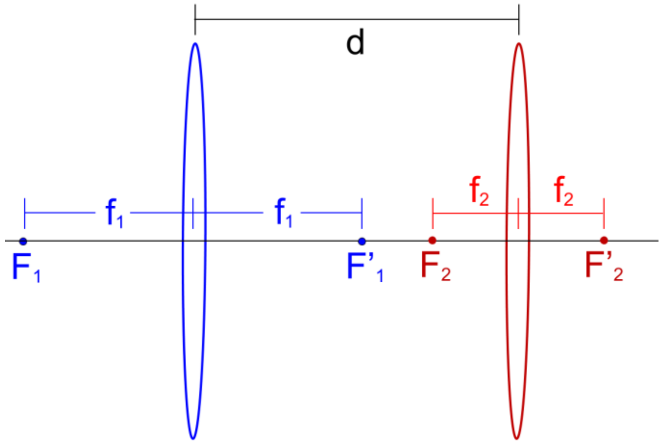
HW-F1. Front and Back Focal Lengths. Find the front focal length f_f and the back focal length f_b in terms of f_1, f_2, and d for the two-lens system below.



$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \qquad \frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \qquad s_{o2} = d - s_{i1}$$

For the back focal length, we take $s_{o1} \to \infty$, Then $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \to \frac{1}{s_{i1}}$ and we have $s_{i1} = f_1$.

The object distance for the second lens is $s_{o2} = d - s_{i1} = d - f_1$.

Substituting this $s_{o2} = d - f_1$ into $\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$ gives

$\frac{1}{f_2} = \frac{1}{d - f_1} + \frac{1}{s_{i2}}$ with s_{i2} now being the back focal length $s_{i2} = f_b$.

We want to solve $\frac{1}{f_2} = \frac{1}{d - f_1} + \frac{1}{f_b}$ for f_b.

$$\frac{1}{f_2} - \frac{1}{d - f_1} = \frac{1}{f_b}$$

$$\frac{1}{f_b} = \frac{1}{f_2} - \frac{1}{d - f_1}$$

$$\frac{1}{f_b} = \frac{(d - f_1) - f_2}{f_2(d - f_1)} \quad \text{and} \quad f_b = \frac{f_2(d - f_1)}{(d - f_1) - f_2}$$

$$\boxed{f_b = \frac{f_2(d - f_1)}{d - f_1 - f_2}}$$

For the front focal length (SHORTCUT): Turn the system around so $f_1 \leftrightarrow f_2$. Then

$$\boxed{f_f = \frac{f_1(d - f_2)}{d - f_1 - f_2}}$$

For the back focal length (LONG WAY): start again with

$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \qquad \frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \qquad s_{o2} = d - s_{i1}.$$

and take $s_{i2} \to \infty$ so that $s_{o1} \to f_f$. Using $s_{i2} \to \infty$, $\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \to \frac{1}{s_{o2}}$ with $s_{o2} = f_2$.

But $s_{o2} = d - s_{i1}$. First substitute $s_{o2} = f_2$ in $s_{o2} = d - s_{i1}$.

Then $f_2 = d - s_{i1}$ and $s_{i1} = d - f_2$.

We can substitute into $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$ the two replacements $s_{o1} = f_f$ and $s_{i1} = d - f_2$.

$$\frac{1}{f_1} = \frac{1}{f_f} + \frac{1}{d - f_2}$$

$$\frac{1}{f_f} = \frac{1}{f_1} - \frac{1}{d - f_2}$$

$$\frac{1}{f_f} = \frac{(d - f_2) - f_1}{f_1(d - f_2)}$$

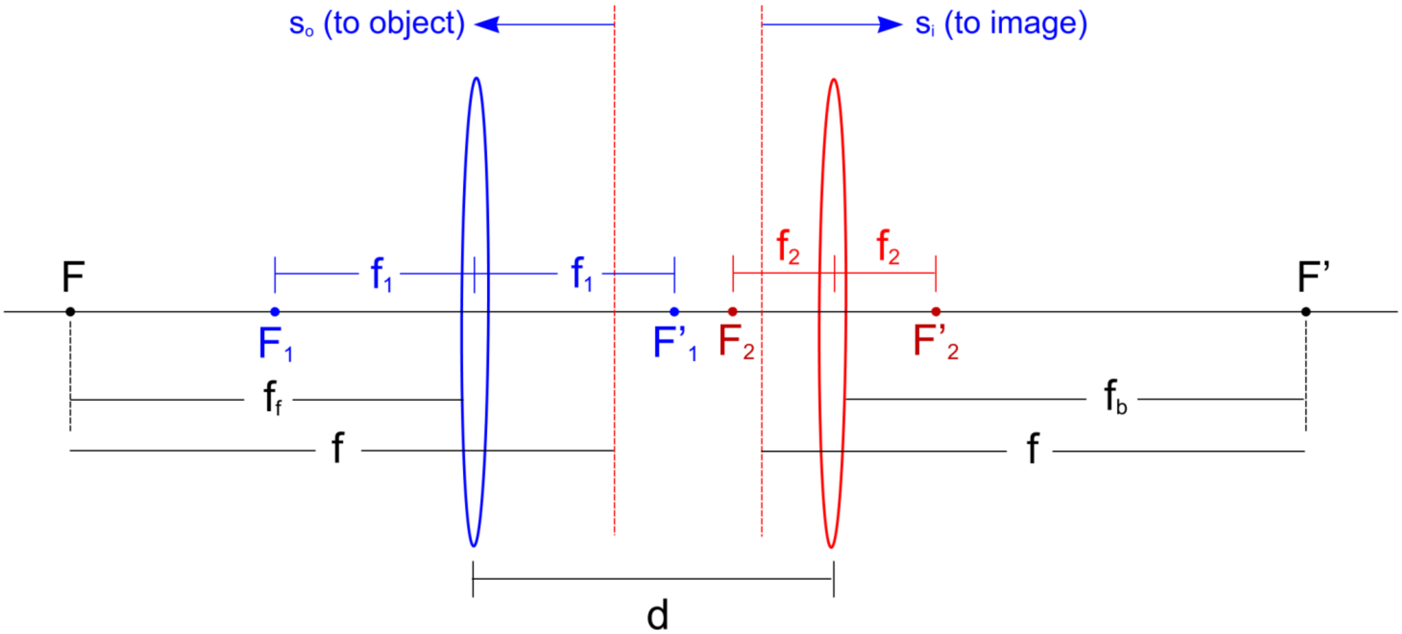
$$\frac{1}{f_f} = \frac{d - f_2 - f_1}{f_1(d - f_2)} \qquad \boxed{f_f = \frac{f_1(d - f_2)}{d - f_1 - f_2}}$$

