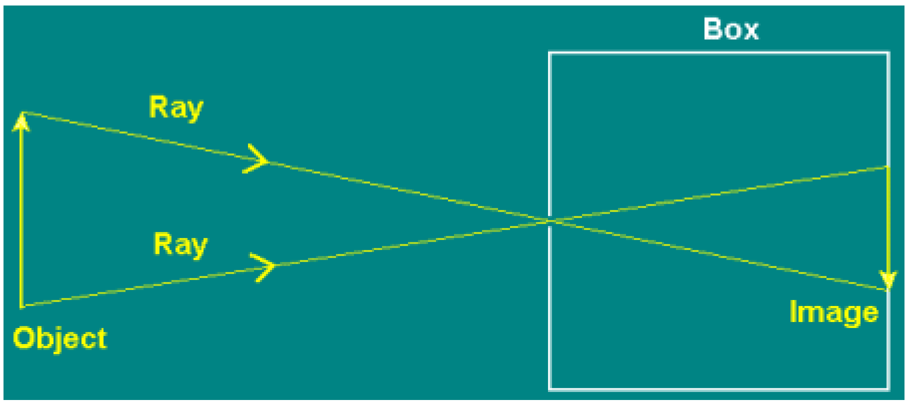
G1. The Camera Obscura or the Pinhole Camera.

The discovery of the ***camera obscura***, also known as the ***pinhole camera***, had to come before the invention of photography. Here is a sketch of the camera obscura.

Camera Obscura



Four individuals played important roles in the discovery and analysis of the camera obscura. The first was the Chinese philosopher Mozi or Motzu c. 400 BCE), who observed the effect in a room and referred to it as a "locked treasure room."

Not long afterward (c. 350), the Greek philosopher Aristotle observed a solar eclipse as sunlight passed through openings in foliage.

Date	Individual	Location	Comment
c. 400 BCE	Mozi	China	"Locked Treasure Room"
c. 350 BCE	Aristotle	Greece	Safe Viewing of Solar Eclipse with Tree Foliage
c. 1000	Alhazen	Persia	Full Explanation with Correct Application to Eye
1490	da Vinci	Italy	Ray-Diagram Sketches

The first recorded explanation of the camera obscura was provided c. 1000 by the Arab scientist Abu Ali Hasan Ibn Al-Haitham, known as Alhazen in the West.

Alhazen was born in what is today known as Iraq, but then was part of Persia. He studied images on a wall as light from lanterns passed through a small opening in a room. He explained that light rays from the lanterns traveled in straight lines forming images on the wall opposite to the wall with the opening.

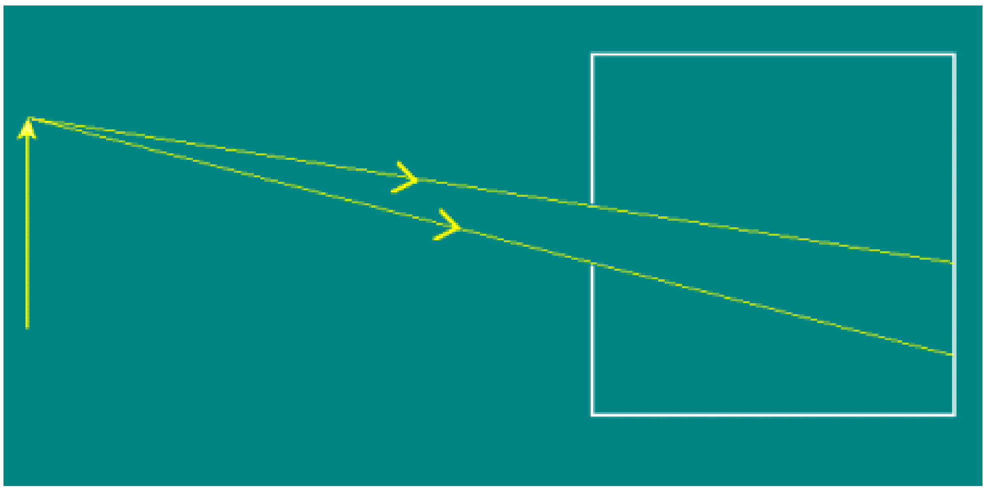
Alhazen also correctly made an analogy with vision, stating that the room was like the eye. Light enters and forms images in the eye. This challenged the rather popular, but erroneous idea that something emanates from the eye and mingles with the changing air due to objects.

The Italian artist-scientist Leonardo da Vinci gave ray diagrams in his notebook in 1490 and 1519. See Alhazen below. A self-portrait of Leonardo also appears below.

Alhazen

da Vinci

The immediate problem with the pinhole is that it is too small. Not much light can enter.



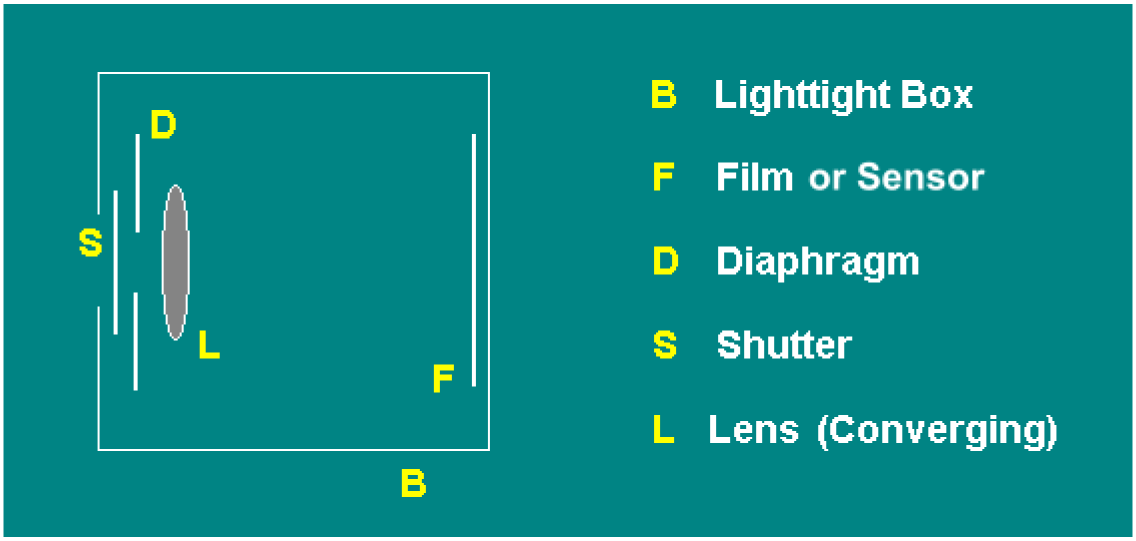
If you make the pinhole larger, as illustrated in the figure, the image becomes blurred. The solution is to use a converging lens. So far, the history of lenses is somewhat mysterious.

The Italian explorer Marco Polo reported marvels after his trips to Asia in the late 1200s. He reported the Chinese use of lenses as spectacles after a trip c. 1270. The Chinese indicated that they got the idea from the Arabs c. 1000.

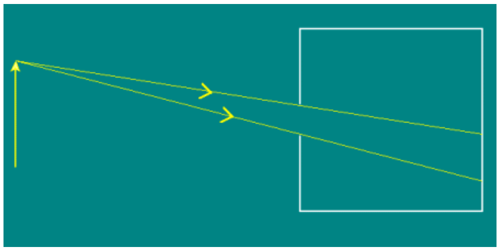
Spectacles were appearing in Europe by 1300. Circa 1550 came the brilliant idea to attach a converging lens (convex lens) to the camera obscura. Now the hole could be made larger. The application of the improved camera obscura for assistance in drawing became widespread.

Now all we need to do is wait a few more centuries for the film to be invented in the first half of the 1800s. That last piece of the puzzle was all chemistry. The basic camera includes a diaphragm to control the size of the opening and a shuttle to determine how much time the light can enter. We return to these in a later chapter. The shutter is near the film in most modern cameras.

Basic Camera

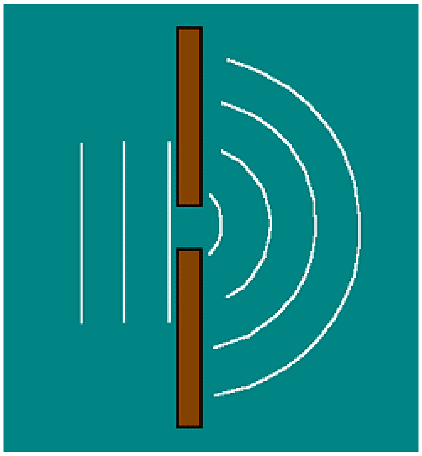


However, pinhole photography is a popular art form. Photographers take photos with the camera obscura. There is no lens. Besides the blur you get if the pinhole is too large, you also



get blur if the pinhole is too small. While we can understand the blur of the large hole by geometrical optics as we sketch rays landing on more than one place in the film, we need to rely on physical optics to explain the blur when the hole is too small. The physics phenomenon is call diffraction. For holes comparable to the wavelength of light, the light will spread out after passing through the opening.

As already mentioned, this spreading out of waves going through openings or around obstacles is called ***diffraction***. The figure at the right depicts a linear wave moving through an opening and turning into a circular wave. Think of water waves. This phenomenon occurs when the wavelength of the wave is comparable to the size of the opening. When the wavelength is extremely small, compared to the opening, there is extremely little diffraction. In order to observe diffraction with visible light waves, which have extremely small wavelengths, you need to send light through a very small opening - a slit. Otherwise, the light passes straight through

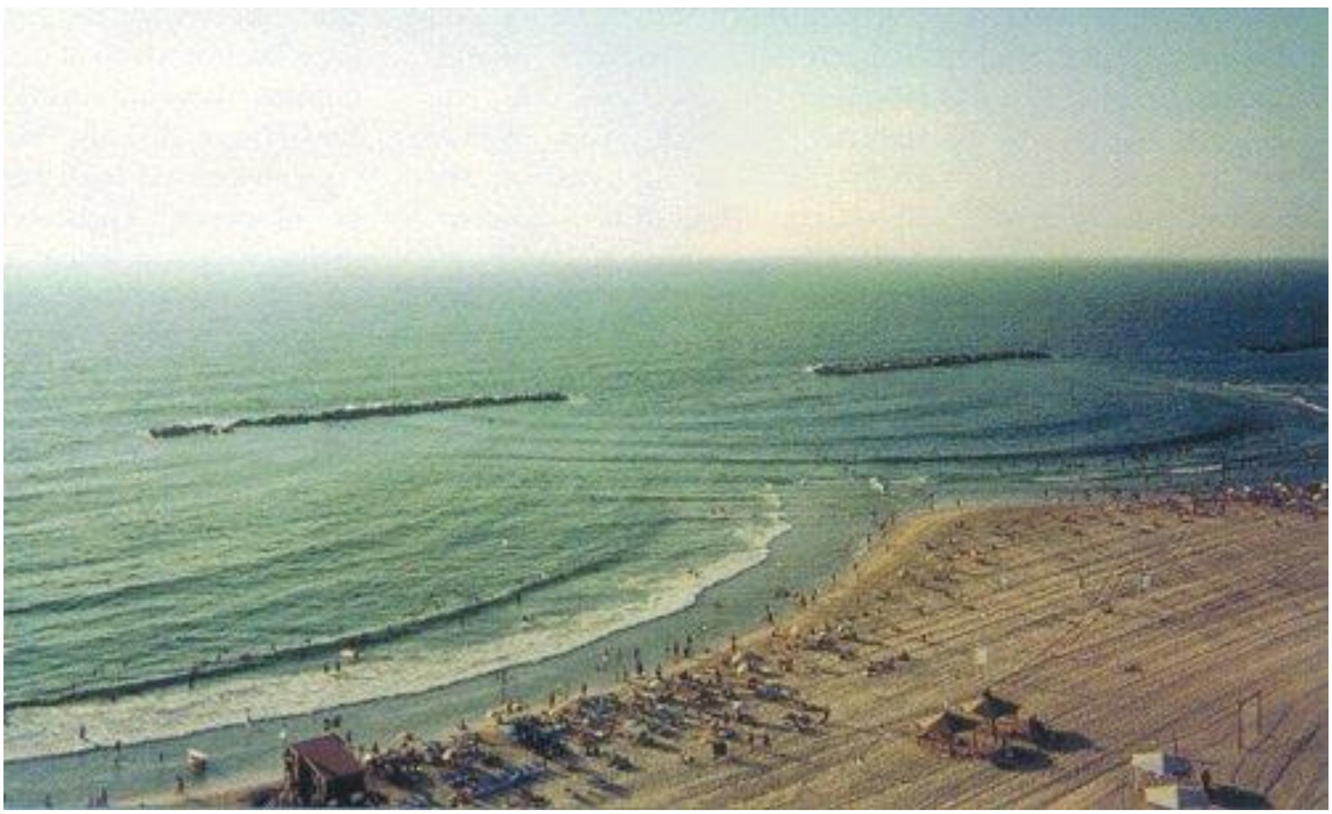


the opening without bending.

