Class G Ideal Gas Law + Thermodynamics

G1. Ideal Gas Law

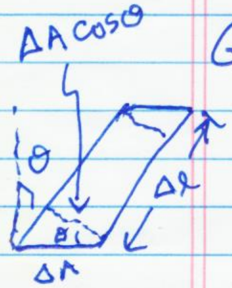
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \left\{ \begin{array}{llll} \text{Boyle} & P \sim \frac{1}{V} & P_1V_1 = P_2V_2 & \Delta T = 0 \\ \text{Charles} & V \sim T & \frac{V_1}{T_1} = \frac{V_2}{T_2} & \Delta P = 0 \\ \text{Gay-Lussac} & P \sim T & \frac{P_1}{T_1} = \frac{P_2}{T_2} & \Delta V = 0 \end{array} \right.$$

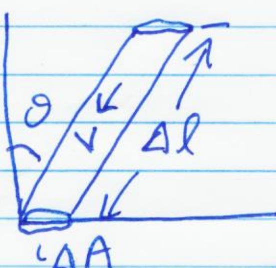
T (Kelvin) └ $T=0$ absolute zero

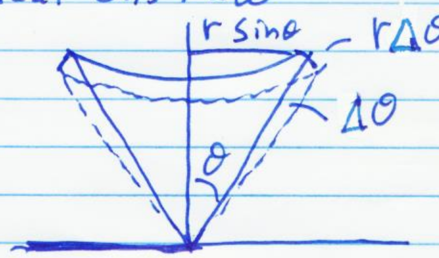
$PV = nRT$ $\qquad$ $PV = NKT$

└ # moles $n \equiv \frac{N}{N_A}$ ← # particles; N_A ← Avogadro's No. └ 6.022×10^{23}

G2. "Derivation" of the Ideal Gas Law







Bit of pressure

$F = \frac{\Delta p}{\Delta t}$ — momentum $p = mv$ $\qquad$ Pressure $P = \frac{F}{A}$

Involves number

$$\Delta P = \underbrace{\frac{N}{V}}_{\text{Total}} \left(\Delta A \Delta l \cos\theta\right) \left[\tfrac{1}{2} \underbrace{f(v)\Delta v}_{\text{Prob. } v \text{ to } v+\Delta v}\right] \left[\frac{2\pi r \sin\theta \, r\Delta\theta}{2\pi r^2}\right] \left[\frac{2mv\cos\theta}{\Delta t}\right] \frac{1}{\Delta A}$$

$\frac{1}{2}$ going up / down

(hemisphere fraction between θ + $\theta + \Delta\theta$)

per Area

Force

Change in momentum is $2mv\cos\theta$

Note: $\frac{\Delta l}{\Delta t} = v$

Note: Quantum Mechanics later teaches us that the lowest energy state has minimal motion.

$$\Delta P = \frac{N}{V} \frac{\Delta A}{\Delta A} \frac{\Delta l}{\Delta t} \frac{1}{2} f(v) \, \Delta v \cos^2\theta \sin\theta \, \Delta\theta \, 2mv$$

← No Relativity (could replace with c) Speed of Light

Temperature

$T \sim \frac{1}{2} m\overline{v^2}$ average kinetic energy of particles

This definition gives an absolute scale.

No Motion ⟹ $T=0$ "classical"

$$P = \frac{N}{V} m \underbrace{\int_0^\infty v^2 f(v)\,dv}_{\overline{v^2}} \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta$$

$$-\frac{\cos^3\theta}{3}\Big|_0^{\pi/2} = \frac{1}{3}$$

$PV = \frac{N}{3} m\overline{v^2}$ $\qquad$ Define T so $PV = NKT$

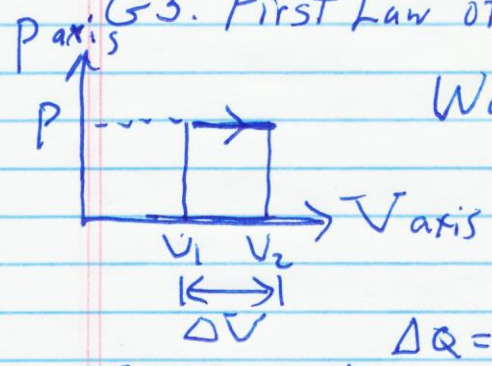
$KT = \frac{1}{3} m\overline{v^2}$ $\qquad$ $\frac{1}{2} m\overline{v^2} = \frac{3}{2} KT$

