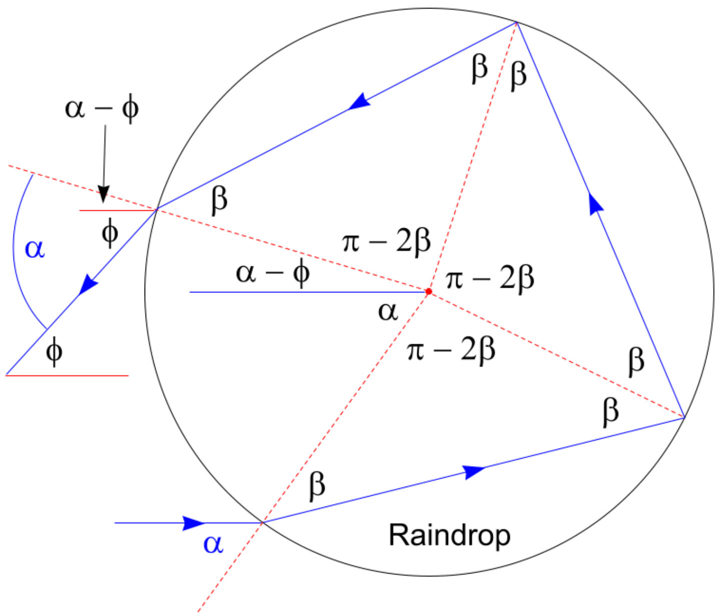
HW-C1. Secondary Rainbow. Determine the rainbow angles for the secondary rainbow.



First sum the angles in the center of the raindrop.

$$\alpha - \phi + 3(\pi - 2\beta) + \alpha = 2\pi$$

Solve for ϕ.

$$\phi = 2\alpha + 3(\pi - 2\beta) - 2\pi$$

$$\phi = 2\alpha - 6\beta + \pi$$

Set $\dfrac{d\phi}{d\alpha} = 0$ to find the rainbow angle.

But we need $\beta = \beta(\alpha)$.

Snell's Law at the entry point: $n_{\text{air}} \sin\alpha = n_{\text{water}} \sin\beta$, where $n_{\text{air}} = 1$ and $n_{\text{water}} = n$.

Then $\sin\alpha = n\sin\beta$, $\sin\beta = \dfrac{\sin\alpha}{n}$, and $\beta = \sin^{-1}\left[\dfrac{\sin\alpha}{n}\right]$.

Using $\phi = 2\alpha - 6\beta + \pi$,

$$\boxed{\phi(\alpha) = 2\alpha - 6\sin^{-1}\left[\frac{\sin\alpha}{n}\right] + \pi}$$

$$\frac{d\phi(\alpha)}{d\alpha} = 2 - 6\frac{d}{d\alpha}\sin^{-1}\left[\frac{\sin\alpha}{n}\right] = 0$$

Let $u = \dfrac{\sin\alpha}{n}$ and use $\dfrac{d\sin^{-1}u}{dx} = \dfrac{1}{\sqrt{1-u^2}}$.

$$\frac{d\phi(\alpha)}{d\alpha} = 2 - 6\frac{d}{du}\sin^{-1}[u]\frac{du}{d\alpha} = 0$$

Then $\frac{d\phi(\alpha)}{d\alpha} = 2 - 6\frac{1}{\sqrt{1-u^2}}\frac{du}{d\alpha} = 0$, where $u = \frac{\sin\alpha}{n}$.