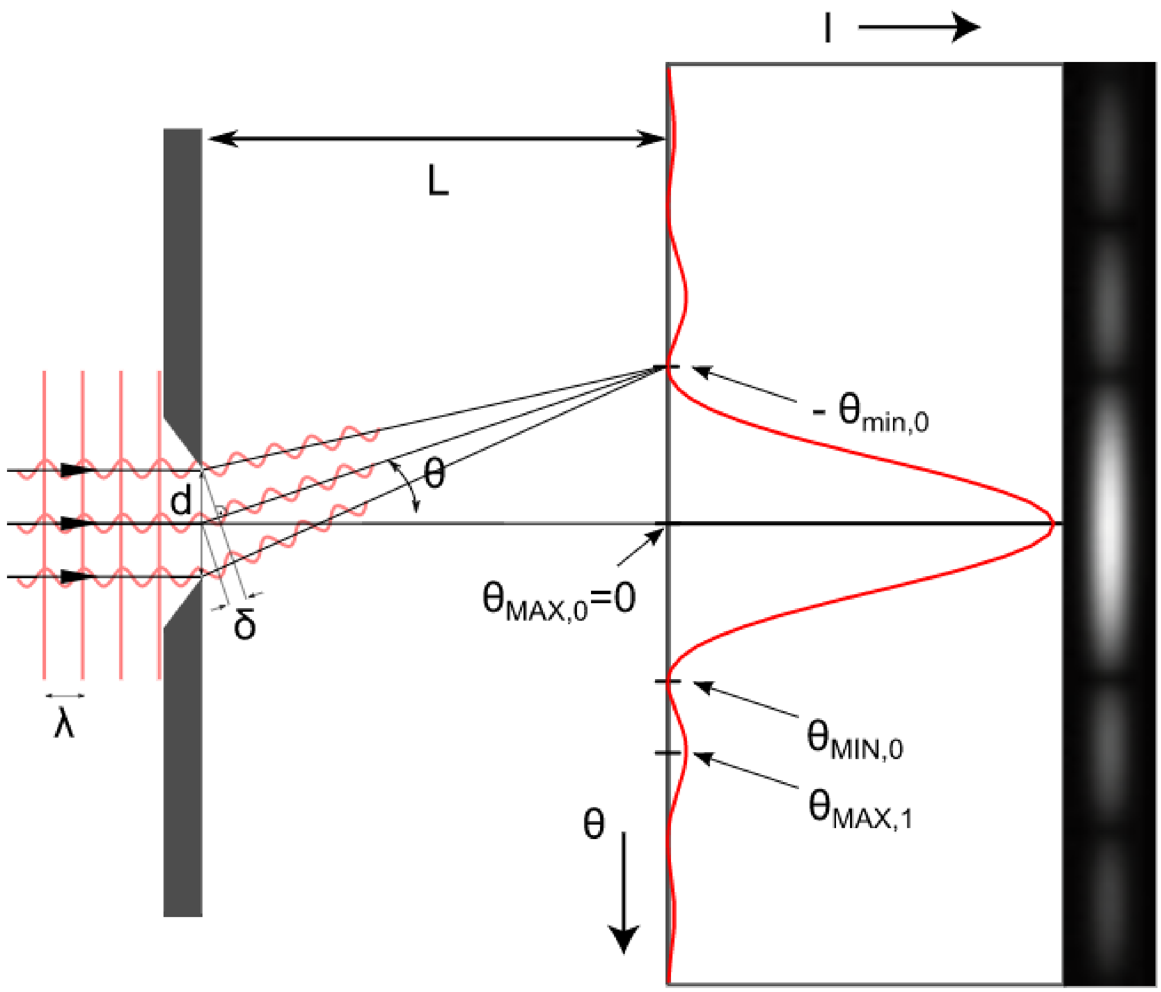
Consider standing in a room where a friend is walking down a nearby corridor. You call after your friend. She hears you and turns. Even though she is down the corridor, she hears the sound due to diffraction. However, after she turns around, she cannot see you since the light doesn't diffract through large openings like doorways. The reason is that the sound waves have wavelengths comparable to the opening of the door (1 meter), while the wavelength of visible light is less than a thousandth of a millimeter!

Below is a fascinating photo taken by Sabina Zigman, as a high school sophomore in a physics honors course (Benjamin N. Cardozo High School, Bayside, New York). She took the photo at a beach in Tel Aviv, Israel. Water waves from the ocean diffract through openings, turning into to nice circular waves. In fact, the beach itself has been shaped by the impact of the circular waves over time. Note the circular beach regions. The photo is published in the January 1999 issue of ***The Physics Teacher*** (submitted by her teacher Sheldon Wortzman).

Diffraction at a Beach



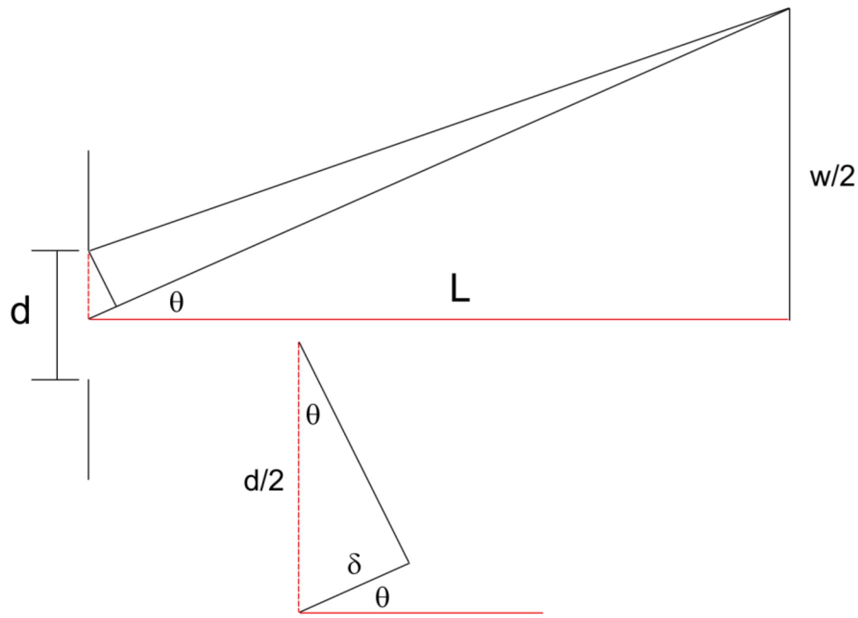
Physical optics provides for the crests and troughs of waves. See the figure below.



Courtesy Wikipedia: jkrieger

That spread out of light at the right screen is not good for taking sharp pinhole photos. Note that at the angle θ shown waves cancel and there is a dark region at the screen at that angle. Let's calculate a formula to find that angle in terms of d , L , and the wavelength λ **for a slit**.

It is a classic formula often taught in introductory physics courses, even in high school. You may have already seen it in an earlier course. The trick is to note that is the change in optical path length δ is just right, the uppermost wave cancels the middle wavy. But the one just below the uppermost wave (not show) will cancel with the one just below the middle one (again not shown). Such a cancellation is called destructive interference. The waves interfere with each other. Crests meets trough and the waves cancel. Such a pair is said to be out of phase. When they travel along that angle and reach the screen, the cancelation occurs at the screen. There is a dark spot there. Then if the maximum of a nearby image falls at this dark place, we can barely make out the two. So this becomes a criterion for us in determining the diameter of our pinhole.



The condition for the wave at the top of the aperture to a corresponding wave at the center of the aperture as it reaches the screen is

$$\delta = \frac{\lambda}{2}.$$

Then a crest will meet up with a trough at the far right screen. The waves are out of phase by one-half wavelength, i.e., 180° out of phase.

The difference in optical path is the parameter δ. From the figure,

$$\delta = \frac{d}{2}\sin\theta.$$

Therefore, the condition to find that first minimum on the screen due to a slit is

$$\delta = \frac{d}{2}\sin\theta = \frac{\lambda}{2},$$

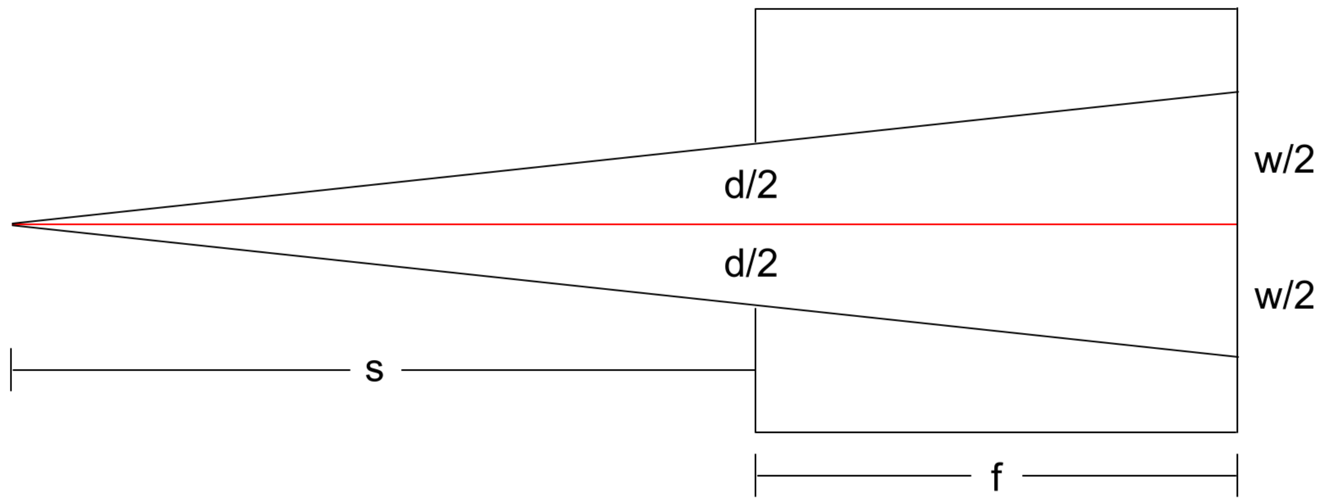
$$d\sin\theta = \lambda.$$

For small angles we can approximate $\sin\theta \approx \tan\theta = \frac{w/2}{L}$. Then,

$$d\,\frac{w/2}{L} = \lambda,$$

$$d = \frac{2\lambda L}{w}.$$

Consider a pinhole camera where we have exaggerated the aperture of the pinhole to help us set up ratios.



