touches the image's right check.
(C) The central image's right hand touches the image's left check.
(D) The central image's right hand touches the image's right check.
(E) The central image's left hand slaps you in the face.

Answer MC5: Choice D. I drew in a nose to help me get left and right correct for all the images.



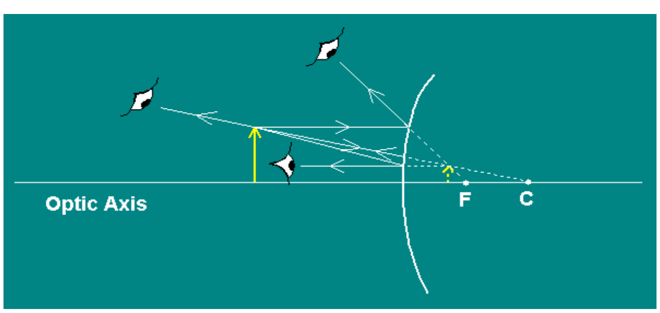
Looking at the image straight in front of you is seeing yourself how others see you. If you move your right hand, the central image moves its right hand. S we pick the choice that matches what the object does.

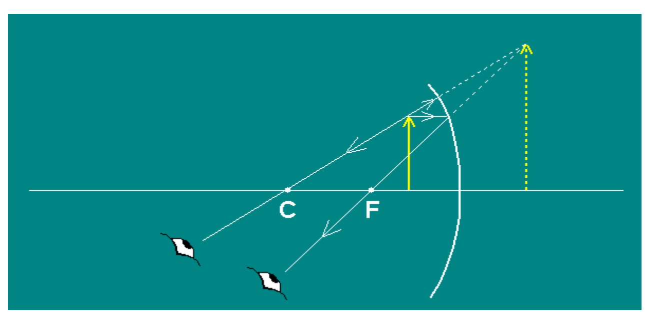


MC6. The image of the photographer is

(A) between the photographer-observer and the mirror.
(B) on the mirror.
(C) behind the mirror.

Answer MC6: Choice C. Say I am not sure if we have a convex or concave mirror here. It doesn't matter. The image is behind the mirror in each case.



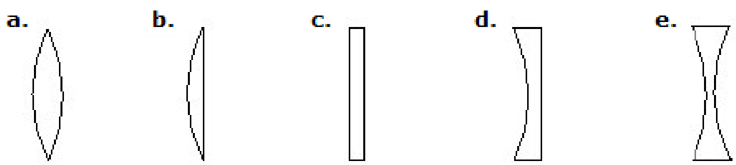


MC7. Which is true for an upright object and vertically-oriented mirror whether that mirror be convex, concave, or planar.

(A) Some real images formed are upright.
(B) All virtual images formed are behind the given mirror being used.
(C) The virtual images formed in some cases are inverted.
(D) All real images, when formed by the appropriate mirror, are smaller.

Answer MC7: Choice B. We rule out A since for a single concave mirror all real images are inverted. Convex and plane mirrors only produce virtual images. B is the one. We can rule out C since all virtual images produced by a single mirror are upright. We can rule out D since real images can be smaller, the same size, or larger than the object.

MC8. Which lens has the shortest positive focal length?



Answer MC8: Choice A. The surfaces both have converging effects, reinforcing convergence. Lens B is weaker due to the neutral planar surface. The focal length for B is longer. Lens C has an infinite focal length as parallel rays pass straight through remaining parallel. Lenses D and E are diverging and therefore have negative focal lengths.

MC9. The wooden rail of the deck is 2 meters from the lens. The image is

(A) between the observer and lens and less than 2 meters from the lens.

