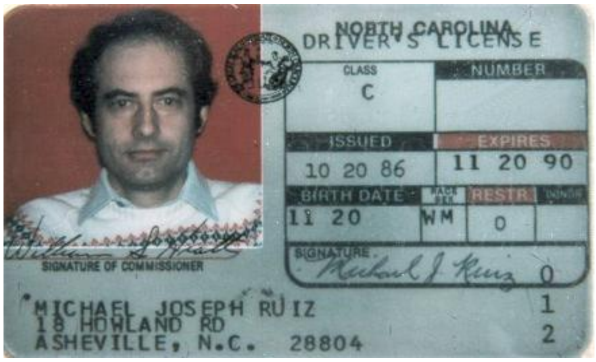
HW-E1. Special Camera Setup



Given: Place a license in front of a 60-mm lens to achieve a magnification of $M = -3$. Note that the lens is a converging lens since we need to produce a real image of the license on the film. Therefore $f = +60 \text{ mm}$. And real images on film are inverted. Therefore, you need to insert a minus sign in front of the 3.

The interest is the 30 mm x 30 mm photo section of the 85 mm x 50 mm license. Therefore $h_o = 30 \text{ mm}$.

(a) Math

$$M = -\frac{s_i}{s_o} = -3 \quad \text{and} \quad \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

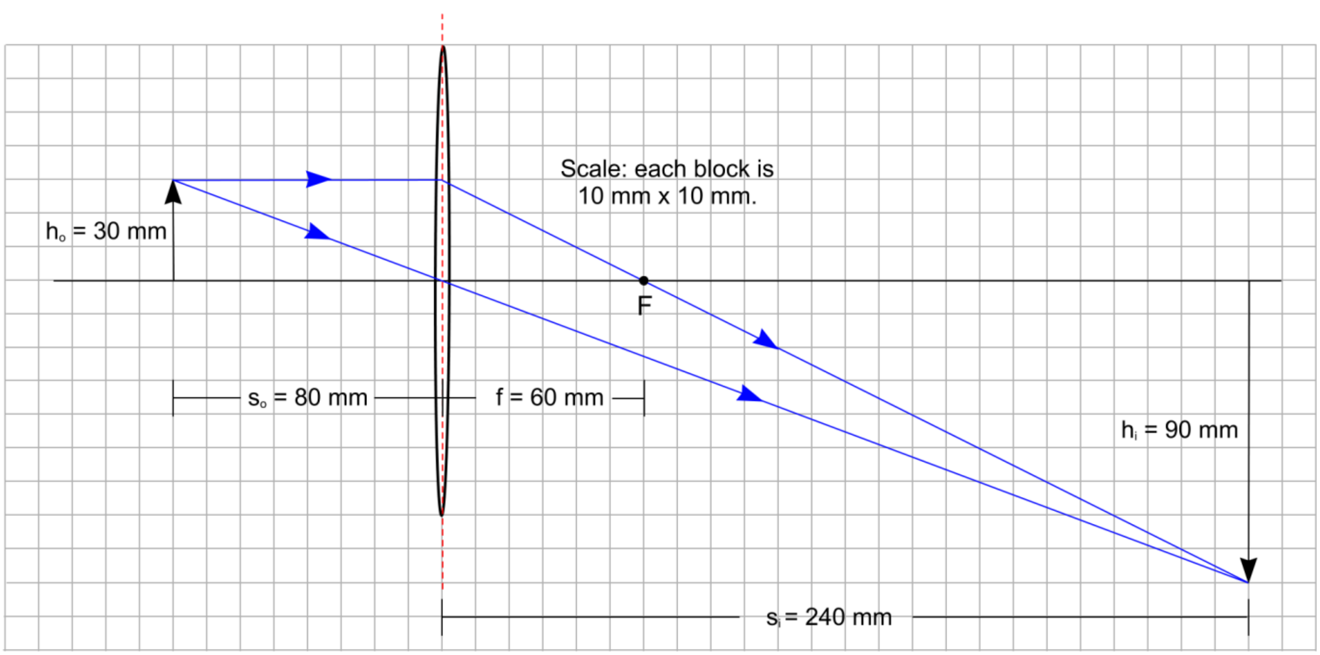
From the M equation $\frac{s_i}{s_o} = 3$, i.e., $s_i = 3s_o$.

Then $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$ becomes $\frac{1}{60} = \frac{1}{s_o} + \frac{1}{3s_o} = \frac{1}{s_o}\left[1 + \frac{1}{3}\right] = \frac{4}{3}\frac{1}{s_o}$ and $s_o = \frac{4}{3} \cdot 60 = 80 \text{ mm}$.