Many pinhole photographers like to call the distance for the aperture to the film (read of the box) as f = focal length. Technically there is no focusing as there is no lens. But for the ideal pinhole size everything is fairly sharp at the read screen or film. So we can at least say that we are in focus. From the geometry,

$$\frac{d}{s} = \frac{w}{s+f} \, .$$

For distances far away compared to the small pinhole camera box, $s >> f$ and we can write

$$s + f \approx s \quad \text{and} \quad \frac{d}{s} \approx \frac{w}{s}$$, which leads to

$$d \approx w \, .$$

Combining the formulas $d = \dfrac{2\lambda L}{w}$ and $d \approx w$, we obtain

$$d = \frac{2\lambda L}{d} \quad \text{and} \quad d^2 = 2\lambda L \, ,$$

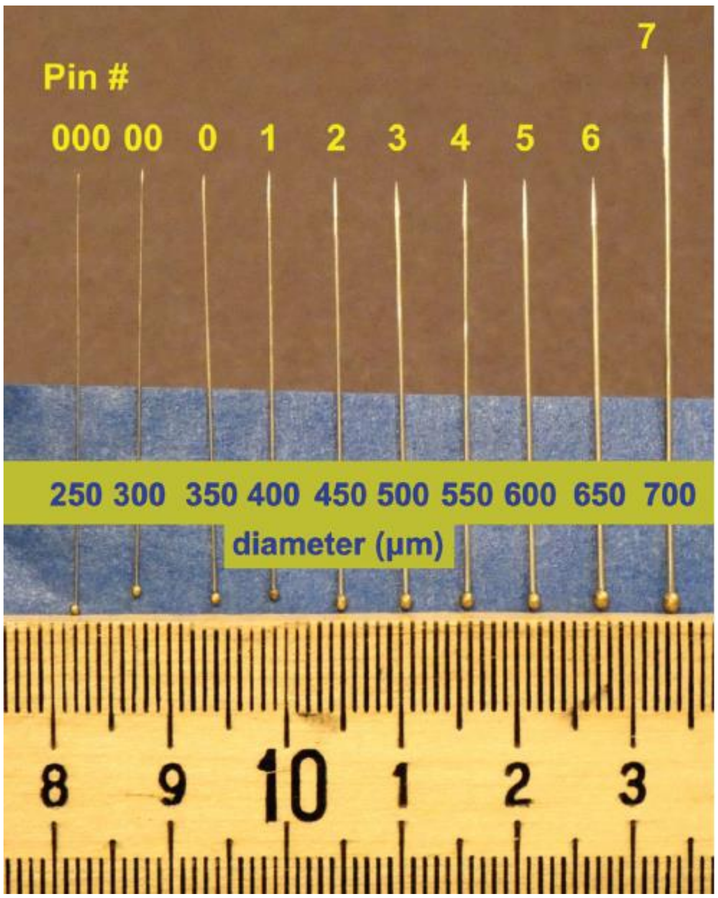
But comparing the above figures, $L = f$. Therefore, $d^2 = 2\lambda f$ and for the slit model,

$$d = \sqrt{2\lambda f} \, .$$

This formula serves as a guide in finding the optimum aperture diameters. You don't want to be smaller than this aperture size. There are different models in arriving at the criterion formula for the aperture. They are of the form similar to ours,

$$d = \sqrt{k \lambda f} \ ,$$

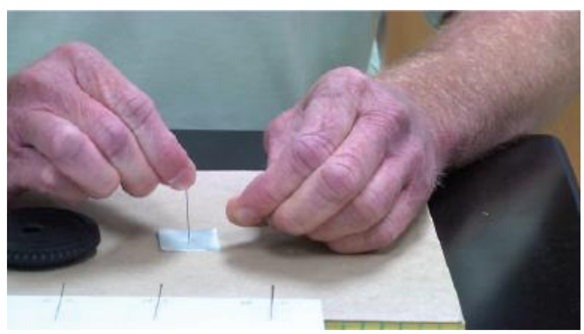
where k depends on the criterion used. There is no criterion that stands out as the correct one. There k varies. One criterion based on the diffraction disk, but analyzed with differential equations, leads to the Airy disk at the screen and k is found to be $k = 2.44$. Another approach uses a formula Rayleigh published once with $k = 3.66$. Deriving these theoretical values would be a nice undergraduate research project.



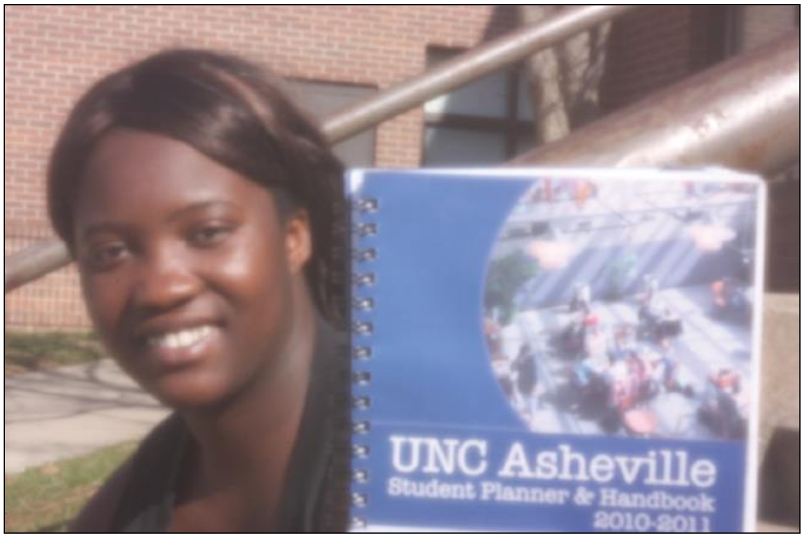
At UNCA, Gerson Morales, an engineering student, did experimental research under my direction in order to find the optimum pinhole size. Thanks to another coauthor, Herb Pomfrey in biology, we learned about insect pins with standard diameter specs. These pins are used by entomologist to mount insects on boards.

The standard insect pins are seen in the figure at the left. Professor Perkins, Chair of our Department of Physics and Astronomy, was also part of the project and a coauthor.

Below, Herb Pomfrey is making a pinhole.



Gerson went out with his girl friend Suzette Ogbon to experiment and find the best pinhole size for our digital camera body. By the way, later Suzette went on to earn a Masters in Civil Engineering from the University of Idaho and Gerson went on to earn a BS in Electrical and Computer Engineering at North Carolina State University. And they got married!



The optimum sharpness was found with the $d = 250\ \mu\text{m}$ pin. For our camera setup

$$f = 37\ \text{mm}.$$

When using $d = \sqrt{k\lambda f}$, one picks the wavelength where the eye is most sensitive. The most sensitive wavelength is in the yellow green at

$$\lambda = 550\ \text{nm} = 0.550\ \mu\text{m}$$

The predicted optimum apertures according to the Airy and Rayleigh values are respectively

$$d_{\text{Airy}} = \sqrt{2.44\lambda f} = \sqrt{2.44\cdot(0.550\ \mu\text{m})\left(37\cdot 10^{-3}\ \mu\text{m}\right)} = 233\ \mu\text{m}$$

and

$$d_{\text{Rayleigh}} = \sqrt{3.66\lambda f} = \sqrt{3.66\cdot(0.550\ \mu\text{m})\left(37\cdot 10^{-3}\ \mu\text{m}\right)} = 273\ \mu\text{m}.$$

And guess what? The average

$$d = \frac{d_{\text{Airy}} + d_{\text{Rayleigh}}}{2} = \frac{233\ \mu\text{m} + 273\ \mu\text{m}}{2} = 248\ \mu\text{m} = \ 250\ \mu\text{m}.$$

the diameter of the pin that Gerson found experimentally to be the best aperture.

Below is a link to our publication, which includes a video abstract.

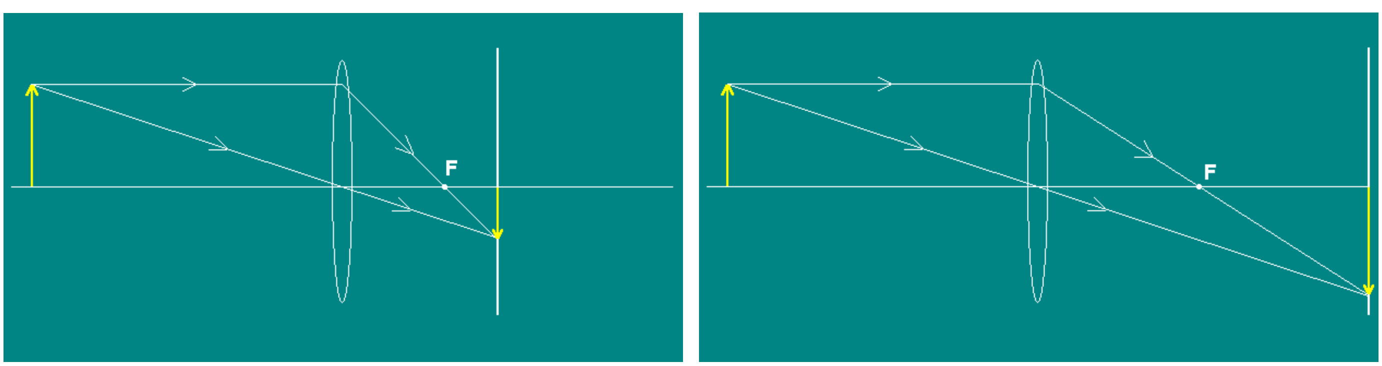
Gerson Morales (UNCA Engineering Student at Time of Research), James Perkins, Herb Pomfrey, and Michael J. Ruiz, "Accurate Pinhole Camera Apertures Using Insect Pins," *Physics Education* **54**, 025002 (March 2019). **pdf** and **Video Abstract**

G2. Aperture and f/#.

A neat set of photos are bound below illustrating six different aperture settings on a camera. The labels you see with the apertures are called f-numbers and an f-number is written as f/#.

Courtesy Wikipedia: KoeppiK

Why is something like an f-number needed? Why not just give the diameter for the aperture such as d = 10 mm? The difficulty of using just the diameter is revealed below.



Suppose we have the same aperture for the two lenses above. Then, the area on the film for the short focal length is smaller than the area for the long focal length. The case with the longer focal length will have the light spread out over a larger area, making the image dimmer. To compensate for this dimmer image, the aperture will have to be larger to compensate. The problem we are trying to solve is finding a method to determine the equivalent area we will in each case so that each image is equally bright.

In the typical arrangement for taking photographs, the object distances are large compared to the size of the camera. Therefore $s_o >> f$. From the lens formula

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f},$$

we find that the image is very close to the focal plane,

$$s_i \approx f \ .$$

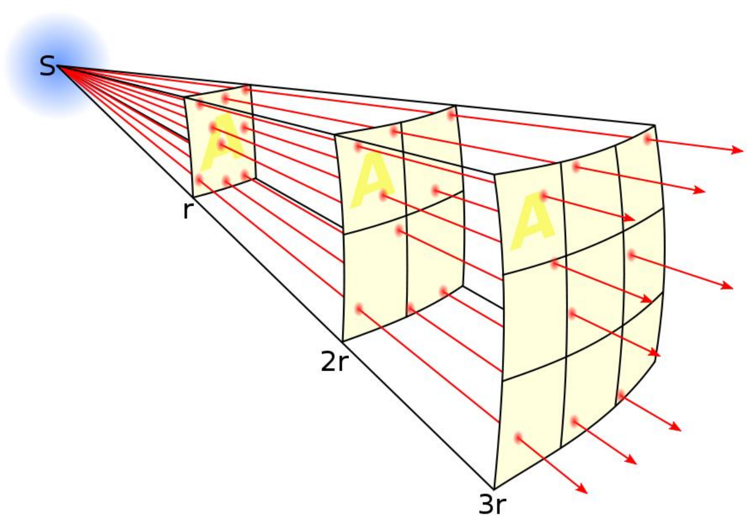
The area of the image at the film is proportional to the square of the image size:

$$A_i \sim h_i^2 \sim s_i^2 \approx f^{\,2} .$$

The intensity of the image decrease as an inverse-square law.

$$I_i \sim \frac{1}{f^{\,2}} \ .$$

Below is the classic inverse-square law from a point source of light.