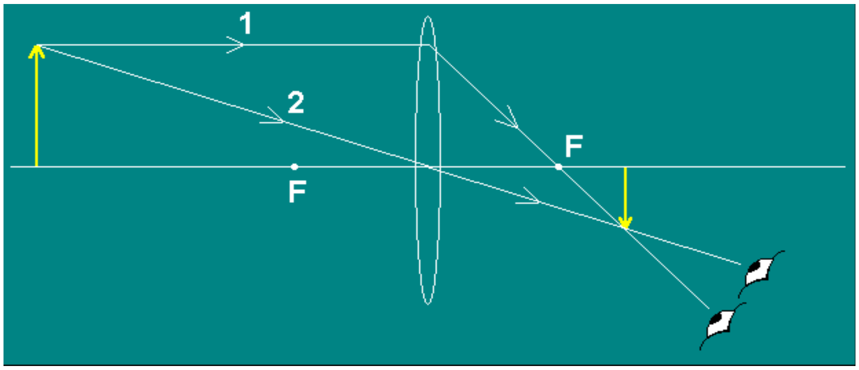
(B) between the observer and lens and greater than 2 meters from the lens .

(C) between the lens and wooden rails.

(D) beyond the wooden rails.

(E) none of these.

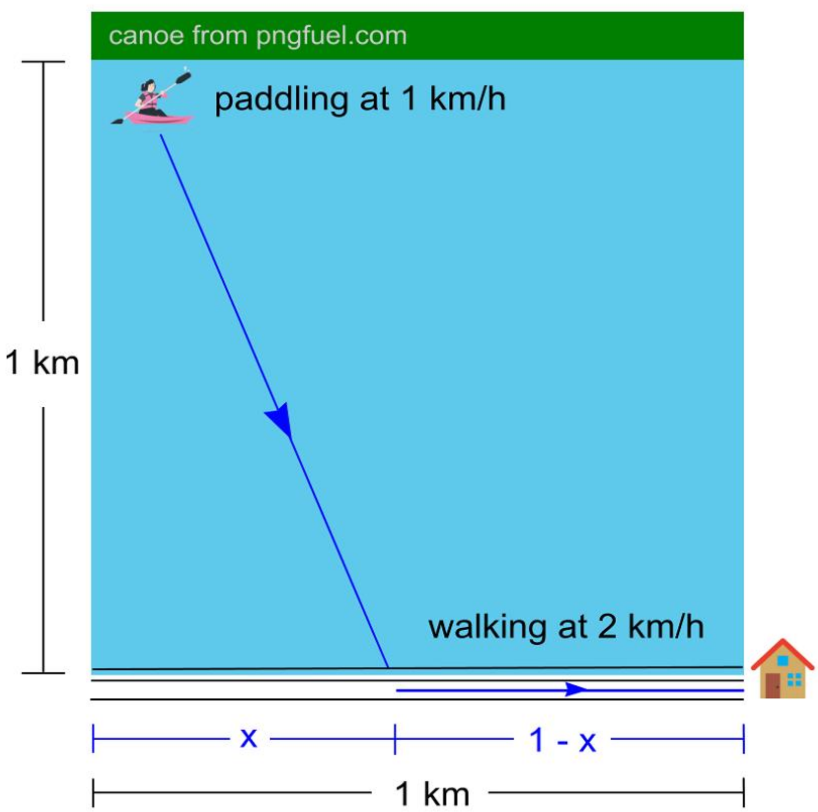
Answer MC9: Choice A. Since the rail is inverted and we have a one-lens problem, the image must be real. All real images are in positive image space, the space in this case, being between the lens and us the observer. Since the image is smaller, the object must be at a distance from the lens greater than 2f and the associated image must be between f and 2f in the space between the lens and us. Therefore, s_o = 2 meters means 2 meters > 2f and f < 1 meter. Since we agreed that the image is at s_i where $f < s_i < 2f$, then si < 2 meters, choice A. We have a case similar to the figure below, where the real image is smaller than the object.



MC10. For a given scene a good picture is taken at f/11 at 1/30 second. What f/# should be used at 1/60 second where the film or sensor sensitivity is the same?

