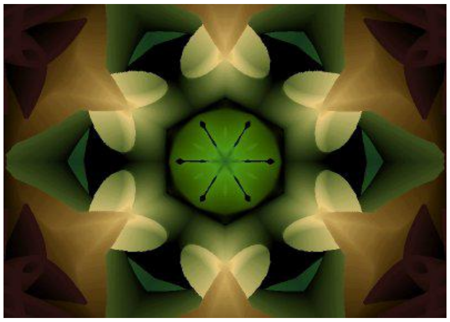
Modern Optics, Prof. Ruiz, UNCA **Fall 2020**

Exam 3 Closed Book, Closed Notes, Closed Everything, HONOR CODE
NOTE EXCEPTION: For this specific exam, a calculator is permitted.

Place a Box around all Final Answers for Each Part of the Subjective Questions.
For Full Credit in the Subjective Questions Show All Work.



[30 pts] Multiple Choice GRE Style.

MC1. What is the angle between the two mirrors for this kaleidoscope? If you believe there is more than one angle that could be the answer, select the one with the smallest angle.

(A) 30° (B) 36° (C) 45° (D) 60° (E) 90°

MC2. You have access to three types of polarizes:

V – a polarizer aligned for transmission of vertically polarized light
H – a polarizer aligned for transmission of horizontally polarized light
S – slanted polarizer aligned such that its polarization is 45° with respect to either of the other polarizers V and H.

The incident light from the Sun is unpolarized. You can say it has mixed polarization, i.e., polarizations in all possible directions superimposed. The light enters three polarizers that are placed one behind the other. Below are 5 cases of such filter arrangements where in two of the cases you use two of one type of the polarizers. For which case below will some light manage to get through all three filters?

(A) VVH (B) VHV (C) VHS (D) VSH (E) SVH

MC3. Unpolarized light with an intensity of 1000 units enters a polarizer. It then passes through a second polarizer that is rotated such that its polarization axis is 45° with respect to that of the first. The intensity of the light after passing through the two polarizers is

(A) 0 units (B) 250 units (C) 500 units (D) 750 units (E) 1000 units.

MC4. Four classical scenarios of a free electron in a laboratory setting are described below.

I. An electron is at rest.
II. An electron is traveling along the x-axis at constant speed.
III. An electron is accelerating along the x-axis.
IV. An electron is traveling along a circular path, i.e., along a circumference.

For which of the following cases does the electron emit electromagnetic radiation.

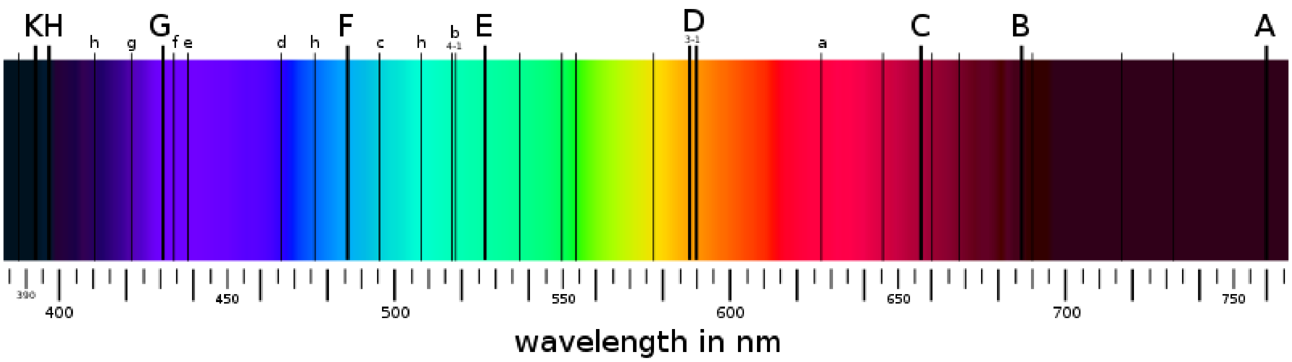
(A) I and II only (B) II, III, and IV only (C) II and III only
(D) III only (E) III and IV only

MC5. Which vector below has zero divergence for all x and y?

