**Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), [DoctorPhys] on YouTube.**
**Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.**

**Chapter F. Enthalpy.** We have seen the first law of thermodynamics discussed in the previous chapter as

$$dU = \delta Q - \delta W \text{ ,}$$

where we can include $\delta W = pdV$ for a gas.

The first law is then

$$dU = \delta Q - pdV \text{ .}$$

We divide by the $m$ of the gas and introduce some notational definitions.

$$dU = \delta Q - pdV \quad \Rightarrow \quad \frac{dU}{m} = \frac{\delta Q}{m} - p\frac{dV}{m} \quad \Rightarrow \quad du = \delta q - pd\alpha$$

This equation is labeled (13) from the last chapter.

Solve for $\delta q$.

$$\delta q = du + pd\alpha$$

Remember that we employ the delta symbol $\delta$ with $q$ to remind us that heat $q$ is not a variable of the gas state. Heat can enter or leave the gas, but it is the energy of the gas that is a valid variable to describe the actual state of the gas when things settle down. We say that the gas is in equilibrium. But note that the variables $u$, $p$, and $\alpha$ are all acceptable variables that can be assigned to the gas. We can make differentials of these with no problem: $du$, $dp$, and $d\alpha$. We call these standard differentials exact differentials, while $\delta q$ is referred to as an inexact differential.

We would like to have variable of the gas state that is closely aligned with heat. Since $\delta q = du + pd\alpha$ we note that the right side $du + pd\alpha$ suggests that we consider $du + pd\alpha + \alpha dp$.

The combination $du + pd\alpha + \alpha dp$ can be expressed as $du + d(\alpha p)$, which we can readily recognize as an exact differential $d(u + \alpha p)$. We define $u + \alpha p$ as the *enthalpy* $h$.

$$h = u + \alpha p$$

Enthalpy is a variable of the gas state as it is defined as a combination of the state variables $u$, $\alpha$, and $p$. All state variables can be expressed in differential form, i.e., exact differentials.

The definition of the enthalpy state variable brings us to our next review equation.

$$(13)\ \boxed{h = u + p\alpha}$$

Note that there is a well-defined enthalpy for each equilibrium state of a gas. Remember that equilibrium means constant temperature and pressure throughout the gas. No matter what path you take in a thermodynamic process to go from an initial point $i$ to a final point $f$, the change in enthalpy is the difference of enthalpies: $\Delta h = h_f - h_i$. It is path independent. This difference behaves as any state variable difference such as the change in specific pressure $\Delta p = p_f - p_i$ or specific volume $\Delta\alpha = \alpha_f - \alpha_i$

Now consider our first law of thermodynamics $du = \delta q - pd\alpha$ and $dh = d(u + p\alpha)$.

$$du = \delta q - pd\alpha \quad => \quad \delta q = du + pd\alpha$$

$$dh = d(u + p\alpha) \quad => \quad dh = du + pd\alpha + \alpha dp \quad => \quad du = dh - pd\alpha - \alpha dp$$

We substitute $du = dh - pd\alpha - \alpha dp$ in $\delta q = du + pd\alpha$ next.

$$\delta q = du + pd\alpha \quad => \quad \delta q = dh - \cancel{pd\alpha} - \alpha dp + \cancel{pd\alpha} \quad => \quad \delta q = dh - \alpha dp$$

We arrive at our 14th equation.

$$(14)\ \boxed{\delta q = dh - \alpha dp}$$

While we cannot talk about heat $q$ content in a gas since it is not a state variable, we can talk about enthalpy of the gas, which is closely related to heat. In fact, for cases of no change in pressure, $\delta q = dh - \alpha dp = dh - \alpha \cdot 0 = dh$, and $\delta q$ becomes an exact differential!