(A) f/4 (B) f/5.6 (C) f/8 (D) f/11 (E) f/22

Answer MC10: Choice C. If we go from 1/30 s to 1/60 s, we are halving the time and thus, the amount of light that gets to the film. So we need to compensate by doubling the area of the aperture Then, twice the amount of light entering, but for half as long, results in the same amount of total light bathing the film during the exposure. Changing f-number by one stop in the direction of increasing aperture does the trick. So we click from f/11 to f/8. Note that the answer choices here the f-numbers without any being skipped. Smaller openings are to the right in the answer array and larger apertures are to the left.



[20 pts] A Trip Home.

A lady is paddling in a canoe at 1 km/h and then will walk the rest of the way home at 2 km/h. For the shortest trip in time, what should x be when she touches land?

Give your answer in simplest form with whatever you have, e.g., fractions, radicals, etc. Be sure that the answer is in simplest form.

Then give your answer to two significant figures.

Always include units with answers.

Neglect the width of the street.

SOLUTION

We will use distance = velocity x time.

$$d = vt$$

Time for trip: $t = \frac{\sqrt{x^2+1}}{1} + \frac{1-x}{2}$, using $t = \frac{d}{v}$ for each part of the trip.

Minimize: $0 = \frac{dt}{dx} = \frac{1}{2}\frac{2x}{\sqrt{x^2+1}} - \frac{1}{2}$

$$\frac{x}{\sqrt{x^2+1}} = \frac{1}{2}$$

$$\frac{x^2}{x^2+1} = \frac{1}{4}$$

$$4x^2 = x^2 + 1$$

$$3x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$\boxed{x = \frac{1}{\sqrt{3}}}$$

$$x = 0.5774$$

$$\boxed{x = 0.58 \text{ km}}$$

[20 pts] The Boy and His Balloon. My son Evan is holding a balloon at a party held for the UNCA Science Faculty and Staff back in the 1980s. He sees a reflection of himself due to the inflated balloon.

In any distance units of your choice, calculate the focal length of the balloon's reflecting surface by making appropriate estimates of the relevant parameters you need for the basic optics formula for curved reflecting surfaces.

Explain the reasoning behind each estimate you make as you examine the photo. What is the radius of curvature for the reflecting surface? Give your focal length and radius of curvature to two significant figures.

SOLUTION. Well, if I hold out my arm like the boy, my hand is about 50 cm = 20 inches, i.e., about a foot and a half from my face. For the boy I am going to use 30 cm = 12 inches = 1 foot.

$$s_o = 30 \text{ cm}$$

By holding up my little finger to the image on my large screen, I estimate that the linear dimension of the boy's face is 2.5 times the linear dimension of the image.

$$M = -\frac{s_i}{s_o} = +\frac{1}{2.5}$$

$$-\frac{s_i}{30} = +\frac{1}{2.5} \quad \Rightarrow \quad s_i = -\frac{30}{2.5} = -\frac{30}{5/2} = -\frac{30 \cdot 2}{5} = -12 \text{ cm}$$

