Lapse Rate. Prof. Ruiz (Doc), UNC-Asheville (1978-2021), DoctorPhys on YouTube.
Prerequisites: Chemistry, Physics I, II. Calculus I, II, III. Thermodynamics.

Chapter T. Entropy and Most Air with Water Droplets. In this short chapter we consider moist air with water droplets undergoing an adiabatic process. The total heat transfer within the system is

$$\delta Q = m_d \delta q_d + m_v \delta q_v + m_l \delta q_l = 0$$

since the process is adiabatic, i.e., there is no net heat transfer to the external environment. But we need to work with the state variable closely related to heat transfer, namely, the entropy $ds = \dfrac{\delta q}{T}$. Express this relation as $\delta q = Tds$.

Note that an infinitesimal adiabatic change, i.e., $\delta q = 0$,

means that $ds = 0$ since $\delta q = Tds$.

Therefore, we can change our description from heat transfer δQ to a change in entropy, $dS = \dfrac{\delta Q}{T}$.

$$\delta Q = m_d \delta q_d + m_v \delta q_v + m_l \delta q_l = 0 \quad \Rightarrow \quad dS = \frac{\delta Q}{T} = m_d \frac{\delta q_d}{T} + m_v \frac{\delta q_v}{T} + m_l \frac{\delta q_l}{T} = 0$$

$$\Rightarrow \quad dS = m_d ds_d + m_v ds_v + m_l ds_l = 0 .$$

The bonus here is that we can integrate the entropy $\int dS = m_d \int ds_d + m_v \int ds_v + m_l \int ds_l$ and obtain

$$S = m_d s_d + m_v s_v + m_l s_l + const .$$

We can choose the constant to be zero.

$$\boxed{S = m_d s_d + m_v s_v + m_l s_l}$$

Working directly with the state variable entropy puts us on solid ground.

$$(32) \quad \boxed{S = m_d s_d + m_v s_v + m_l s_l}$$

We have derived out last review equation.

We now proceed to our big derivation: the reversible saturated adiabatic lapse rate.