Using $\frac{du}{d\alpha} = \frac{d}{d\alpha}\left[\frac{\sin\alpha}{n}\right] = \frac{\cos\alpha}{n}$,

$$\frac{d\phi(\alpha)}{d\alpha} = 2 - 6\frac{1}{\sqrt{1-\left[\frac{\sin\alpha}{n}\right]^2}}\frac{\cos\alpha}{n} = 0 .$$

$$2 - 6\frac{1}{\sqrt{n^2 - \sin^2\alpha}}\cos\alpha = 0 \qquad 1 = 3\frac{1}{\sqrt{n^2 - \sin^2\alpha}}\cos\alpha$$

$$\sqrt{n^2 - \sin^2\alpha} = 3\cos\alpha \qquad n^2 - \sin^2\alpha = 9\cos^2\alpha$$

$$n^2 = 9\cos^2\alpha + \sin^2\alpha$$

$$n^2 = 8\cos^2\alpha + \cos^2\alpha + \sin^2\alpha$$

$n^2 = 8\cos^2\alpha + 1$ and $\cos^2\alpha = \frac{n^2-1}{8}$

$$\alpha = \cos^{-1}\sqrt{\frac{n^2-1}{8}} \qquad \beta = \sin^{-1}\left[\frac{\sin\alpha}{n}\right] \qquad \phi = 2\alpha - 6\beta + \pi$$

$$n_{700\text{ nm}} = 1.331 \quad \alpha = \cos^{-1}\sqrt{\frac{1.331^2-1}{8}} = 71.91° \quad \beta = \sin^{-1}\left[\frac{\sin 71.91°}{1.331}\right] = 45.58°$$

$$\phi_{700\text{ nm}} = 2\alpha - 6\beta + \pi = 2(71.91°) - 6(45.58°) + 180° = 50°$$

$$n_{400\text{ nm}} = 1.345 \quad \alpha = \cos^{-1}\sqrt{\frac{1.345^2-1}{8}} = 71.46° \quad \beta = \sin^{-1}\left[\frac{\sin 71.46°}{1.345}\right] = 44.82°$$

$$\phi_{400\text{ nm}} = 2\alpha - 6\beta + \pi = 2(71.46°) - 6(44.82°) + 180° = 54°$$

Secondary Rainbow: **Red** is on bottom at **50°** and **Violet** is on top with rainbow angle **54°**.

This problem requested answers to two significant figures.

HW-C2. Glass Bow. Raining crown glass beads.

Dispersion Table for Several Transparent Solids

Medium	Color, Wavelength (nm), and Index of Refraction					
	Violet 410	Blue 470	Green 550	Yellow 580	Orange 610	Red 660
Ice	1.3170	1.3136	1.3110	1.3087	1.3080	1.3060
Crown Glass	1.5380	1.5310	1.5260	1.5225	1.5216	1.5200
Quartz	1.5570	1.5510	1.5468	1.5438	1.5432	1.5420
Light Flint	1.6040	1.5960	1.5910	1.5875	1.5867	1.5850
Dense Flint	1.6980	1.6836	1.6738	1.6670	1.6650	1.6620
Diamond	2.4580	2.4439	2.4260	2.4172	2.4150	2.4100

Fundamentals of Optics (4th ed.), Jenkins and White (McGraw Hill, NY, 1976).

Primary Rainbow (from text): $\alpha = \cos^{-1}\sqrt{\dfrac{n^2-1}{3}}$, $\beta = \sin^{-1}\left[\dfrac{\sin\alpha}{n}\right]$, $\phi = -2\alpha + 4\beta$.

$$\alpha_{410} = \cos^{-1}\sqrt{\frac{1.538^2-1}{3}} = 47.57° \qquad \beta = \sin^{-1}\left[\frac{\sin 47.57°}{1.538}\right] = 28.68°$$

$$\phi_{410} = -2(47.57°) + 4(28.68°) = 19.6°$$

$$\alpha_{660} = \cos^{-1}\sqrt{\frac{1.52^2-1}{3}} = 48.63° \quad \beta = \sin^{-1}\left[\frac{\sin 48.63°}{1.52}\right] = 29.59°$$

$$\phi_{660} = -2(48.63°) + 4(29.59°) = 21.1°$$

This problem asked for three significant figures.

Range is from **19.6° (violet)** to **21.1° (red)**