Courtesy Wikimedia: Borb

Think of the energy from the point source S being distributed over a sphere as the light spreads out in all directions. The intensity times the surface area of a sphere then remains constant due to conservation of energy.

$$I(4\pi r^2) = \text{const}$$

$$I(r) = \frac{\text{const}}{r^2} \sim \frac{1}{r^2} .$$

Returning to

$$I_i \sim \frac{1}{f^2} .$$

If you double the focal length, the intensity of the image drops to 1/4. So we need to quadruple the area of the aperture to compensate. Therefore, in general, to compensate for the $I_i \sim \frac{1}{f^2}$ drop, we need the area of the aperture to increase as follows:

$$A \sim f^2 .$$

But the area of a circular aperture is related to the radius and diameter by

$$A = \pi r^2 = \pi \left[\frac{d}{2}\right]^2 = \frac{1}{4}\pi d^2 \sim d^2 .$$

This relation means we want $d^2 \sim f^2$, i.e.,

$$d \sim f .$$

If you double the focal length, you need to double the diameter of your opening in order to compensate so that the image has the same brightness on the film. The parameter that defines the opening cannot be the diameter itself, but a ratio

$$\frac{f}{d}$$

Let’s call this parameter the f-number #. Then

$$\# = \frac{f}{d} .$$

To see a specific case, take $\# = 4$. Then the diameter

$$d = \frac{f}{\#} = \frac{f}{4}$$

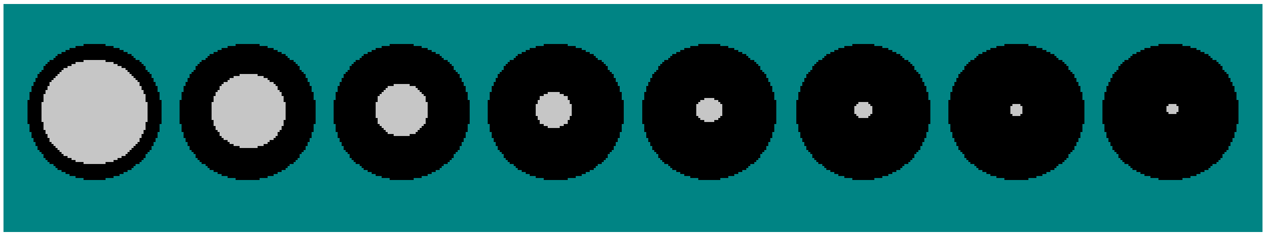
will always be one-fourth the focal length. When you adjust the opening to be $\frac{f}{4}$ the diameter of the aperture will always correspond to one-fourth of the focal length, regardless of the lens, For a 100-mm focal length lens, the opening diameter will be d = f/4 = 100/4 = 25 mm. For a 200-mm focal length lens, the corresponding diameter will be d = f/4 = 200/4 = 50 mm, and so on. The optical engineer designs the lenses so that when you click the f/4 choice into place, the diameter will be coordinated with the corresponding focal length. See f-numbers below where 4 is a choice, in between the 5.6 and 2.8 choices.

Courtesy Wikimedia: MarkSweep

We would like to derive the specific f-number sequence used with lenses. Note our definitions

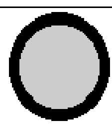
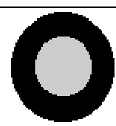
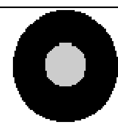
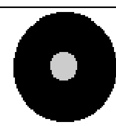
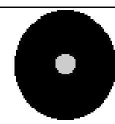
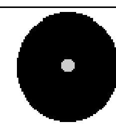
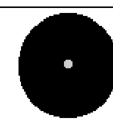
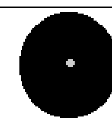
$$d = \frac{f}{\#} \quad \text{and} \quad \# = \frac{f}{d}.$$

The sequence chosen in photography is based on openings that get smaller by 1/2.



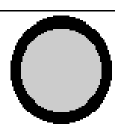
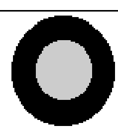
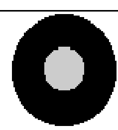
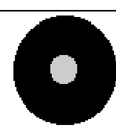
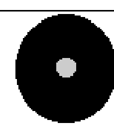
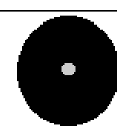
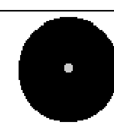
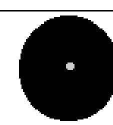
One also uses the term stops. You stop more and more light from reaching the film as you proceed to the right in the above series of areas. Each adjacent area to the right is 1/2 the area of the previous one. If you select a specific opening and look at the neighbor to the left, that neighbor has twice the area compared to specific opening you selected at the start.

Let the left opening above, the largest one, be the case where the f-number $\# = 1$. Then the sequence representing the areas as you go from left right is given below.

Aperture								
Relative Area	1	1/2	1/4	1/8	1/16	1/32	1/64	1/128

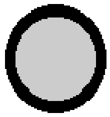
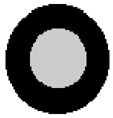
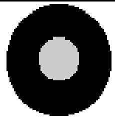
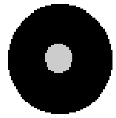
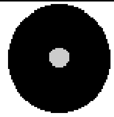
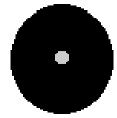
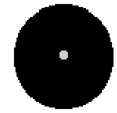
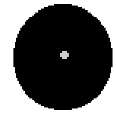
But since the diameter $d = \frac{f}{\#}$ appears in the f-number equation, we want relative diameters.

$$d \sim \sqrt{A}$$

Aperture								
Relative Area	1	1/2	1/4	1/8	1/16	1/32	1/64	1/128
Relative Diameter	$\sqrt{1}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{1}{16}}$	$\sqrt{\frac{1}{32}}$	$\sqrt{\frac{1}{64}}$	$\sqrt{\frac{1}{128}}$

We are almost there. Note that $d = \frac{f}{\#}$ means that the special $\#$ is $\# = \frac{f}{d}$, i.e.,

$$\# \sim \frac{1}{d}.$$

Aperture								
Relative Area A	1	1/2	1/4	1/8	1/16	1/32	1/64	1/128
Relative Diameter d	$\sqrt{1}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{1}{8}}$	$\sqrt{\frac{1}{16}}$	$\sqrt{\frac{1}{32}}$	$\sqrt{\frac{1}{64}}$	$\sqrt{\frac{1}{128}}$
$\# \sim \frac{1}{d}$	$\sqrt{1}$	$\sqrt{2}$	$\sqrt{4}$	$\sqrt{8}$	$\sqrt{16}$	$\sqrt{32}$	$\sqrt{64}$	$\sqrt{128}$
#	1	1.4	2	2.8	4	5.6	8	11

The actual diameter is calculated from the formula

$$d = \frac{f}{\#}.$$

A UNCA student taught me that you can quickly write down the f-number sequence by starting with 1 and 1.4. Then you double the 1 to get the next number which is 2. You then double the 1.4 to get the next 2.8. Then you double the 2, etc. You are skip jumping as you double.

1	**1.4**	**2**	**2.8**	**4**	**5.6**	**8**	**11**	**16**	**22**

In the above sequence all the blue numbers are doubled as you proceed skip jumping from 1. Similarly, all the red numbers are doubled as you skip jump along your journey.

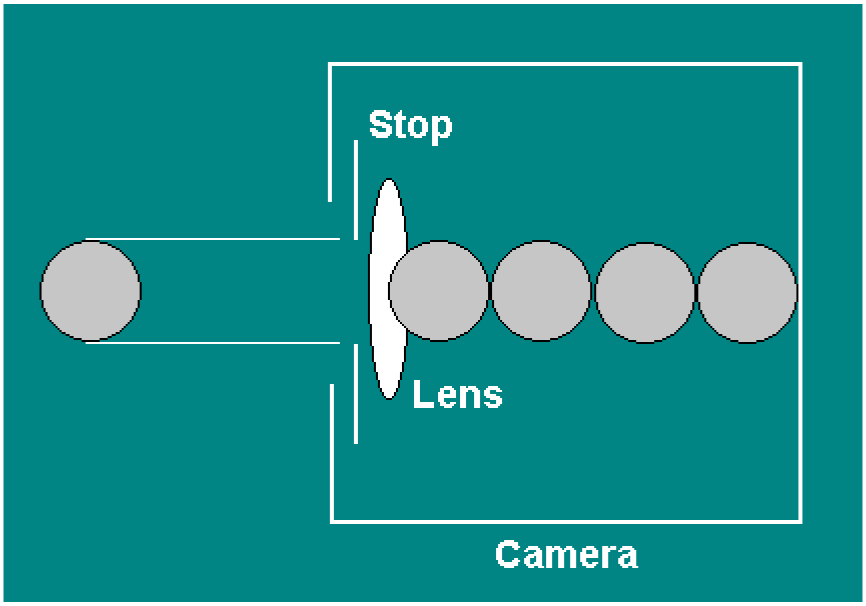
NOTE: The bigger the $\#$ in the above sequence, the smaller the diameter since $d = \frac{f}{\#}$.

Courtesy George Hodan at publicdomainpictures.net

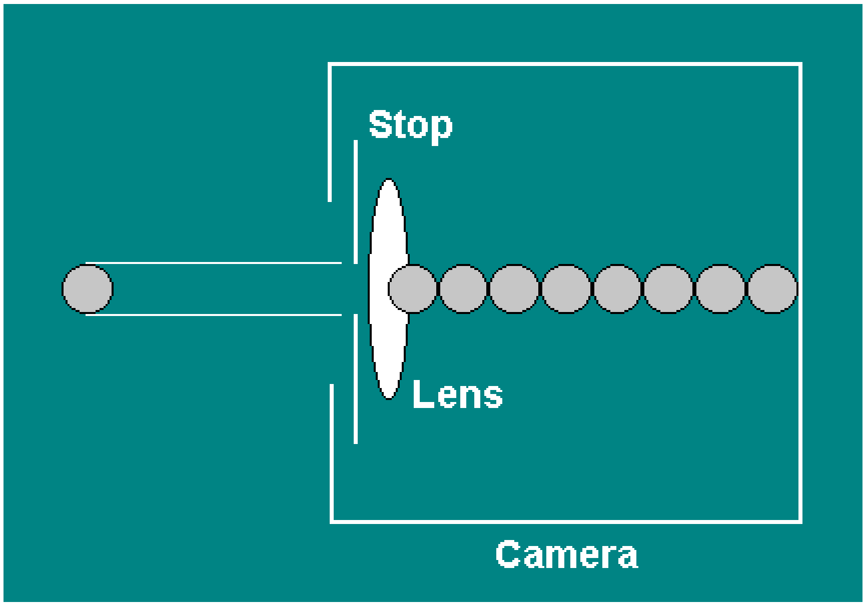
Below is a wonderful visualization that another UNCA student told me was taught by artist Professor Dan Millspaugh in his photography course at UNCA. Dan has since retired. Dan draws a picture that expresses the formula

$$\# = \frac{f}{d} .$$

The case f / 4 since f = 4 d giving d = f / 4.



The case f / 8 since f = 8 d giving d = f / 8.



G3. Full Stops and Fractional Stops.