(A) $x\hat{i} + y\hat{j}$ (B) $x\hat{i} - y\hat{j}$ (C) $x^2 y\hat{i}$ (D) $\cos x\hat{i} + \cos y\hat{j}$ (E) $e^x\hat{i} - e^y\hat{j}$

MC6. An observer holds up a diffraction grating to view a white light source across the room. The observer then looks at one of the first maxima which appear on either side of the central maximum, i.e., left or right. The color in the first maximum that is closest to the central beam is (A)blue (B)green (C) red.

MC7. A spectrum can be observed using a spectrometer, where one type of design employs a diffraction grating. The ability to distinguish closely spaced lines is called the resolution.



Suppose that two nearby wavelengths λ_1 and $\lambda_2 > \lambda_1$ can just be distinguished, i.e. barely resolved. Define $\lambda_{avg} = \dfrac{\lambda_1 + \lambda_2}{2}$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Which of the following could be used as a measure of resolving ability or power? A high resolving power means you can distinguish close wavelengths better.

(A) $\lambda_{avg} + \Delta\lambda$ (B) $\lambda_{avg} - \Delta\lambda$ (C) $\lambda_{avg}\Delta\lambda$ (D) $\dfrac{\lambda_{avg}}{\Delta\lambda}$ (E) $\dfrac{\Delta\lambda}{\lambda_{avg}}$

MC8. For $\alpha = \dfrac{5\pi}{2N}$ where N is very large, the approximation for $\left[\dfrac{\sin(N\alpha)}{N\sin\alpha}\right]^2$ is

(A) $\left[\dfrac{2}{5\pi}\right]^2$ (B) $\left[\dfrac{2N}{5\pi}\right]^2$ (C) $\left[\dfrac{5\pi}{2}\right]^2$ (D) $\left[\dfrac{5\pi}{2N}\right]^2$ (E) $\left[\dfrac{10}{\pi}\right]^2$

MC9. A wave has an amplitude $E_p = E_o(1+i)$. Which answer below gives the irradiance?

(A) $\dfrac{E_o^2}{4}$ (B) $\dfrac{E_o^2}{2}$ (C) E_o^2 (D) $2E_o^2$ (E) $4E_o^2$

MC10. The irradiance due to blocking all Fresnel zones of a wavefront except the first zone is found to be $I_1 > 0$ at some point on the central axis of the wavefront. Now you block all zones except the first two. The irradiance will (A) decrease (B) remain the same (C) increase.

[20 pts] P1. Fresnel Equations. In class we derived the following.

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \qquad r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

For this problem light is going from air ($n_1 = 1$) into a medium with index of refraction $n_2 = n$.

(a) Calculate $R_s = r_s^2$ in terms of n for $R_p = r_p^2 = 0$.

(b) What is R_s as an exact fraction in its most reduced form if n is exactly $\frac{3}{2}$?

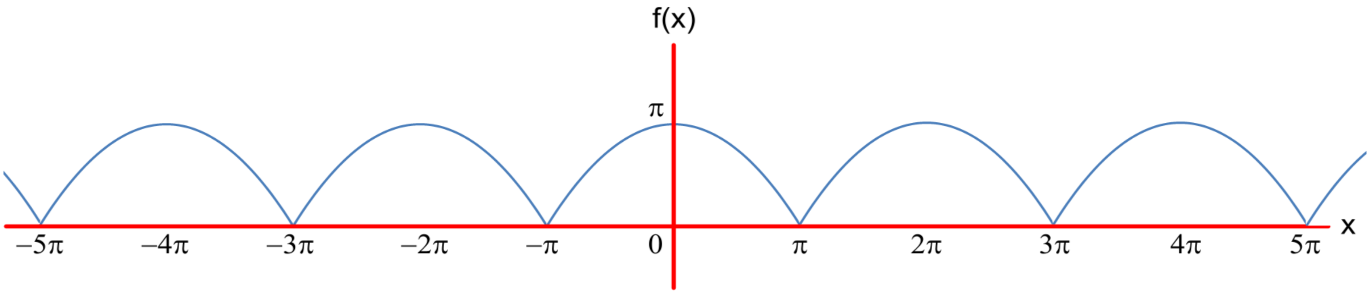
(c) Give your answer to (b) to three significant figures.

