Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

Chapter O. Mixing Ratios. The mixing ratio of vapor to dry air is defined as

$$r_v \equiv \frac{m_v}{m_d},$$

where m_v and m_d are respectively the masses of vapor and dry air in a volume of moist air.

The following equations will be helpful for us at this time.

$$(7)\ \boxed{p = p_d + e} \qquad (8)\ \boxed{p_d = \rho_d R_d T} \qquad (9)\ \boxed{e = \rho_v R_v T}$$

Write the ideal gas laws (8) and (9) with the densities on the left side.

$$\rho_d = \frac{p_d}{R_d T} \qquad \rho_v = \frac{e}{R_v T}$$

Since the dry air and vapor exist in the same volume V, the densities are $\rho_d = \frac{m_d}{V}$ and $\rho_v = \frac{m_v}{V}$.

Therefore, $m_d = \rho_d V$ and $m_v = \rho_v V$. Remember that v as a subscript refers to vapor and the upper-case V is the volume. The above equations lead to the following.

$$r = \frac{m_v}{m_d} = \frac{\rho_v V}{\rho_d V} = \frac{\rho_v}{\rho_d} = \frac{\frac{e}{R_v T}}{\frac{p_d}{R_d T}} \quad => \quad r = \frac{\frac{e}{R_v}}{\frac{p_d}{R_d}} \quad => \quad r = \frac{e}{p_d}\frac{R_d}{R_v}.$$

From (6) $\boxed{\varepsilon = \frac{R_d}{R_v} = \frac{M_v}{M_d}}$,

$$r = \frac{e}{p_d}\frac{R_d}{R_v} \quad => \quad r = \frac{e}{p_d}\varepsilon.$$

To emphasize that the mixing ratio is for the vapor, we can add the subscript v for vapor.

$$\boxed{r_v = \frac{\varepsilon e}{p_d}}$$

This equation give us the mixing ratio in terms of the partial pressures e and p_d.

$$(25)\ \boxed{r_v = \frac{\varepsilon e}{p_d}}$$

Now we consider the mixing ratio in terms of the vapor pressure and total pressures. Since the total pressure of the moist air is equal to the sum of the dry-air partial pressure and vapor partial pressure, we have our earlier equation

$$(7)\ \boxed{p = p_d + e}.$$

The partial pressure of the dry ais is then $p_d = p - e$.

We can substitute $p - e$ for p_d in $r_v = \dfrac{\varepsilon e}{p_d}$ and obtain $r_v = \dfrac{\varepsilon e}{p - e}$.

$$(26)\ \boxed{r_v = \frac{\varepsilon e}{p - e}}$$

A very common notation for mixing ratio is w, where w stands for water. Other authors prefer r to avoid the clash when w is employed for vertical wind speed. Still other authors follow standards set by classic texts where w has been the notation for years. I use r in this course following the Glossary of the American Meteorological Society. But I do use w in my aviation thermodynamics course.