The standard f-number sequence is a full stop sequence where the area change by a factor of 2 or 1/2. We can write a formula for the sequence

$$\# = \sqrt{2^n} = (2)^{n/2} = \left[(2)^{1/2}\right]^n = (1.4142)^n.$$

For the full-stop sequence n is an integer. You get the next one by multiplying by 1.4142.

n	-2	-1	0	1	2	3	4	5	6	7	8	9	10
#	0.5	0.7	1	1.4	2	2.8	4	5.6	8	11	16	22	32

For half stops, use

$$\# = \sqrt{2^{n/2}} \equiv \sqrt{2^x} = (2)^{n/4} = \left[(2)^{1/4}\right]^n = (1.1892)^n$$

You get the next one by multiplying by 1.1892.

n	-2	-1	0	1	2	3	4	5	6	7	8	9	10
x	-2/2	-1/2	0	1/2	2/2	3/2	7/2	5/2	6/2	7/2	8/2	9/2	10/2
#	0.7	0.8	1	1.2	1.4	1.7	2	2.4	2.8	3.4	4	4.8	5.6

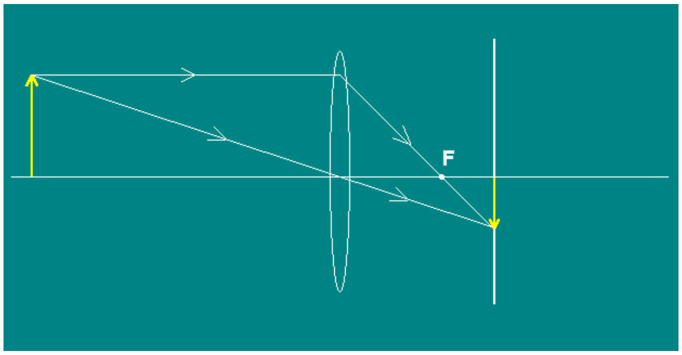
For steps in third stops use

$$\# = \sqrt{2^{n/3}} = \sqrt{2^x} = \left[(2)^{1/6}\right]^n = (1.1225)^n$$

You get the next one by multiplying by 1.1225.

n	-2	-1	0	1	2	3	4	5	6	7	8	9	10
x	-2/3	-1/3	0	1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/2	9/3	10/3
#	0.8	0.8	1	1.1	1.3	1.4	1.6	1.8	2	2.2	2.5	2.8	3.2

G4. Depth of Field.

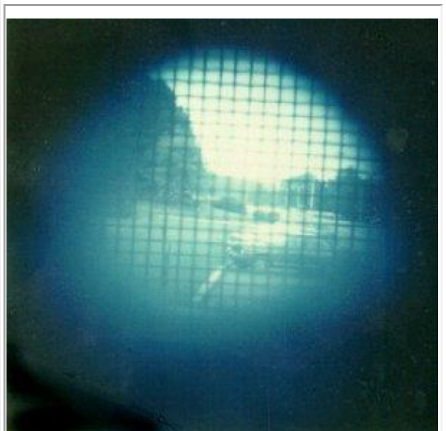


Strictly speaking, there is one distance from the camera which is in focus on the film. Each object distance s_o has a unique image distance s_i .

However, the human eye has limitations and can tolerate some deviation from perfect focus. Therefore, there is a range of distances that appears to be in acceptable or tolerable focus. We call the range of subject distances that are perceived to be in focus the ***depth of field***. You also know from pinhole physics that for the pinhole camera everything is sharp since there is no need for focusing. Therefore, the pinhole camera has an ***infinite depth of field***.

Below is a comparison between a pinhole camera and the classic single-lens reflex (SLR) camera. The pinhole photo at the left shows the screen in focus along with the distant cars. The right photo was taken with an SLR camera focused on the screen. In the right photo, the screen is too small to see even though the camera was focused there. The distant cars are very blurry since the camera was not focused on them and the depth of field did not reach that far.

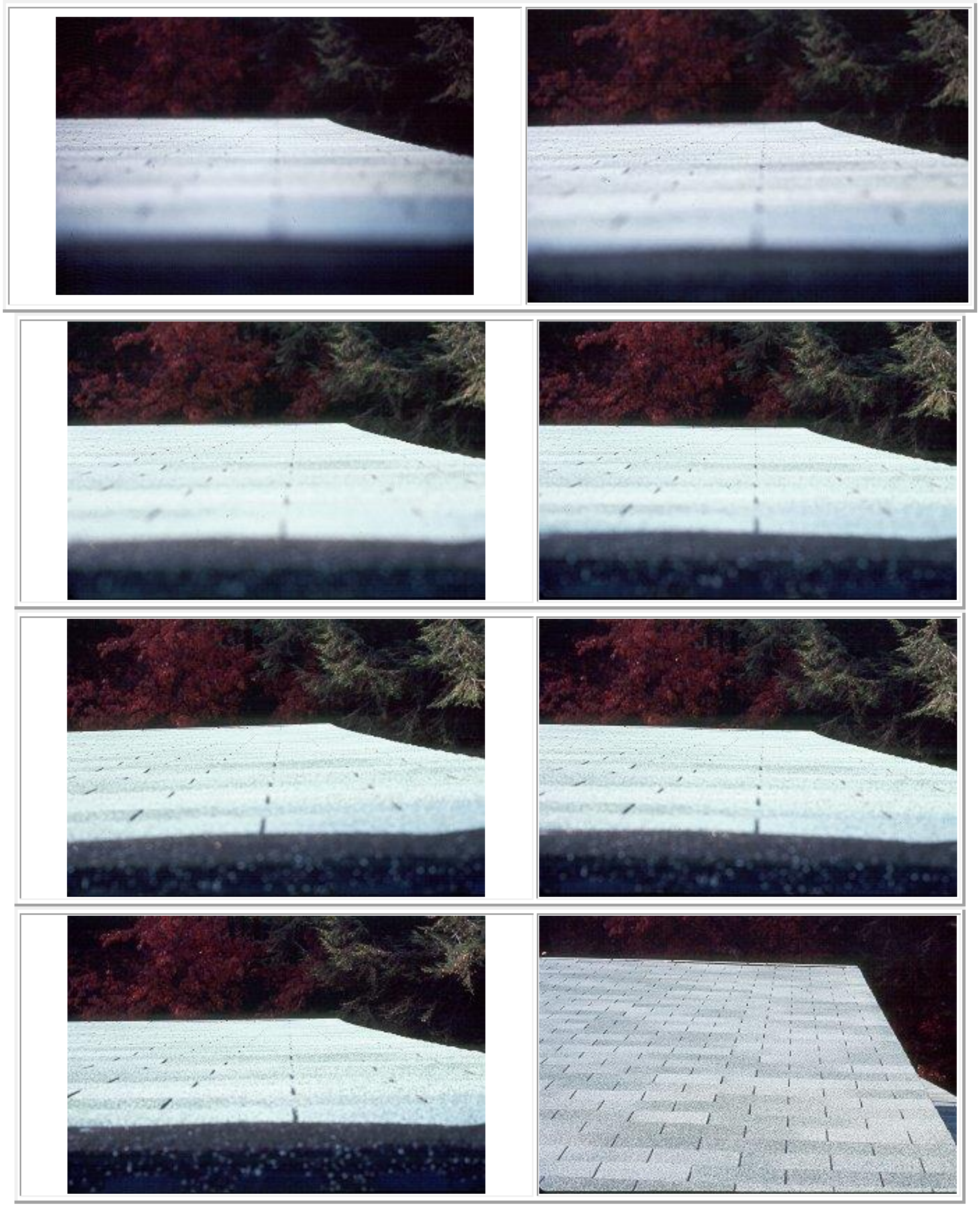
Pinhole Camera and SLR Photographs

Pinhole Camera (Infinite Depth of Field)	**SLR Focused on Window Screen**
	

Photos by Prof. Ruiz (c. 1980), Old Office in Basement of Rhoades Hall

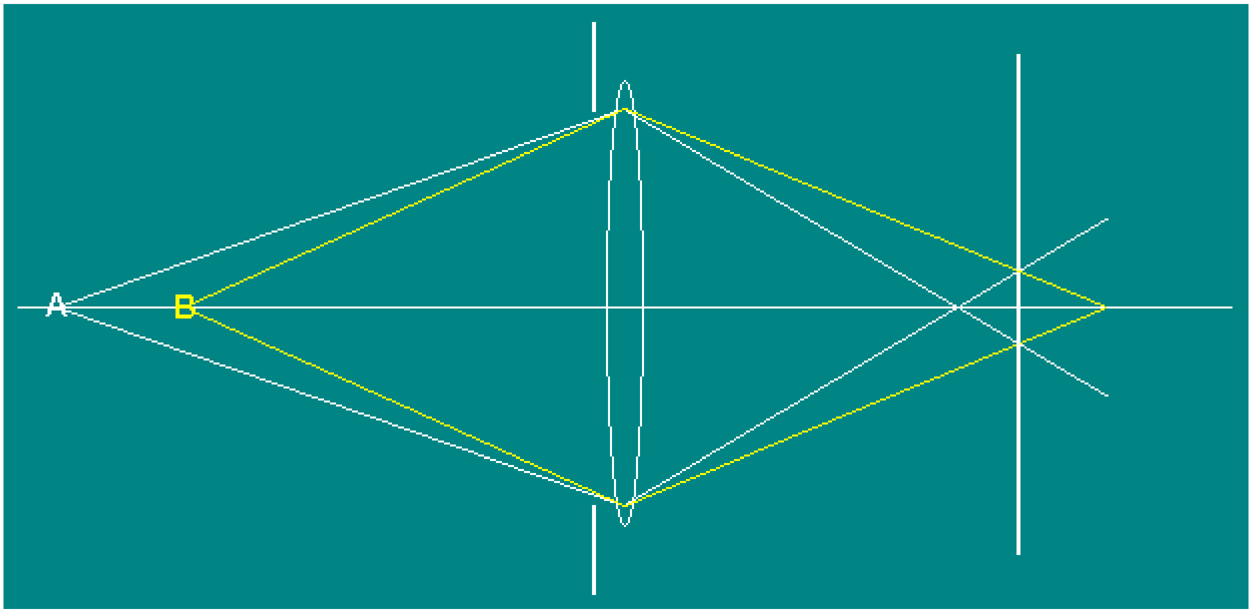
Below are a series of photos taken near the apex of a roof looking at new shingles just tacked on by your instructor. The first photo was taken at about f / 1.8 and the last two photos at f / 22. It is not surprising then that the depth of field improves when aperture size decreases, becoming more like a pinhole. Your instructor-photographer stood up for the very last photo, making it much easier to focus on the entire roof.

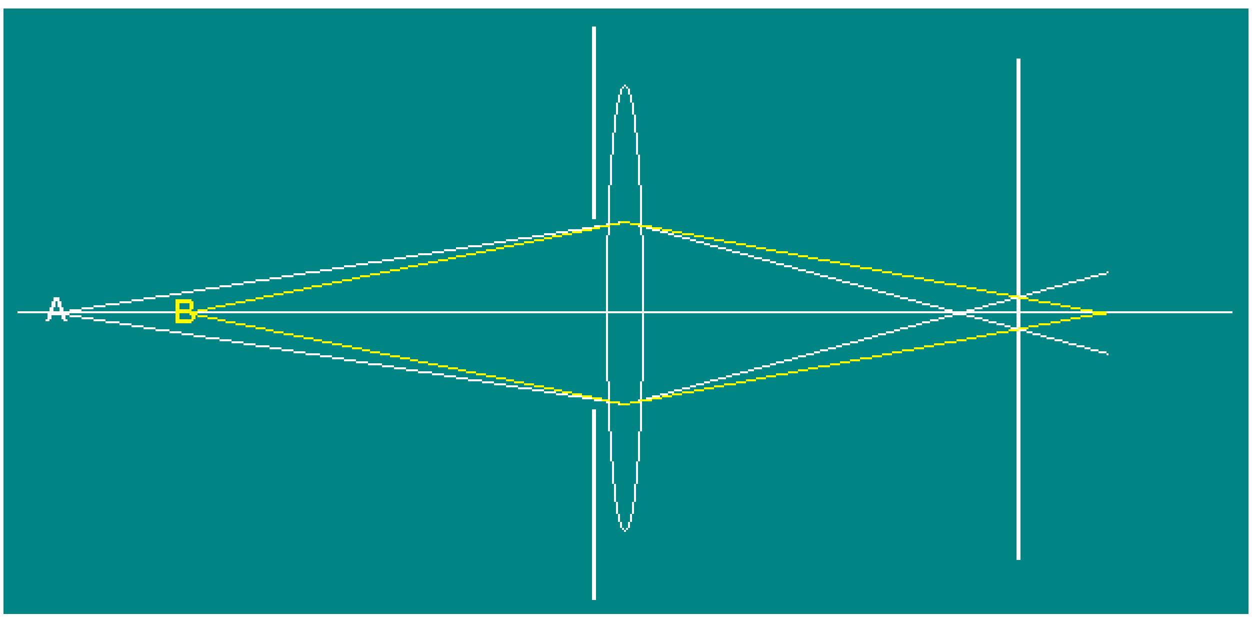
Depth of Field Increasing with Smaller Openings
The Photographer Stood Back for Last Photo



Photos of Prof. Ruiz's Old House on Howland Road Taken by the Prof. (c. 1983)

The following ray diagram illustrates why depth of field is better for smaller apertures. The different points A and B in the figure are focused at different distances behind the lens, as expected by physics. A circle of confusion arises if you try to capture both points on the film. To photo both points, you want the film at the least circle of confusion, i.e., the best compromise. Click on the radio button to reduce the aperture and notice what happens to the circle of confusion. It reduces considerably. With the reduced opening, we can make the separation distance between A and B larger if the former circle of confusion was tolerable.