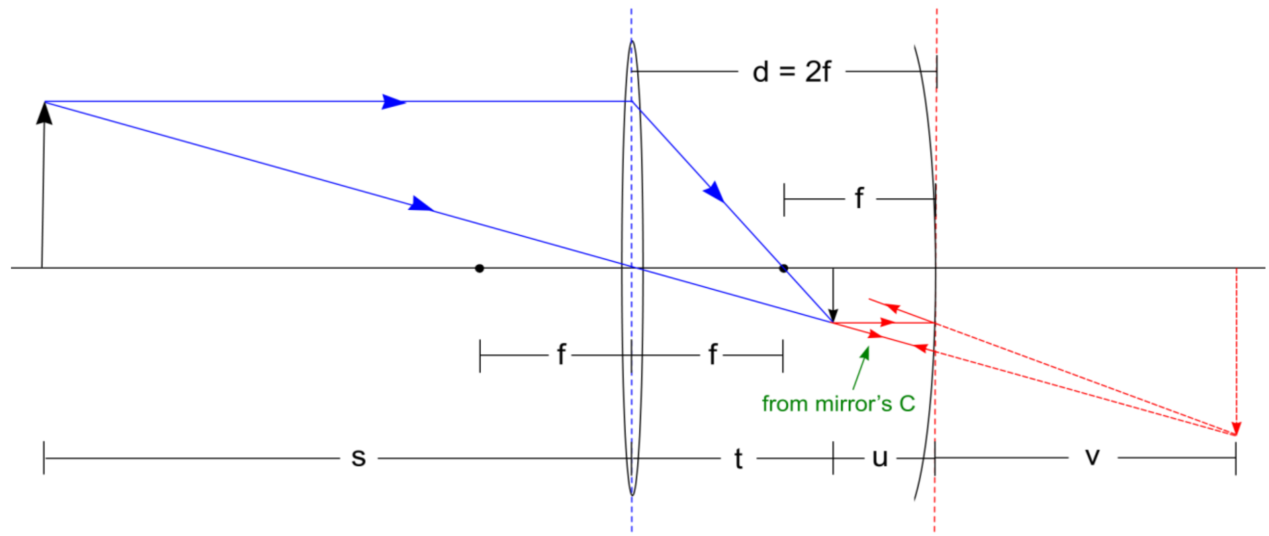
$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{30} + \frac{1}{(-12)} = \frac{12-30}{(30)(12)} = -\frac{18}{360} = -\frac{1}{20}$$

$f = -20 \text{ cm}$ $\quad$ $R = 2f = -40 \text{ cm}$ $\quad$ For R, will accept absolute magnitude.

To 2 significant figures: $\boxed{f = -20 \text{ cm}}$ $\quad$ $\boxed{R = 40. \text{ cm}}$

These answers correspond to f = 8 inches and R = 2f = 16 inches.

[30 pts] Lens-Mirror System. The diagram below is only for a specific object distance s.



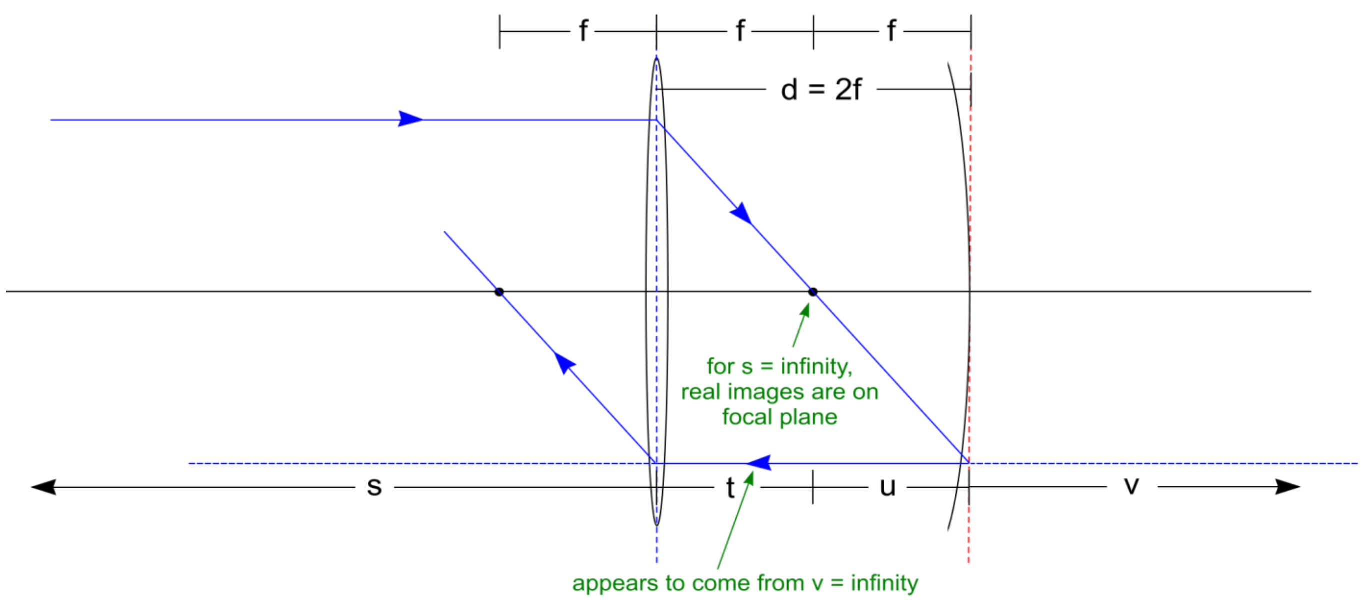
(a) Find v when $s \to \infty$. [5 pts]

