Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

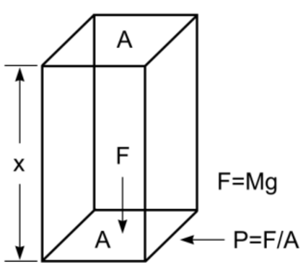
Chapter D. Temperature, Pressure, and Height.

1. Lapse Rate. The goal of this course is to derive a general lapse-rate formula. The lapse rate refers to temperature decreasing as we increase our attitude in the troposphere. The Greek uppercase gamma is used: Γ. It is defined below.

$$(10)\ \boxed{\Gamma = -\frac{dT}{dz}}$$

Consider the above equation a definition. The lapse rate is defined as the negative of $\frac{dT}{dz}$ since a lapse is positive when the derivative is negative. Remember it this way. When you go up in altitude, the temperature drops. Therefore $\frac{dT}{dz}$ is negative. The lapse rate will then be positive, which is what you want for lapse since lapse has decrease built into its meaning. Therefore, the lapse rate is given by (10).

2. Hydrostatic Equilibrium. The Another important equation involves the change is pressure as height is increased. Blaise Pascal gave us the formula for pressure as you proceed lower and lower in a fluid, which can be either a liquid or gas. We will derive his formula below.



Here x is the depth and the fluid above our little patch of area has volume $V = Ax$. This fluid presses down on the fluid below it. We want the pressure at depth x. Pressure is force per unit area. The force of the block of fluid with mass m due to gravity is $F = mg$ from intro physics.

$$p = \frac{F}{A} = \frac{Mg}{A}$$

Since mass M equals density ρ times volume V, the pressure is $p = \frac{Mg}{A} = \frac{\rho Vg}{A}$. The volume is given by $V = Ax$. With this substitution

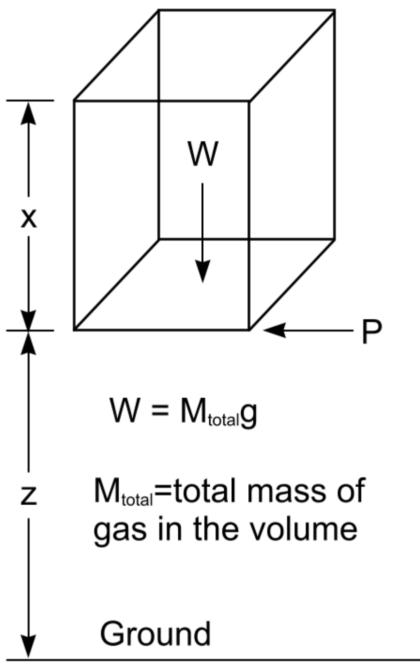
$$p = \frac{\rho Vg}{A} = \frac{\rho Axg}{A} = \rho gx .$$

The differential pressure formula is

$$dp = \rho g dx .$$

Pascal's Law also applies to the atmosphere as since the law applies to fluids, which can be liquids or gases. For the moment, we take the atmosphere to have the same temperature at all heights, which of course is an oversimplification. We also take the atmosphere to be in equilibrium called hydrostatic equilibrium, where gravity has causes the atmosphere to settle down and there are no winds.

Let the top of the atmosphere be at the top of the column of air shown in the figure. Consider measuring from the ground up so that



$$x + z = \text{const},$$

a constant. Then,

$$dx + dz = 0$$

$$dz = -dx$$

We would like to measure from the ground up so we focus on the variable z. Then, for hydrostatic equilibrium using Pascal's law:

$$p = \rho g x \quad \text{and} \quad dp = \rho g dx = -\rho g dz.$$

Since this equation is in differential form and the temperature can be considered constant in the dz slice of atmosphere, the equation holds in general.

Atmospheric Hydrostatic Equilibrium: $dp = -\rho g dz$.

(11) $\boxed{\dfrac{dp}{dz} = -\rho g}$