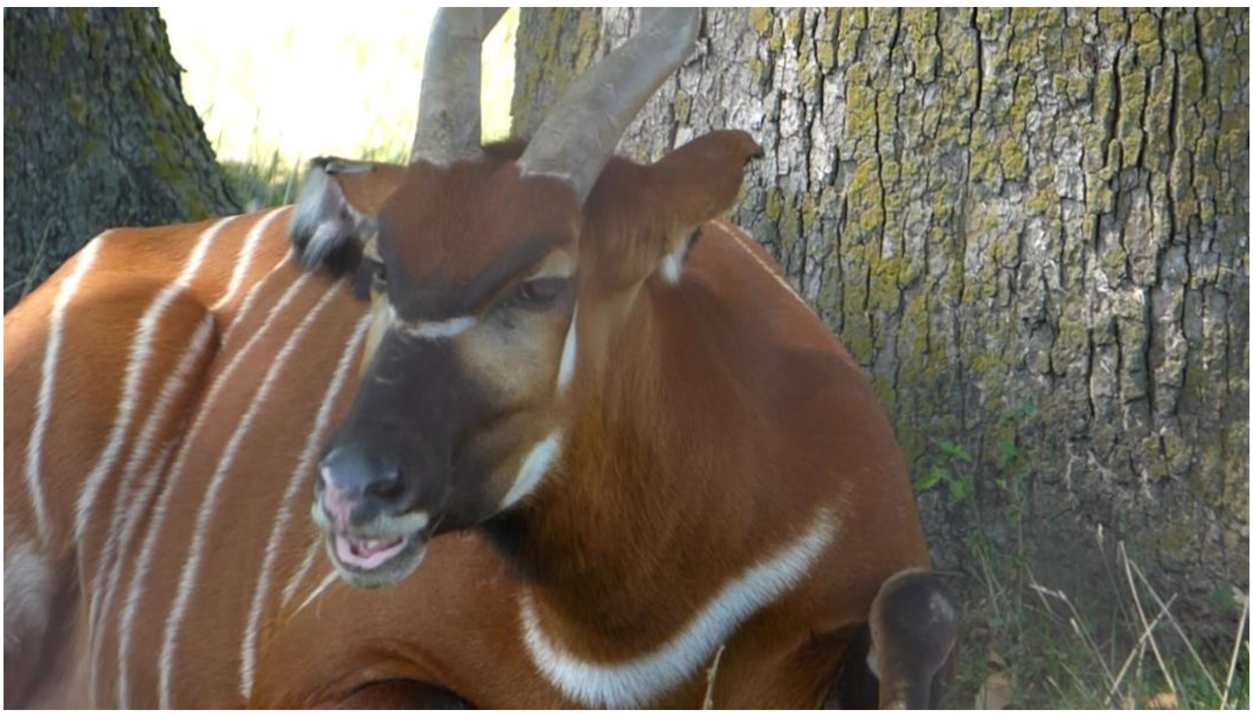


The distance from A to B in tolerable focus is called the **depth of field**. The corresponding distance between the images on the right side of the lens is called the **depth of focus**.

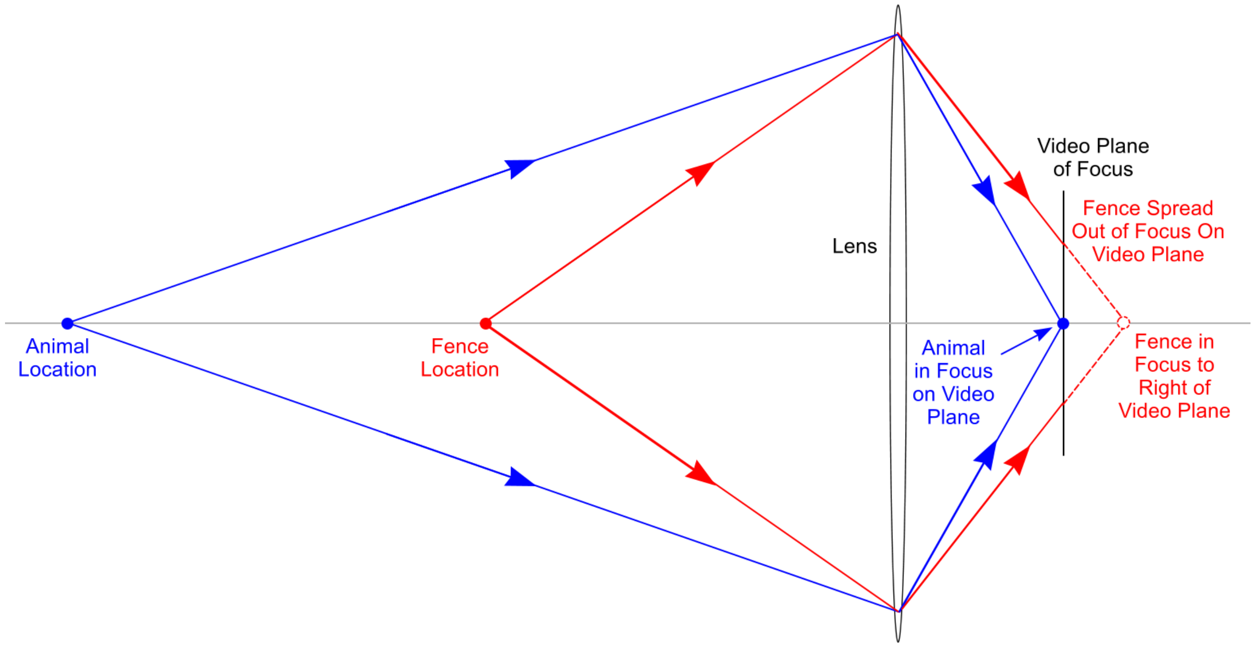
G5. The Vanishing Fence.

Michael J. Ruiz, "Depth of Field and the Vanishing Fence," *Physics Education* **53**, 055015 (September 2018). **pdf** and **Video Abstract**

The Experimental Observation



The Theoretical Explanation



The camera was focused on the African bongo and the image of the fence was blurred so bad that the faint unfocused light from the fence is not even seen.

You actually see through the fence. Check out that light from the animal coming from behind the fence. It is blocked by the fence if you look horizontally straight on. But some of the light from the animal goes above and below the fence reaching the lens, which then focuses it in the video plane of focus.

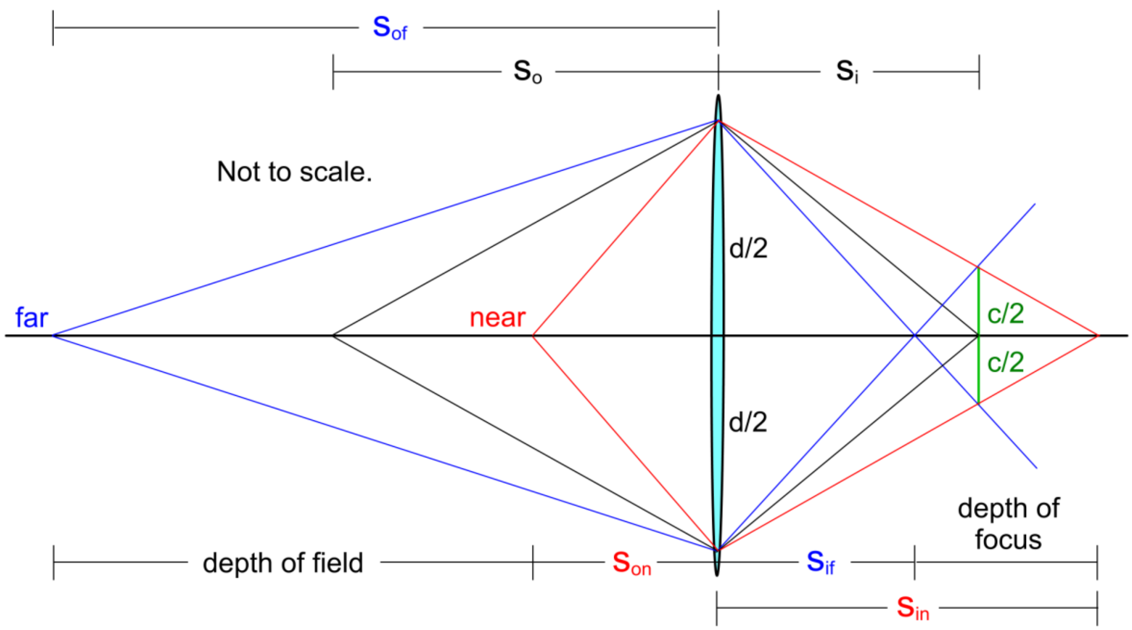
The Camera Sees Through the Fence!

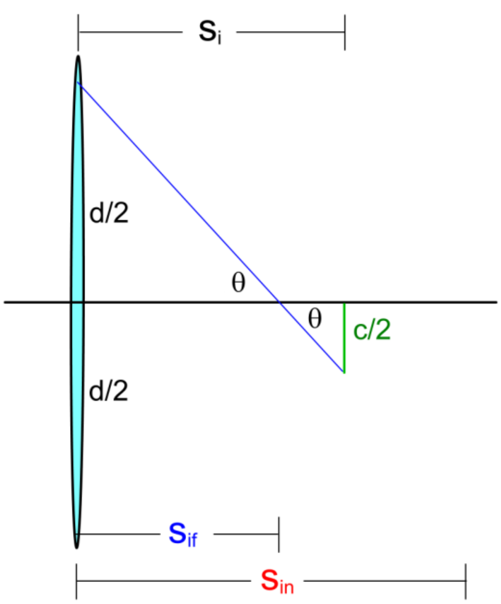
Like the fictitious "X-Ray" Vision of a Superhero, but REAL in our case.

But it doesn't always work. The fence was too close to the lion below.

G6. Depth of Field Formula.

The focus for an object at sa distance of s_o is the image distance s_i. But since the eye can tolerate a little blur, we find that an far object and near object are in tolerable focus. The film is at s_i where now see see what is called the least circle of confusion. Putting the film at other distances results in larger circles of confusion. So we have the best setup below. We would like to solve for the depth of field. Anything within this range will be in tolerable focus. The corresponding image points give the limits for the depth of focus in the image space.





