Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

Chapter R. Entropy, Dry Air, and Moist Air. In this chapter we determine the differential change in specific entropy for dry air and moist air. The form of the first law of thermodynamics with the specific enthalpy is

(14) $\boxed{\delta q = dh - \alpha dp}$.

Using $\delta q = Tds$, we obtain

$$Tds = dh - \alpha dp\,.$$

From (16) $\boxed{c_p = \frac{\delta q_p}{dT} = \frac{dh}{dT}}$, we can write $dh = c_p dT$.

Using $dh = c_p dT$, the equation $Tds = dh - \alpha dp$ can be expressed as follows.

$$Tds = dh - \alpha dp \quad \Rightarrow \quad Tds = c_p dT - \alpha dp$$

Dividing by the temperature, $ds = \frac{c_p}{T} dT - \frac{\alpha}{T} dp$.

From the ideal gas law $p\alpha = RT$ (2), we have $\frac{\alpha}{T} = \frac{R}{p}$.

Therefore, $ds = \frac{c_p}{T} dT - \frac{\alpha}{T} dp$ with this substitution leads to

$$ds = \frac{c_p}{T} dT - \frac{R}{p} dp\,.$$

We will use this general result first for dry air and then vapor.

Applying this equation to dry air gives us $ds_d = \frac{c_{pd}}{T} dT - \frac{R_d}{p_d} dp_d$.

(29) $\boxed{ds_d = \frac{c_{pd}}{T} dT - \frac{R_d}{p_d} dp_d}$

We apply our general result $ds = \frac{c_p}{T} dT - \frac{R}{p} dp$ next for the vapor.

$$ds = \frac{c_p}{T} dT - \frac{R}{p} dp \quad \Rightarrow \quad ds_v = \frac{c_{pv}}{T} dT - \frac{R_v}{e} de$$

Including the subscript "s" with e reinforces that we are talking about saturation. The result is

$$ds_v = \frac{c_{pv}}{T} dT - \frac{R_v}{e_s} de_s .$$

This equation is our 30th review equation.

(30) $$\boxed{ds_v = \frac{c_{pv}}{T} dT - \frac{R_v}{e_s} de_s}$$

This equation gives the differential change in specific entropy for water vapor at saturation.