9Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

Chapter C. Partial Pressure. Consider a mixture of gases such as air in a closed container. For the gas mixture we can write

$pV = nR^*T$, where the container has volume V .

We consider gases do not interact chemically with each other. Let n_i be the number of moles for the i[th] gas constituent. The total number of moles is then given by

$$n = n_1 + n_2 + \ldots .$$

For ideal gases we treat each particle of the gas as a point particle that interacts with the walls of the container. The total pressure can be expressed as a sum of the pressures due to each gas constituent. This addition of pressures is called *Dalton's law of partial pressures*. Let p_i be the pressure due to the i[th] gas constituent. The total pressure is then given by

$$p = p_1 + p_2 + \ldots$$

Substituting $p = p_1 + p_2 + \ldots$ and $n = n_1 + n_2 + \ldots$ in $pV = nR^*T$ gives us

$$(p_1 + p_2 + \ldots)V = (n_1 + n_2 + \ldots)R^*T$$

since each gas constituent has the same volume V of the container.

We can rewrite the above equation as

$$p_1V + p_2V + \ldots = n_1R^*T + n_2R^*T + \ldots .$$

Since the constituents are independent of each other, we have

$p_iV = n_iR^*T$ for each of the constituents.

We now perform the same trick we did in the previous chapter.

$$p_iV = n_iR^*T \quad \Rightarrow \quad p_i\frac{V}{m_i} = \frac{n_i}{m_i}R^*T \quad \Rightarrow \quad p_i\alpha_i = \frac{n_i}{m_i}R^*T \quad \Rightarrow \quad p_i\alpha_i = \frac{R^*}{m_i/n_i}T$$

$$p_i\alpha_i = \frac{R^*}{m_i/n_i}T \quad \Rightarrow \quad p_i\alpha_i = \frac{R^*}{M_i}T \quad \Rightarrow \quad p_i\alpha_i = R_iT$$

The i[th] gas has molecular weight M_i and gas constant R_i.

1. Total Pressure for Moist Air. Meteorologists break air into two constituents: dry air and water vapor. The dry air in turn has constituents (mainly nitrogen and oxygen) and its own associated molecular weight M_d. The vapor has molecular weight given by M_v. Note that the reciprocal of α_i is the density $\rho_i = \frac{m_i}{V}$.

$$\alpha_i = \frac{V}{m_i} \quad \Rightarrow \quad \frac{1}{\alpha_i} = \frac{1}{V/m_i} \quad \Rightarrow \quad \frac{1}{\alpha_i} = \frac{m_i}{V} \quad \Rightarrow \quad \frac{1}{\alpha_i} = \rho_i$$

The ideal gas law $p\alpha = RT$ can be written as $p = \rho RT$.

$$p_i\alpha_i = R_iT \quad \Rightarrow \quad p_i = \rho_i R_i T$$

For dry air and water vapor we can write

$p_d = \rho_d R_d T$ and $p_v = \rho_v R_v T$, where the total pressure is $p = p_d + p_v$.

The practice is to call the water vapor partial pressure $p_v = e$ due to historical reasons.

We come to three new equations for our review list that we are compiling.

(7) $\boxed{p = p_d + e}$ Total pressure of moist air.

We next look at the partial pressures of the dry air and water vapor.

2. Ideal Gas Law for Dry Air. We are ready to give the ideal gas law for dry air which includes the partial pressure for dry air.

(8) $\boxed{p_d = \rho_d R_d T}$ The ideal gas law for the dry air is $p_d\alpha_d = R_dT$. Substitute α_d with $\frac{1}{\rho_d}$.

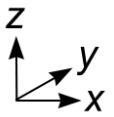
3. Ideal Gas Law for Water Vapor. Next is the ideal gas law for water vapor which includes the partial pressure for water vapor.

(9) $\boxed{e = \rho_v R_v T}$ The ideal gas law for the vapor is $p_v\alpha_v = R_vT$. Substitute α_v with $\frac{1}{\rho_v}$.

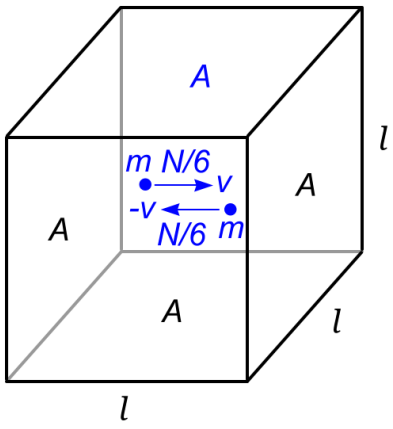
4. Pressure and the Microscopic. This very important section makes a connection between the microscopic (particle collisions against the container walls for a confined gas) and macroscopic quantities such as pressure, volume, and temperature. We reveal the intimate relationship between physics and

chemistry, giving us insight that the temperature (macroscopic) is due to the average kinetic energy of the gas particles (microscopic).

I will present a super simple model for a gas where the particles all move at the same speed v with $\frac{1}{3}$ of the particles moving in each of the three directions x, y, and z.



With this model, I will arrive at important equations for the energy of an ideal gas of point particles and macroscopic quantities such as pressure, volume, and temperature. And are results are the same found when the particles have different speeds and can travel in slanted directions!



