HW-B1. Index of Refraction for Air.

$n = 1 + A(\lambda)\cdot 10^{-8}\dfrac{p}{T+273.15}$, where here T is in degrees Celsius, p is in pascals, and

$$\frac{A(\lambda)}{0.0028425} = 8342.54 + 2406147\left[130 - \frac{1}{\lambda_o^2}\right]^{-1} + 15998\left[38.9 - \frac{1}{\lambda_o^2}\right]^{-1}$$ with λ_o in μm.

a) Show that at 633 nm the index of refraction formula is $n(\lambda = 633 \text{ nm}) = 1 + \dfrac{78.6\,p}{T+273}10^{-8}$.

$$\frac{A(0.633)}{0.0028425} = 8342.54 + 2406147\left[130 - \frac{1}{0.633^2}\right]^{-1} + 15998\left[38.9 - \frac{1}{0.633^2}\right]^{-1}$$

$$\frac{A(0.633)}{0.0028425} = 8342.54 + 2406147\left[130 - 2.4957\right]^{-1} + 15998\left[38.9 - 2.1957\right]^{-1}$$

We keep four significant figures to the right of the decimal so as not to round off prematurely.

$$\frac{A(0.633)}{0.0028425} = 8342.54 + 2406147\left[127.5043\right]^{-1} + 15998\left[36.4043\right]^{-1}$$

$$\frac{A(0.633)}{0.0028425} = 8342.54 + \frac{2406147}{127.5043} + \frac{15998}{36.4043}$$

$$\frac{A(0.633)}{0.0028425} = 8342.54 + 18{,}871 + 439.45$$

$$\frac{A(0.633)}{0.0028425} = 27{,}653$$

$$A(0.633) = 0.0028425 \times 27{,}653 = 78.6$$

$$n_{633} = 1 + A(0.633)\cdot 10^{-8}\frac{P}{T+273.15} = 1 + \frac{78.6\,p}{T+273.15}10^{-8}$$

b) Repeat for n_{455} and n_{532}.

$$\frac{A(0.455)}{0.0028425} = 8342.54 + 2406147\left[130 - \frac{1}{0.455^2}\right]^{-1} + 15998\left[38.9 - \frac{1}{0.455^2}\right]^{-1}$$

$$\frac{A(0.455)}{0.0028425} = 8342.54 + 2406147\left[130 - 4.8303\right]^{-1} + 15998\left[38.9 - 4.8303\right]^{-1}$$

$$\frac{A(0.455)}{0.0028425} = 8342.54 + 2406147\left[125.1697\right]^{-1} + 15998\left[34.0697\right]^{-1}$$

$$\frac{A(0.455)}{0.0028425} = 8342.54 + \frac{2406147}{125.1697} + \frac{15998}{34.0697}$$

$$\frac{A(0.455)}{0.0028425} = 8342.54 + 19,223 + 469.57$$

$$\frac{A(0.455)}{0.0028425} = 28,035$$

$$A(0.455) = 0.0028425 \times 28,035 = 79.7$$

$$n_{455} = 1 + A(0.455) \cdot 10^{-8} \frac{P}{T + 273.15} = 1 + \frac{79.7\,p}{T + 273.15} 10^{-8}$$

The green case.

$$\frac{A(0.532)}{0.0028425} = 8342.54 + 2406147\left[130 - \frac{1}{0.532^2}\right]^{-1} + 15998\left[38.9 - \frac{1}{0.532^2}\right]^{-1}$$

$$\frac{A(0.532)}{0.0028425} = 8342.54 + 2406147\left[130 - 3.5333\right]^{-1} + 15998\left[38.9 - 3.5333\right]^{-1}$$

$$\frac{A(0.532)}{0.0028425} = 8342.54 + 2406147\left[126.4667\right]^{-1} + 15998\left[35.3667\right]^{-1}$$

$$\frac{A(0.532)}{0.0028425} = 8342.54 + \frac{2406147}{126.4667} + \frac{15998}{35.3667}$$

$$\frac{A(0.532)}{0.0028425} = 8342.54 + 19,026 + 452.35$$

$$\frac{A(0.532)}{0.0028425} = 27,821$$

$$A(0.532) = 0.0028425 \times 27,821 = 79.1$$

$$n_{532} = 1 + A(0.532) \cdot 10^{-8} \frac{P}{T + 273.15} = 1 + \frac{79.1p}{T + 273.15} 10^{-8}$$

HW-B2. The Mirage Angle and Wavelength. Find $\alpha = \sqrt{2(1-\frac{n_{\text{hot}}}{n_{\text{cold}}})}$ for extreme violet and red:

$n(\lambda = 400\text{ nm}) = 1 + \frac{80.4\,p}{T + 273.15}10^{-8}$ and $n(\lambda = 700\text{ nm}) = 1 + \frac{78.2\,p}{T + 273.15}10^{-8}$, where

Temperature Cold: $T_{\text{cold}} = 27.5\ ^\circ\text{C}$ (ambient air temperature)

Temperature Hot: $T_{\text{hot}} = 53.0\ ^\circ\text{C}$ (air temperature near hot road surface)

Pressure: was standard atmospheric pressure to 3 significant figures, $p = 1.01\times10^{5}\text{ Pa}$

The Violet Calculation:

$$n_{\text{hot}}(\lambda = 400\text{ nm}) = 1 + \frac{80.4\,p}{T_{\text{hot}} + 273.15}10^{-8} = 1 + \frac{80.4(1.01\times10^{5})}{53 + 273.15}10^{-8} = 1.000249$$

$$n_{\text{cold}}(\lambda = 400\text{ nm}) = 1 + \frac{80.4\,p}{T_{\text{cold}} + 273.15}10^{-8} = 1 + \frac{80.4(1.01\times10^{5})}{27.5 + 273.15}10^{-8} = 1.000270$$

$$\alpha_{400} = \sqrt{2\left[1 - \frac{n_{\text{hot}}(400\text{ nm})}{n_{\text{cold}}(400\text{ nm})}\right]} = \sqrt{2\left[1 - \frac{1.000249}{1.000270}\right]} = 0.006480\text{ rad}$$

$$\alpha_{400} = 0.006480\text{ rad}\cdot\frac{180^\circ}{\pi} = 0.37^\circ$$

The Red Calculation:

$$n_{\text{hot}}(\lambda = 700\text{ nm}) = 1 + \frac{78.2\,p}{T_{\text{hot}} + 273.15}10^{-8} = 1 + \frac{78.2(1.01\times10^{5})}{53 + 273.15}10^{-8} = 1.000242$$

$$n_{\text{cold}}(\lambda = 700\text{ nm}) = 1 + \frac{78.2\,p}{T_{\text{cold}} + 273.15}10^{-8} = 1 + \frac{78.2(1.01\times10^{5})}{27.5 + 273.15}10^{-8} = 1.000263$$

$$\alpha_{700} = \sqrt{2\left[1 - \frac{n_{\text{hot}}(700\text{ nm})}{n_{\text{cold}}(700\text{ nm})}\right]} = \sqrt{2\left[1 - \frac{1.000242}{1.000263}\right]} = 0.006480\text{ rad}$$

$$\alpha_{700} = 0.006480\text{ rad}\cdot\frac{180^\circ}{\pi} = 0.37^\circ$$

Conclusion: $\alpha_{400} = \alpha_{700}$

The mirage angle is not very sensitive to wavelength. It is the same for all colors.