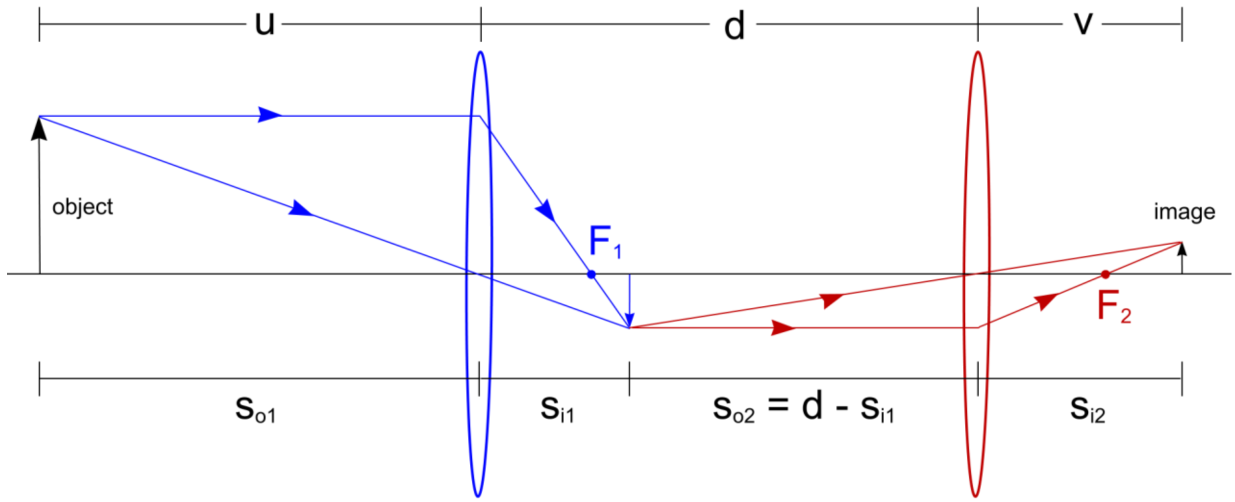
$$s_i = 3s_o = 3 \cdot 80 = 240 \text{ mm}$$

The license should be placed 80 mm in front of the lens.
The film should be placed 240 mm behind the lens.

(b) Ray Diagram



HW-E2. Engineering Design of a Magic Illusion



Design Specs Given: u = 6 ft, d = 10 ft, v = 4 ft, M = -1/36. Find the focal lengths f_1 and f_2.

For Lens 1: $$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \quad \text{and} \quad M_1 = -\frac{s_{i1}}{s_{o1}}$$

For Lens 2: $$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \quad \text{and} \quad M_2 = -\frac{s_{i2}}{s_{o2}}, \text{ where } s_{o2} = d - s_{i1}.$$

We also know that $M = M_1 M_2 = \left[-\frac{s_{i1}}{s_{o1}}\right]\left[-\frac{s_{i2}}{s_{o2}}\right] = \frac{s_{i1}}{s_{o1}}\frac{s_{i2}}{s_{o2}} = \frac{s_{i1}}{s_{o1}}\frac{s_{i2}}{d - s_{i1}} = \frac{1}{36}$.

Also given is $s_{o1} = u = 6$, $s_{i2} = v = 4$, and $d = 10$. Using this information, we can solve for s_{i1}.

$$M = \frac{s_{i1}}{s_{o1}}\frac{s_{i2}}{d - s_{i1}} = \frac{1}{36} \text{ becomes } M = \frac{s_{i1}}{6}\frac{4}{10 - s_{i1}} = \frac{1}{36}.$$

$$s_{i1}\frac{4}{10 - s_{i1}} = \frac{1}{6}$$

$$4s_{i1} = \frac{1}{6}(10 - s_{i1})$$

$$24s_{i1} = 10 - s_{i1}$$

$$25s_{i1} = 10$$

$$s_{i1} = \frac{2}{5}$$

Then $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$ with our numbers becomes $\frac{1}{f_1} = \frac{1}{6} + \frac{1}{(2/5)} = \frac{1}{6} + \frac{5}{2} = \frac{1+15}{6} = \frac{16}{6} = \frac{8}{3}$.

$$\boxed{f_1 = \frac{3}{8}}$$

$$\boxed{f_1 = 0.375 \text{ ft}}$$

Summary of what we know so far: $s_{o1} = 6$, $s_{i1} = \frac{2}{5}$, $s_{i2} = 4$. We also know

$$s_{o2} = d - s_{i1} = 10 - \frac{2}{5} = \frac{50-2}{5} = \frac{48}{5}.$$

Then, we can calculate the remaining focal length: $\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$.

$$\frac{1}{f_2} = \frac{1}{(48/5)} + \frac{1}{4}$$

$$\frac{1}{f_2} = \frac{5}{48} + \frac{1}{4}$$

$$\frac{1}{f_2} = \frac{5 \cdot 4 + 48}{48 \cdot 4}$$

$$\frac{1}{f_2} = \frac{20 + 48}{48 \cdot 4}$$

$$\frac{1}{f_2} = \frac{68}{48 \cdot 4}$$

$$\frac{1}{f_2} = \frac{17}{48}$$

$$\boxed{f_2 = \frac{48}{17}}$$

$$\boxed{f_2 = 2.82 \text{ ft}}$$