Summary: $\boxed{f_b = \frac{f_2(d - f_1)}{d - f_1 - f_2}}$ and $\boxed{f_f = \frac{f_1(d - f_2)}{d - f_1 - f_2}}$.

For a thick lens, in the text: $\boxed{f_b = \frac{f_2(d - nf_1)}{d - n(f_1 + f_2)}}$ and $\boxed{f_f = \frac{f_1\left(d - nf_2\right)}{d - n(f_1 + f_2)}}$.

Let n =1 and you get our formulas, which makes sense since air separates our effective surfaces.

HW-F2. Effective Focal Length. Find the effective focal length f in terms of f_1, f_2, and d.



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \qquad M = -\frac{s_i}{s_o}$$

$$\frac{1}{s_{o1}} + \frac{1}{s_{i1}} = \frac{1}{f_1} \qquad \frac{1}{s_{o2}} + \frac{1}{s_{i2}} = \frac{1}{f_2}$$

$$M_1 = -\frac{s_{i1}}{s_{o1}} \qquad M_2 = -\frac{s_{i2}}{s_{o2}}$$

$$M = M_1 M_2 = \left[-\frac{s_{i1}}{s_{o1}}\right]\left[-\frac{s_{i2}}{s_{o2}}\right] = \frac{s_{i1}}{s_{o1}}\frac{s_{i2}}{s_{o2}}$$ must match $M = -\dfrac{s_i}{s_o}$.

In general $\dfrac{1}{s_o} + \dfrac{1}{s_i} = \dfrac{1}{f}$. Solve for s_i.

$$\frac{s_o + s_i}{s_o s_i} = \frac{1}{f}$$

$$f(s_o + s_i) = s_o s_i$$

$$fs_o + fs_i = s_o s_i$$

$$fs_o = s_i(s_o - f)$$

$$\boxed{s_i = \frac{fs_o}{s_o - f}}$$

Therefore, we have the following pair of equations.

$$s_{i1} = \frac{f_1 s_{o1}}{s_{o1} - f_1} \qquad\qquad s_{i2} = \frac{f_2 s_{o2}}{s_{o2} - f_2}$$

To find the effective focal length take $s_{o1} \to \infty$ in $\frac{1}{s_{o1}} + \frac{1}{s_{i1}} = \frac{1}{f_1}$. Then $s_{i1} \to f_1$.

$$s_{o2} = d - s_{i1} \to d - f_1$$

From $s_{i2} = \frac{f_2 s_{o2}}{s_{o2} - f_2}$

$$s_{i2} = \frac{f_2 s_{o2}}{s_{o2} - f_2} \to \frac{f_2(d - f_1)}{(d - f_1) - f_2} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

To keep our bearing, we summarize the parameters. For large s_o, $s_{o1} = s_o$.

$$s_{o1} = s_o \qquad s_{i1} = f_1 \qquad s_{o2} = d - f_1 \qquad s_i = f$$

Then $M = \frac{s_{i1}}{s_{o1}} \frac{s_{i2}}{s_{o2}} = -\frac{s_i}{s_o}$, with the above values gives

$\frac{s_{i1}}{s_{o1}} \frac{s_{i2}}{s_{o2}} = \frac{f_1}{s_o} \frac{s_{i2}}{d - f_1}$ for the left side and $-\frac{s_i}{s_o} = -\frac{f}{s_o}$ for the right side.

Putting these together, $\frac{f_1}{s_o} \frac{s_{i2}}{d - f_1} = -\frac{f}{s_o}$, simplifying to $f_1 \frac{s_{i2}}{d - f_1} = -f$.

The result $f_1 \frac{s_{i2}}{d - f_1} = -f$ leads to $s_{i2} = -\frac{f(d - f_1)}{f_1}$.

We need our former formula $s_{i2} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$ and use it with $s_{i2} = -\frac{f(d - f_1)}{f_1}$.

$$\frac{f_2(d - f_1)}{d - (f_1 + f_2)} = -\frac{f(d - f_1)}{f_1}$$

$$\frac{f_2(d - f_1) f_1}{d - (f_1 + f_2)} = -f(d - f_1)$$

The $(d - f_1)$ factors cancel.

$$\frac{f_2 f_1}{d - (f_1 + f_2)} = -f$$

$$f = -\frac{f_1 f_2}{d - (f_1 + f_2)}$$

$$f = \frac{f_1 f_2}{(f_1 + f_2) - d}$$

$$\boxed{f = \frac{f_1 f_2}{f_1 + f_2 - d}}$$

$$\frac{1}{f} = \frac{f_1 + f_2 - d}{f_1 f_2}$$

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$