Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

Chapter S. Entropy and Water Droplets. In this chapter we determine the differential change in specific entropy for liquid water droplets. We need to be careful here to make sure we do not use any equation that applies only to an ideal gas. Therefore, we start with the general equation

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \, .$$

The specific heat $c_p = \dfrac{\delta q_p}{dT}$ by definition. We connect δq_p to ds via $\delta q_p = Tds$. Then,

$$c_p = \frac{\delta q_p}{dT} = \left(\frac{Tds}{dT}\right)_p = T\left(\frac{\partial s}{\partial T}\right)_p .$$

We use a partial derivative since $s = s(T, p)$, depending on two independent variables.

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \quad => \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$$

We have determined one of the components in $ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp$, namely the $\left(\frac{\partial s}{\partial T}\right)_p$.

The change in specific entropy has two contributions, but so far we only know $\left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$.

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \quad => \quad ds = \frac{c_p}{T} dT + \left(\frac{\partial s}{\partial p}\right)_T dp$$

What about the second component $\left(\frac{\partial s}{\partial p}\right)_T dp$? Here is where we use (22) $\boxed{\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial \alpha}{\partial T}\right)_p}$.

The total change in specific entropy using this Maxwell relation (22) brings us to

$$ds = \frac{c_p}{T} dT + \left(\frac{\partial s}{\partial p}\right)_T dp \quad => \quad ds = \frac{c_p}{T} dT - \left(\frac{\partial \alpha}{\partial T}\right)_p dp \, .$$

For a liquid, the specific volume hardly changes as T changes at constant p , i.e., $\left(\frac{\partial \alpha}{\partial T}\right)_p \approx 0$

Therefore, for liquid water $ds_l = \frac{c_{pl}}{T} dT - \left(\frac{\partial \alpha_l}{\partial T} \right)_p dp$ becomes simply

$$ds_l = \frac{c_{pl}}{T} dT ,$$

where c_{pl} is the specific heat at constant pressure for liquid water.

We have our 31st equation.

$$(31) \quad \boxed{ds_l = \frac{c_{pl}}{T} dT}$$

Then above equation gives the differential change in specific entropy of the liquid water droplets as you make a differential change in temperature. In other words, an infinitesimal change in the temperature causes an infinitesimal specific entropy change.