(b) Find v when $s = f$. [5 pts]

(c) Find v when $s = 3f$. [10 pts]

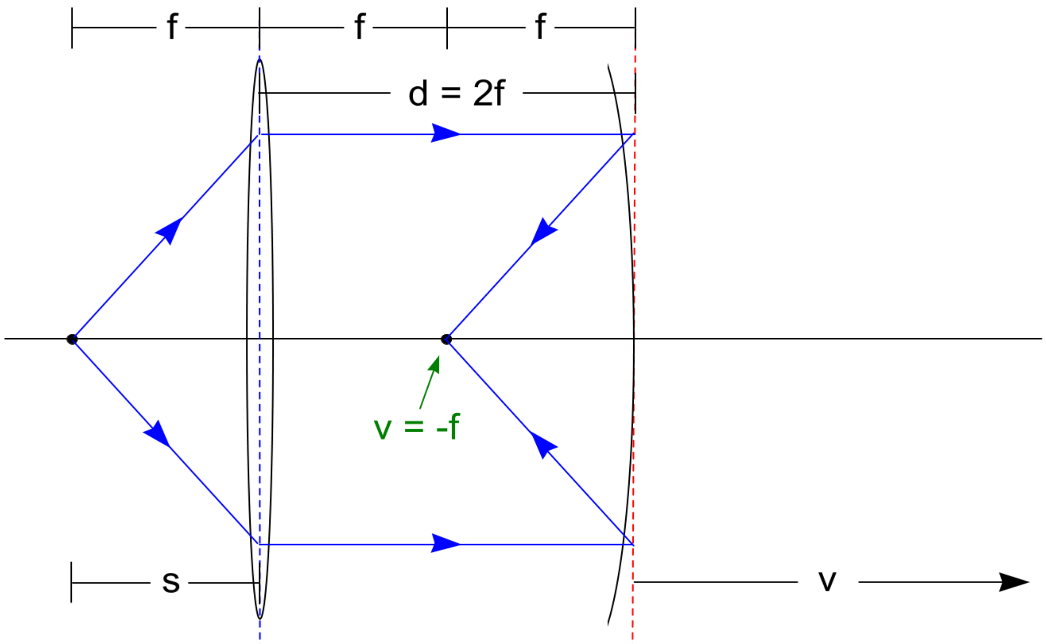
(d) Derive a formula for v in terms of s and f, i.e., find $v = v(s, f)$. [10 pts]

SOLUTION. (a) Find v when $s \to \infty$.



From a quick sketch, the ray leaving the mirror appears to come from $\boxed{v = \infty}$. But since the reflected ray is heading to infinity at the left, you can also give $v = -\infty$. It's as if the infinity to the left meets the infinity to the right. To understand this strange situation, imagine an object at a distance $s_{o2} \to f + \varepsilon$ from the mirror. Then $s_{i2} \to \text{large}$. The image is far to the left in positive image space. Now nudge the object so $s_{o2} \to f - \varepsilon$. Then $s_{i2} \to -\text{large}$, a virtual image far to the right in negative image space. What happens at f? The rays are parallel.

(b) Find v when $s = f$. From the quick sketch, $\boxed{v = -f}$.