[20 pts] P2. Fourier Series. The basic formulas for a Fourier Series are given below.

$$f(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left[a_m \cos(mx) + b_m \sin(mx)\right]$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$



The periodic wave shown above is a repeating section of an inverted parabola.

(a) Use the general formula for a parabola $f(x) = ax^2 + bx + c$ to determine a, b, and c to fit the shown function in the interval from $x = -\pi$ to $x = +\pi$, the needed interval for the Fourier integrations. Write out $f(x)$ with your values for a, b, and c.

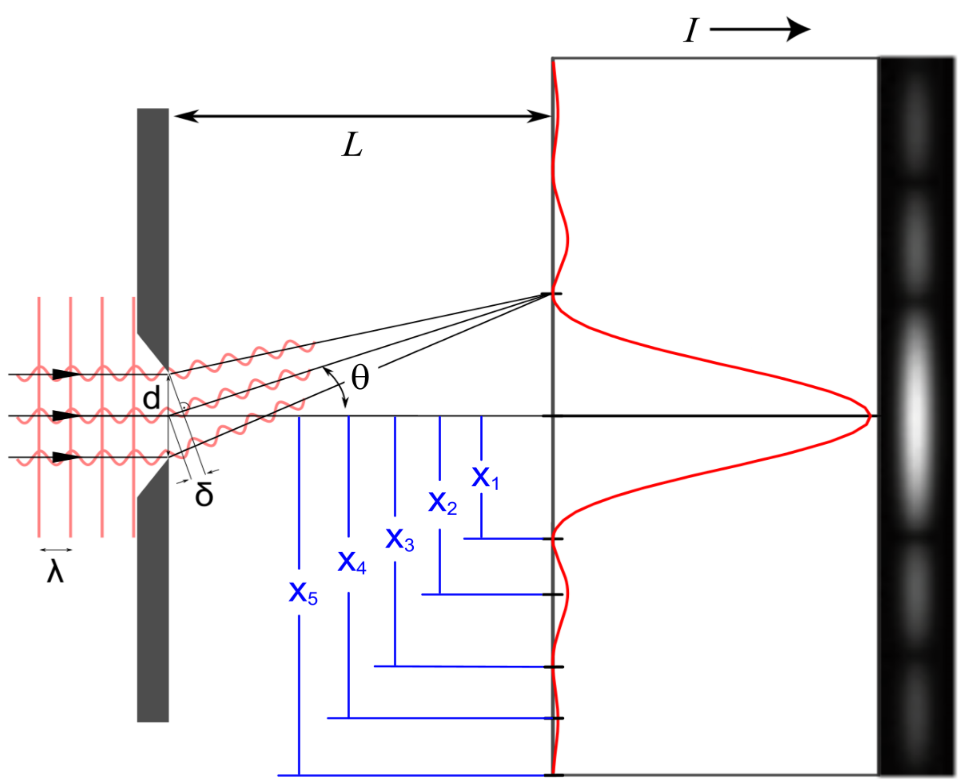
(b) Which of the Fourier coefficients are zero?

(c) Calculate a_0 only and you are finished with this problem. Give your answer in the simplest exact form.