We start by finding s_{of}, which stands for the distance from the lens to the far extreme (blue). Then,

$$\frac{1}{s_{of}} + \frac{1}{s_{if}} = \frac{1}{f}.$$

From the detail at the left we see two similar triangles that give

$$\frac{s_{if}}{d/2} = \frac{s_i - s_{if}}{c/2} \quad \Rightarrow \quad \frac{s_{if}}{d} = \frac{s_i - s_{if}}{c}$$

$$cs_{if} = (s_i - s_{if})d \quad \Rightarrow \quad cs_{if} = s_i d - s_{if} d$$

$$cs_{if} + s_{if} d = s_i d \quad \Rightarrow \quad s_{if}(c+d) = s_i d$$

$$s_{if} = \frac{s_i d}{c+d}$$

$$\frac{1}{s_{of}} + \frac{1}{s_{if}} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{s_{of}} + \frac{1}{\frac{s_i d}{c+d}} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{s_{of}} + \frac{c+d}{s_i d} = \frac{1}{f}$$

$$\frac{s_i d + s_{of}(c+d)}{s_{of} s_i d} = \frac{1}{f} \quad \Rightarrow \quad f s_i d + f s_{of}(c+d) = s_{of} s_i d$$

$$f s_i d + f s_{of} c + f s_{of} d = s_{of} s_i d$$

$$f s_{of} c + f s_{of} d - s_{of} s_i d = -f s_i d$$

$$s_{of}(fc + fd - s_i d) = -f s_i d$$

$$s_{of} = -\frac{f s_i d}{fc + fd - s_i d}$$

But we need the far subject distance in terms of s_o, not s_i .

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{s_i + s_o}{s_o s_i} = \frac{1}{f} \quad \Rightarrow \quad f s_i + f s_o = s_o s_i$$

$$f s_o = s_o s_i - f s_i \quad \Rightarrow \quad f s_o = s_i(s_o - f) \quad \Rightarrow \quad s_i = \frac{f s_o}{s_o - f}$$

$$s_{of} = -\frac{f s_i d}{fc + fd - s_i d} \quad \Rightarrow \quad s_{of} = -\frac{f \frac{f s_o}{s_o - f} d}{fc + fd - \frac{f s_o}{s_o - f} d}$$

Can divide top and bottom by f.

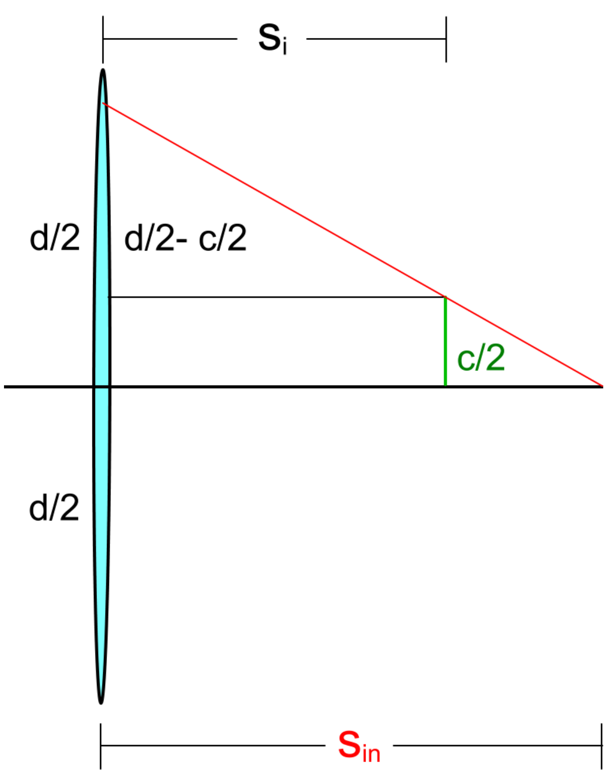
$$s_{of} = -\frac{f\dfrac{s_o}{s_o - f}d}{c + d - \dfrac{s_o}{s_o - f}d} \quad => \quad s_{of} = -\frac{fs_o d}{(s_o - f)(c + d) - s_o d}$$

$$s_{of} = -\frac{fs_o d}{s_o c + s_o d - fc - fd - s_o d}$$

$$s_{of} = -\frac{fs_o d}{s_o c - fc - fd}$$

$$s_{of} = \frac{fs_o d}{-s_o c + fc + fd}$$

$$\boxed{s_{of} = \frac{fs_o d}{fd - c(s_o - f)}}$$



Now for the near extreme distance s_{on} .

$$\frac{1}{s_{on}} + \frac{1}{s_{in}} = \frac{1}{f}$$

From the two similar triangles,

$$\frac{(d/2) - (c/2)}{s_i} = \frac{c/2}{s_{in} - s_i}$$

$$\frac{d - c}{s_i} = \frac{c}{s_{in} - s_i}$$

$$(s_{in} - s_i)(d - c) = cs_i$$

$$s_{in}d - s_{in}c - s_i d + s_i c = cs_i$$

$$s_{in}d - s_{in}c - s_i d = 0$$

$$s_{in}(d - c) - s_i d = 0$$

$$s_{in}(d - c) = s_i d$$

$$s_{in} = \frac{s_i d}{d - c}$$

$$\frac{1}{s_{on}} + \frac{1}{s_{in}} = \frac{1}{f} \qquad s_{in} = \frac{s_i d}{d - c}$$

$$\frac{1}{s_{on}} + \frac{1}{\frac{s_i d}{d - c}} = \frac{1}{f}$$

$$\frac{1}{s_{on}} + \frac{d - c}{s_i d} = \frac{1}{f}$$

$$\frac{s_i d + s_{on}(d - c)}{s_{on} s_i d} = \frac{1}{f}$$

$$f[s_i d + s_{on}(d - c)] = s_{on} s_i d$$

$$f s_i d + f s_{on}(d - c) = s_{on} s_i d$$

$$f s_i d + f s_{on}(d - c) - s_{on} s_i d = 0$$

$$f s_i d + s_{on}[f(d - c) - s_i d] = 0$$

$$s_{on}[f(d - c) - s_i d] = -f s_i d$$

$$s_{on} = -\frac{fs_i d}{f(d-c) - s_i d}$$

$$s_i = \frac{fs_o}{s_o - f} \qquad\qquad s_{on} = -\frac{fs_i d}{f(d-c) - s_i d}$$

$$s_{on} = -\frac{f\,\dfrac{fs_o}{s_o - f}\,d}{f(d-c) - \dfrac{fs_o}{s_o - f}\,d}$$

Divide top and bottom by f as we did before.

$$s_{on} = -\frac{\dfrac{fs_o}{s_o - f}\,d}{(d-c) - \dfrac{s_o}{s_o - f}\,d}$$

$$s_{on} = -\frac{fs_o d}{(s_o - f)(d-c) - s_o d}$$

$$s_{on} = -\frac{fs_o d}{s_o d - s_o c - fd + fc - s_o d}$$

$$s_{on} = -\frac{fs_o d}{-s_o c - fd + fc}$$

$$s_{on} = \frac{fs_o d}{s_o c + fd - fc}$$

$$\boxed{s_{on} = \frac{fs_o d}{fd + c(s_o - f)}}$$

$$\boxed{s_{of} = \frac{f s_o d}{fd - c(s_o - f)}} \qquad \boxed{s_{on} = \frac{f s_o d}{fd + c(s_o - f)}}$$

$$DoF = s_{of} - s_{on}$$

$$DoF = \frac{f s_o d}{fd - c(s_o - f)} - \frac{f s_o d}{fd + c(s_o - f)}$$

$$\frac{DoF}{f s_o d} = \frac{1}{fd - c(s_o - f)} - \frac{1}{fd + c(s_o - f)}$$

$$\frac{DoF}{f s_o d} = \frac{[fd + c(s_o - f)] - [fd - c(s_o - f)]}{[fd - c(s_o - f)][fd + c(s_o - f)]}$$

$$\frac{DoF}{f s_o d} = \frac{fd + c(s_o - f) - fd + c(s_o - f)}{(fd)^2 - c^2(s_o - f)^2}$$

$$\frac{DoF}{f s_o d} = \frac{fd + cs_o - cf - fd + cs_o - cf}{(fd)^2 - c^2(s_o - f)^2}$$

$$\frac{DoF}{f s_o d} = \frac{cs_o - cf + cs_o - cf}{(fd)^2 - c^2(s_o - f)^2}$$

$$\frac{DoF}{f s_o d} = \frac{2cs_o - 2cf}{(fd)^2 - c^2(s_o - f)^2}$$

$$\frac{DoF}{f s_o d} = \frac{2c(s_o - f)}{(fd)^2 - c^2(s_o - f)^2}$$

$$\boxed{DoF = \frac{2fs_o dc(s_o - f)}{(fd)^2 - c^2(s_o - f)^2}}$$

Since typically $s_0 >> f$ and always c is super small compared to the other parameters.

$$DoF = \frac{2fs_o^2 dc(1 - \frac{f}{s_o})}{(fd)^2\left[1 - \frac{c^2(s_o - f)^2}{(fd)^2}\right]} \rightarrow \frac{2fs_o^2 dc}{(fd)^2} = \frac{2s_o^2 c}{fd}$$

Since $d = \frac{f}{\#}$, we can also write $\boxed{DoF = \frac{2s_o^2 c\#}{f^2}}$

Greater depth of field for large #, which means small apertures.

G7. Hyperfocal Length.

The hyperfocal length is defined as $H = s_o$ such that the far extreme in tolerable focus is at infinity, i.e., $s_{of} = \infty$. Then from

$$\boxed{s_{of} = \frac{fs_o d}{fd - c(s_o - f)}}$$

we have $s_{of} = \dfrac{fHd}{fd - c(H - f)} \rightarrow \infty$. The denominator $fd - c(H - f) = 0$.