(c) Find v when $s = 3f$. Whenever you have $s_o = 3f$ for a lens, we get from the master equation $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$, the result $\frac{1}{f} = \frac{1}{3f} + \frac{1}{s_i}$. Now solve for s_i. We find

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{3f} = \frac{1}{f}\left[1 - \frac{1}{3}\right] = \frac{1}{f}\frac{2}{3} \quad \text{and} \quad s_i = \frac{3}{2}f = 1.5f.$$

The image will serve as the object at distance $s_{o2} = 0.5f$ from the mirror.

For the mirror: $$\frac{1}{f} = \frac{1}{s_{2o}} + \frac{1}{s_{2i}} = \frac{1}{0.5f} + \frac{1}{s_{2i}}.$$

$$\frac{1}{s_{2i}} = \frac{1}{f} - \frac{1}{0.5f} \quad => \quad \frac{1}{s_{2i}} = \frac{1}{f} - \frac{2}{f} = -\frac{1}{f} \quad => \quad s_{2i} = -f$$

The image is virtual and behind the mirror as expected.

$$\boxed{v = f}$$

(d) Derive a formula for v in terms of s and f, i.e., find $v = v(s, f)$.

Lens: $\frac{1}{f} = \frac{1}{s} + \frac{1}{t}$ Mirror: $\frac{1}{f} = \frac{1}{u} + \frac{1}{(-v)}$ $u = d - t = 2f - t$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{t} \quad \Rightarrow \quad \frac{1}{f} = \frac{t+s}{st}$$

$$\frac{1}{f} = \frac{t+s}{st} \quad \Rightarrow \quad st = (t+s)f \quad \Rightarrow \quad st = tf + sf$$

$$st - tf = sf \quad \Rightarrow \quad t(s-f) = sf \quad \Rightarrow \quad t = \frac{sf}{s-f}$$

Summary for Lens: $\frac{1}{f} = \frac{1}{s} + \frac{1}{t}$ and $t = \frac{sf}{s-f}$

For Mirror: $\frac{1}{f} = \frac{1}{u} + \frac{1}{(-v)}$ and $v = \frac{-uf}{u-f}$ by analogy to the lens.

The connecting equation is $u = d - t = 2f - t$.

Then $u = 2f - \frac{sf}{s-f}$, where we substituted $t = \frac{sf}{s-f}$.

Eventually, this u will be substituted in $v = \frac{-uf}{u-f}$.

$$u = 2f - \frac{sf}{s-f} = \frac{(s-f)2f - sf}{s-f}$$

$$u = \frac{2sf - 2f^2 - sf}{s-f}$$

$$u = \frac{sf - 2f^2}{s - f}$$

Now we are ready for $v = \dfrac{-uf}{u - f}$. Remember that our goal is $v = v(s, f)$.

$$v = \frac{-\dfrac{sf - 2f^2}{s - f}\, f}{\dfrac{sf - 2f^2}{s - f} - f}$$

Multiply top and bottom by $s - f$.

$$v = \frac{-(sf - 2f^2)f}{sf - 2f^2 - (s - f)f}$$

An f cancels everywhere.

$$v = \frac{-(sf - 2f^2)}{s - 2f - (s - f)}$$

$$v = -\frac{(sf - 2f^2)}{s - 2f - s + f}$$

$$v = -\frac{(sf - 2f^2)}{-f}$$

$$\boxed{v = s - 2f}$$

We are finished, but let's check our master formula with the three cases we answered. We found (a) $s = \infty$, $v = \infty$; (b) $s = f$, $v = -f$; (c) $s = 3f$, $v = f$.

(a) For $s = \infty$ the function $v(s, f) = s - 2f$ gives $v(\infty, f) = \infty - 2f = \infty$.

(b) For $s = f$ the function $v(s, f) = s - 2f$ gives $v(f, f) = f - 2f = -f$.

(c) For $s = 3f$ the function $v(s, f) = s - 2f$ gives $v(3f, f) = 3f - 2f = f$.