[20 pts] P3. Diffraction. Monochromatic light with wavelength λ goes through a slit with width $d = 160\lambda$ and reaches a screen a distance $L = 1\text{ m}$ after passing through the slit. The single-slit diffraction formulas are

$$I_r \equiv \frac{I(\theta)}{I_o} = \frac{\sin^2\beta}{\beta^2} \qquad \text{and} \qquad \beta = \frac{1}{2}kd\sin\theta,$$

where θ is the usual angle measured from the center of the slit referenced to the axis joining the slit center and screen center (see figure). The two maxima x_2 and x_4 are not quite in the exact middle between their neighboring minima, but you may take the two maxima to be exactly in the middle of their neighboring minima to get fast results. Therefore, for example, you can use the very good approximations that $x_2 = \frac{1}{2}(x_1 + x_3)$ and $\beta_2 = \frac{1}{2}(\beta_1 + \beta_3)$.



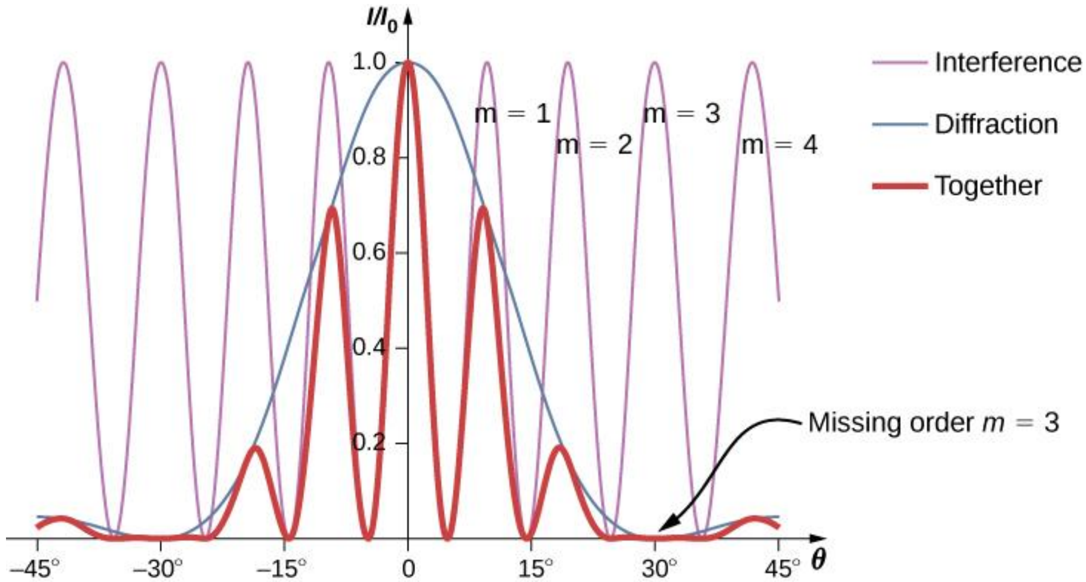
Adapted from Wikipedia: jkrieger.

Complete the table below to two significant figures. You only need to do x_1 and x_2.
But note that you will need β_3 to complete the entry for 2.

	1	2	3	4	5
***x* (in mm)**			not applicable	not applicable	not applicable
I / I_o			not applicable	not applicable	not applicable

[10 pts] P4. Interference. Light with wavelength λ enters two closely spaced slits where the distance between the centers of each slit is $a = 6\lambda$ and the width of each slit is $b = 2\lambda$. The relevant equations for the irradiance and associated parameters are

$$I_r \equiv \frac{I(\theta)}{I_o} = \cos^2\alpha \frac{\sin^2\beta}{\beta^2}, \quad \alpha = \frac{1}{2}ka\sin\theta, \text{ and } \beta = \frac{1}{2}kb\sin\theta.$$



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The first maximum to the right of the central maximum is slightly to the left of location of the first maximum m = 1 due to the interference factor. You may neglect this slight difference and consider the "together" max to be exactly at m = 1. Find the relative irradiance I_r for this "together" peak at the m = 1 interference location. Give your answer to three significant figures.