$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$ → $\frac{1}{2}kT$ for each dimension or degree of freedom

$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}kT$ ← Equipartition Theorem

G3. First Law of Thermodynamics

$$\underbrace{\Delta U}_{\text{Change in Energy}} = \underbrace{\Delta Q}_{\text{Heat Goins in}} - \underbrace{\Delta W}_{\text{work Done by expansion}}$$



$$\text{Work } \Delta W = F\Delta x = PA\Delta x = P\Delta V$$

$$U = N\frac{1}{2}m\overline{v^2} = \frac{3}{2}\underbrace{Nk}_{nR}T = \frac{3}{2}nRT$$

$$\Delta Q = \Delta U + P\Delta V \qquad \Delta Q_V = \Delta U$$

$_V$ ← constant volume

Specific heat at constant volume

$$\hookrightarrow c_V = \frac{1}{n}\frac{\Delta Q_V}{\Delta T} = \frac{1}{n}\frac{\Delta U}{\Delta T} = \frac{1}{n}\frac{3}{2}nR\frac{\Delta T}{\Delta T} = \frac{3}{2}R$$

definition

G4. Four Thermodynamic Processes

1. Isometric $\Delta V = 0 \qquad W = \int P\,dV = 0$

2. Isobaric $\Delta P = 0 \qquad W = \int_{V_1}^{V_2} P\,dV = P(V_2 - V_1) = P\Delta V$

3. Isothermal $\Delta T = 0$

$$W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}dV = nRT\int_{V_1}^{V_2}\frac{1}{V}dV = nRT\ln V\Big|_{V_1}^{V_2}$$

$$W = nRT\ln\frac{V_2}{V_1}$$

$\Delta U = \Delta Q - P\Delta V$ 4. Adiabatic $\Delta Q = 0$

$$\hookrightarrow \overbrace{\Delta Q}^{\text{zero}} = \Delta U + P\Delta V = nc_V\Delta T + P\Delta V$$

$$PV = nRT \Rightarrow P\Delta V + V\Delta P = nR\Delta T$$

from

$$\hookrightarrow nc_V\Delta T = -P\Delta V$$

$$\overbrace{\Delta Q}^{\text{zero}} = \underbrace{nc_V\Delta T + nR\Delta T}_{nc_p\Delta T} - V\Delta P$$

from

$$\hookrightarrow nc_p\Delta T = V\Delta P$$

$$\text{Let } \gamma \equiv \frac{c_p}{c_V} \qquad \frac{nc_p\Delta T}{nc_V\Delta T} = \frac{-V\Delta P}{P\Delta V} \Rightarrow \gamma = -\frac{V\,dP}{P\,dV} \qquad \frac{dP}{P} = -\gamma\frac{dV}{V}$$

$$\int_{P_1}^{P_2}\frac{dP}{P} = -\gamma\int_{V_1}^{V_2}\frac{dV}{V} \qquad \ln P\Big|_{P_1}^{P_2} = -\gamma\ln V\Big|_{V_1}^{V_2} \qquad \ln\frac{P_2}{P_1} = -\gamma\ln\frac{V_2}{V_1} = \ln\left[\frac{V_2}{V_1}\right]^{-\gamma}$$

$$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^{-\gamma} \qquad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad P_1V_1^{\gamma} = P_2V_2^{\gamma} = \text{Const}$$