In the left figure there are N moving gas particles in a box, where each particle has mass m and speed v. Of the $N/3$ particles moving in the horizontal direction (x), half move to the right and half to the left at any given time, neglecting the instant collisions are made with the walls. Therefore, there are $N/6$ particles moving to the left.

Pressure exists on the left wall as the particles bounce off it. Let time t be the time it takes a particle from the right wall to travel all the way to the left wall. In this time, all $N/6$ particles moving left get a chance to reflect off the left wall. We use Newton's second law "$F = ma$" to find the force on the left wall. The acceleration a is the change of velocity with respect to time at the wall collision.

The initial velocity is $v_i = -v$ (moving left) and the final speed (after the collision) is $v_f = +v$ (moving right). The change in velocity is then

$$\Delta v = v_f - v_i = -v - (-v) = 2v\,.$$

The force due to the $N/6$ particles colliding at the left wall in time t can now be found.

$$F = \frac{N}{6}ma \quad \Rightarrow \quad F = \frac{N}{6}m\frac{\Delta v}{t} \quad \Rightarrow \quad F = \frac{N}{6}m\left(\frac{2v}{t}\right) \quad \Rightarrow \quad F = \frac{N}{3}\frac{mv}{t}$$

Now we need to relate the force F to the pressure using via $p = \frac{F}{A}$.

Substitute $F = pA$ in $F = \frac{N}{3}\frac{mv}{t}$.

$$F = \frac{N}{3}\frac{mv}{t} \quad \Rightarrow \quad pA = \frac{N}{3}\frac{mv}{t}$$

At this point we note that $v = \frac{l}{t}$. So $\frac{1}{t} = \frac{v}{l}$.

With $\frac{1}{t} = \frac{v}{l}$, our equation $pA = \frac{N}{3}\frac{mv}{t}$ can be transformed as follows.

$$pA = \frac{N}{3}\frac{mv}{t} \quad \Rightarrow \quad pA = \frac{N}{3}mv\frac{1}{t} \quad \Rightarrow \quad pA = \frac{N}{3}mv\frac{v}{l} \quad \Rightarrow \quad pAl = \frac{N}{3}mv^2$$

Since the volume of the cube is $V = Al$ we can further transform our equation.

$$pAl = \frac{N}{3}mv^2 \quad \Rightarrow \quad pV = \frac{N}{3}mv^2 \quad \Rightarrow \quad pV = \frac{2N}{3}\frac{mv^2}{2}$$

We desire this form since the kinetic energy of a particle with mass m and speed v is $\frac{1}{2}mv^2$.

$$pV = \frac{2N}{3}\frac{mv^2}{2}$$

The energy of the gas is due to the kinetic energy of all the particles.

$$U = N\frac{1}{2}mv^2$$

From $pV = \frac{2N}{3}\frac{mv^2}{2}$ we obtain $N\frac{1}{2}mv^2 = \frac{3}{2}pV$.

Therefore, we can relate the total energy to the pressure and volume:

$$U = N\frac{1}{2}mv^2 \quad \text{and} \quad N\frac{1}{2}mv^2 = \frac{3}{2}pV \quad \text{lead to} \quad U = \frac{3}{2}pV.$$

Finally, from the ideal gas law $pV = NkT$. The energy can be expressed in terms of temperature.

$$U = \frac{3}{2}pV \quad \Rightarrow \quad U = \frac{3}{2}NkT.$$

Since $Nk = nR^*$, the variable n is moles, we have $U = \frac{3}{2}nR^*T$.

Our two important equations are below.

$$U = N\frac{1}{2}mv^2 \qquad U = \frac{3}{2}nR^*T$$

We can improve our model by considering particles with different speeds.

Consider n_1 with speed v_1, n_2 with speed v_2, and so on.

We can now write the total energy as shown below.

$$U = n_1 \frac{1}{2} m v_1^2 + n_2 \frac{1}{2} m v_2^2 + \ldots$$

$$U = N \frac{\left(n_1 \frac{1}{2} m v_1^2 + n_2 \frac{1}{2} m v_2^2 + \ldots \right)}{N} \quad \Rightarrow \quad U = N \frac{1}{2} m \frac{\left(n_1 v_1^2 + n_2 v_2^2 + \ldots \right)}{N}$$

The part $\frac{\left(n_1 v_1^2 + n_2 v_2^2 + \ldots \right)}{N}$ is the average of v^2, which we can write as $\overline{v^2}$.

When we let the particles travel in slanted directions, we get the same result!

$$\boxed{U = N \frac{1}{2} m \overline{v^2}}$$

Summary

$$\boxed{U = N \frac{1}{2} m \overline{v^2}} \qquad \boxed{U = \frac{3}{2} NkT} \qquad \boxed{U = \frac{3}{2} nR * T}$$

These results ate the same you obtain when you consider particles moving at different angles in general. The above equations are remarkable since we connect the microscopic to the macroscopic. Temperature is a macroscope property of the gas while the average square of the particle speeds is a microscopic feature. The temperature (macroscopic) is due to the average kinetic energy of the particles (microscopic).

Temperature is a result of the average kinetic energy of the gas particles! From

$U = N \frac{1}{2} m \overline{v^2} = \frac{3}{2} NkT$, we find $\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$.

$$\boxed{\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT}$$