$$fd = c(H - f) \quad => \quad fd = cH - cf \quad => \quad fd + cf = cH$$

$$H = f + \frac{fd}{c}$$

But since $f << \frac{fd}{c}$

$$H = \frac{fd}{c}$$

and with $d = \frac{f}{\#}$,

$$\boxed{H = \frac{f^2}{c\#}}.$$

Now we find out the near extreme when focused on the hyperfocal distance. We use

$$\boxed{s_{on} = \frac{fs_o d}{fd + c(s_o - f)}}$$

with

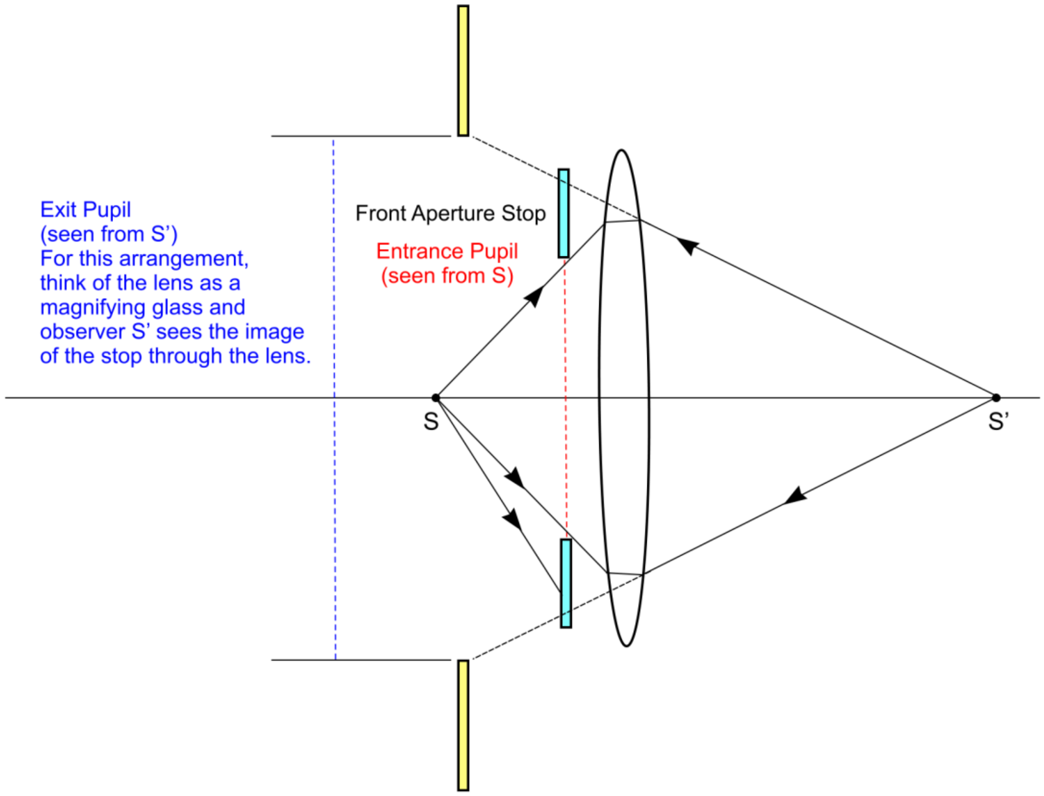
$$s_o = H = \frac{fd}{c}$$

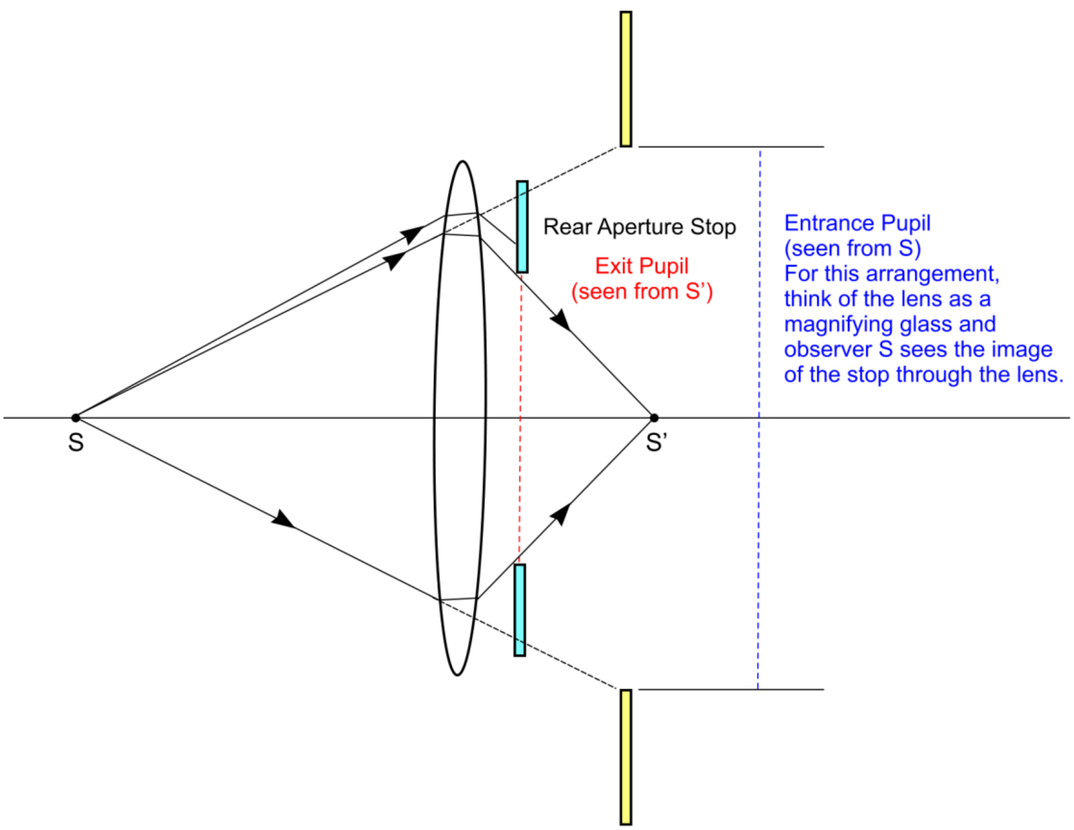
$$s_{on} = \frac{f\frac{fd}{c}d}{fd + c(\frac{fd}{c} - f)}$$

$$s_{on} = \frac{f\frac{fd}{c}d}{fd + fd - cf} \approx \frac{f\frac{fd}{c}d}{fd + fd} = \frac{f^2d^2}{2cfd} = \frac{1}{2}\frac{fd}{c} = \frac{H}{2}$$

$$\boxed{s_{on} = \frac{H}{2}}$$

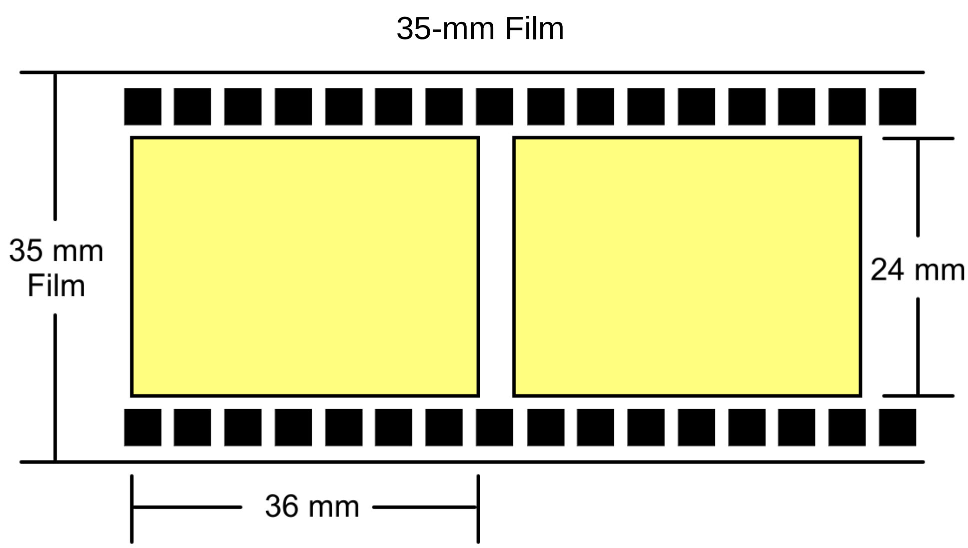
G8. Entrance and Exit Pupils.

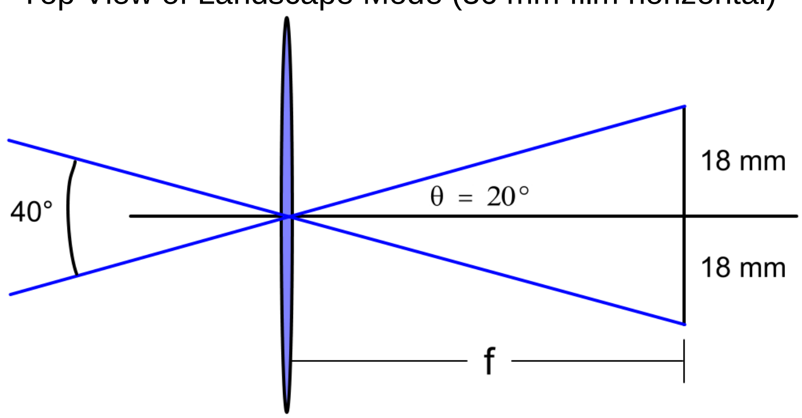




G9. Angle of View.

Here is how you can determine the normal angle of view for the eye. Take one hand and make the peace sign, i.e. the V with two fingers. Then bring the vertex near one eye and close the other eye. That is about the normal angle you take in when you look in front of you. Y=The angle of your V is a little less than 45°. It s more like 40°.





For the classic SLR camera, 35-mm film is used where the picture will fall on a 36 mm x 24 mm section of the film. For the horizontal angle of view we want

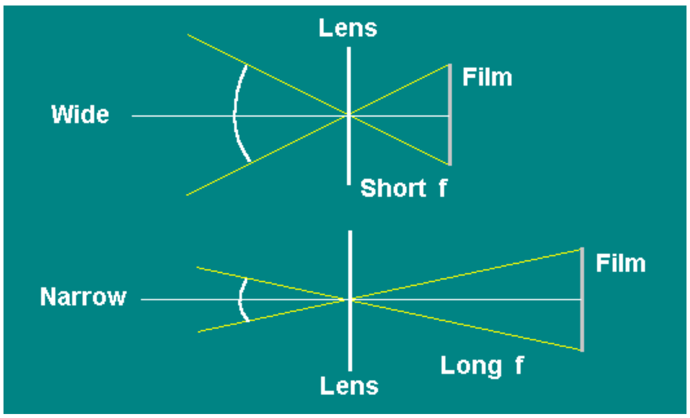
$$\tan 20^\circ = \frac{18}{f}\text{, i.e., } f = \frac{18}{\tan 20^\circ} = 49.46^\circ = 50 \text{ mm}.$$

For 35 mm film, a 50-mm focal length produces the normal angle of view in landscape mode.

Can you show that the vertical angle of view is 27° since the vertical height of the film (24 mm) is less than the horizontal length (36 mm? The diagonal angle of view uses the diagonal measure of 43 mm for the 24 mm x 36 mm film, giving 47°. The convention in our course is to report the horizontal angle of view since the camera is usually held with this orientation in taking pictures.

The figure below illustrates how the angle of view depends on focal length. The professional photographer often prefers to intentionally deviate from the normal angle of view in order to obtain a variety of choices in taking a picture. If you want to photograph a large family sitting on a sofa, you need a wider angle of view than normal. If you want to photograph a bird far away in a tree, you need a narrow angle of view, similar to that of a telescope. A lens with a narrow angle of view is therefore called a telephoto lens. For portrait shots, it is best to get some distance away from the subject and use a gentle telephoto (100 mm) to capture the face;.

Angle of View (Focused at Infinity)



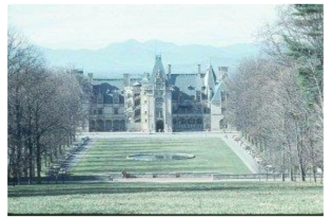
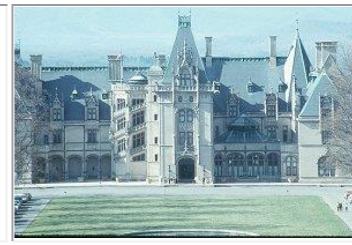
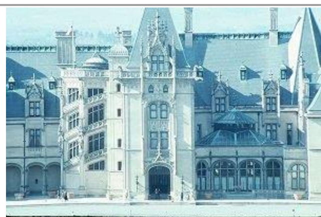
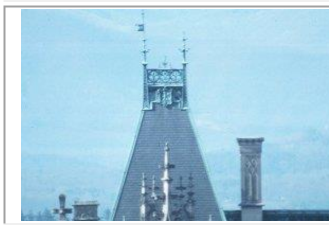
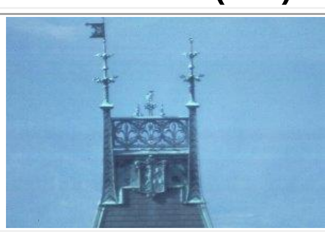
The table below lists lenses with different focal lengths and their horizontal angles of view. Make a mental note that the typical 28-mm lens has a horizontal angle of view of about 60° and that the normal 50-mm gives 40°. When you get to 50 mm and beyond, you can halve the angle when you double the focal length. So going from 50 mm to 100 mm, the angle of view drops from 40° to 20°. To go from 100 mm to 300 mm (which is triple), you reduce from 20° to 20/3, which is roughly 21/3 = 7°.

Lenses by Focal Length for the SLR Camera

Description of Lens	Focal Length (mm)	Horizontal Angle of View
Super Wide Angle	< 24	> 74°
Wide Angle	24 - 35	74° - 54°
Normal	50	40°
Telephoto	80 - 300	25° - 7°
Super Telephoto	> 300	< 7°

The Biltmore House, Asheville, NC with Different Focal Lengths

Camera Optics Publication

		
f = 50 mm (40°)	**f = 100 mm (20°)**	**f = 200 mm (10°)**
		
f = 300 mm (7°)	**f = 400 mm (5°)**	**f = 400 mm (Raising the Camera)**
		
f = 500 mm (4°)	**f = 600 mm (3.4°)**	**f = 800 mm (2.6°)**
		
f = 900 mm (2.3°)	**f = 1200 mm (1.7°)**	**f = 1500 mm (1.7°)**
		
f = 1800 mm (1.4°)	**f = 8 mm (132°)**	