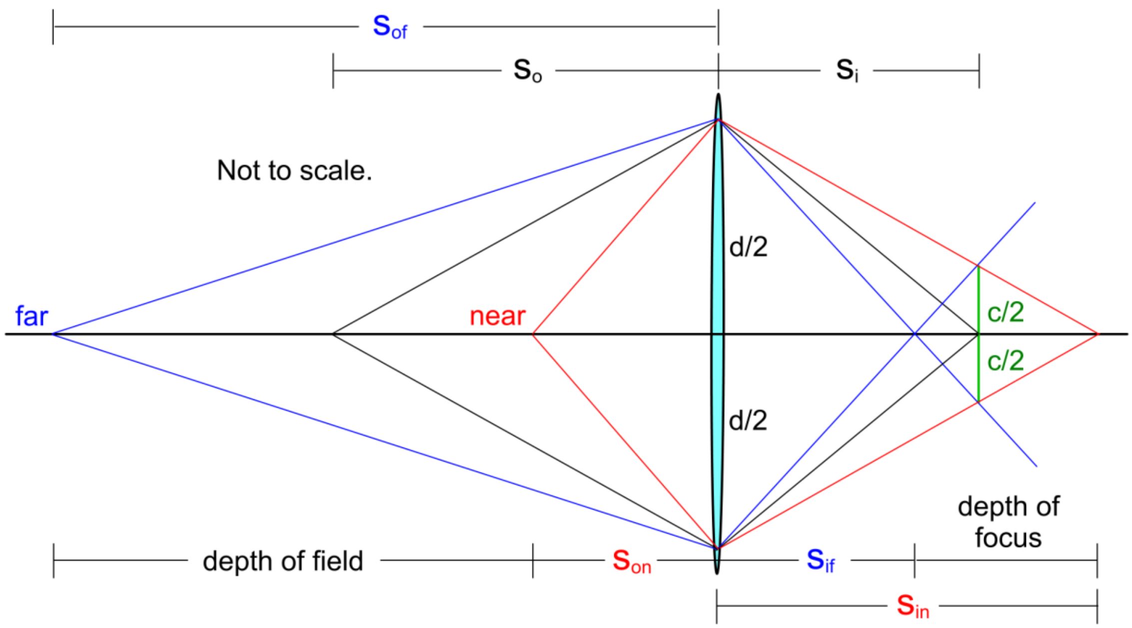
HW-G1. Depth of Focus.



In class we found the formula for the depth of field to be the following.

$$DoF = \frac{2fs_o dc(s_o - f)}{(fd)^2 - c^2(s_o - f)^2}$$

You are to find the formula for the depth of focus: $DoFocus$. Then give the depth of focus from your formula to three significant figures when

$$f = 50 \text{ mm},\ f/\# = f/2,\ s_o = 2 \text{ m},\ c = 50\mu\text{m}.$$

Finally, show that a good approximation to your exact formula is $DoFocus = 2c\#$.

From the book: $s_{in} = \dfrac{s_i d}{d - c}$ $\quad s_{if} = \dfrac{s_i d}{c + d}$. We need to subtract these.

$$DoFocus = s_{in} - s_{if} = \frac{s_i d}{d - c} - \frac{s_i d}{c + d}$$. Note $DoFocus$ must be positive by definition.

$$\frac{DoFocus}{s_i d} = \frac{1}{d - c} - \frac{1}{c + d}$$

$$\frac{DoFocus}{s_i d} = \frac{(c+d)-(d-c)}{(d-c)(c+d)}$$

$$\frac{DoFocus}{s_i d} = \frac{2c}{d^2 - c^2}$$

$$DoFocus = \frac{2cs_i d}{d^2 - c^2}$$, but we need to get rid of that s_i .

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad \frac{s_i + s_o}{s_o s_i} = \frac{1}{f} \quad \Rightarrow \quad fs_i + fs_o = s_o s_i$$

$$fs_o = s_o s_i - fs_i \quad \Rightarrow \quad fs_o = s_i(s_o - f) \quad \Rightarrow \quad s_i = \frac{fs_o}{s_o - f}$$

$$DoFocus = \frac{2cs_i d}{d^2 - c^2} \qquad s_i = \frac{fs_o}{s_o - f}$$

$$DoFocus = \frac{2c\,\frac{fs_o}{s_o - f}\,d}{d^2 - c^2}$$

$$\boxed{DoFocus = \frac{2cfs_o d}{(d^2 - c^2)(s_o - f)}}$$

$f = 50$ mm , $f/\# = f/2$, $s_o = 2$ m , $c = 50\mu$m

$$DoFocus = \frac{2(0.050)(50)(2000)(25)}{(25^2 - 0.050^2)(2000-50)} = \frac{2.5000\times10^5}{1.2187\times10^6} = 0.205 \text{ mm}$$

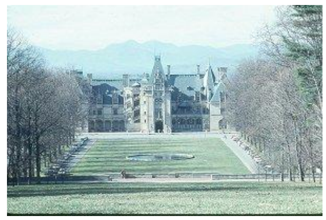
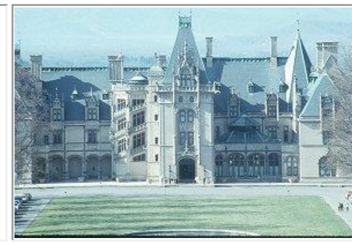
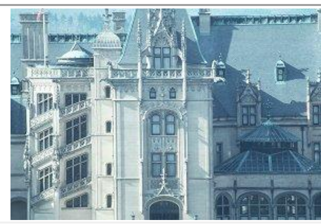
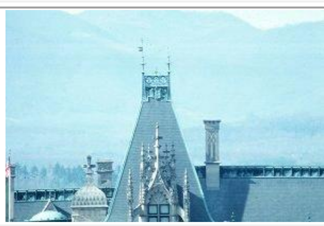
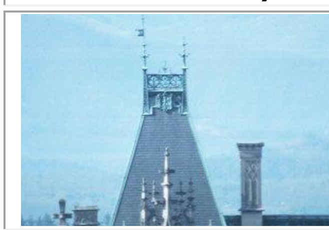
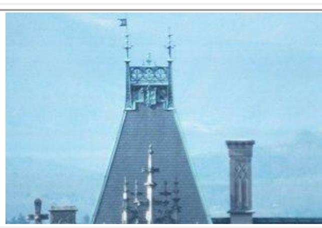
$$DoFocus \approx \frac{2cfs_o d}{(d^2)(s_o)} = \frac{2cf}{d}$$

$$DoFocus = \frac{2cf}{d} = \frac{2cf}{f/\#} = 2c\#$$

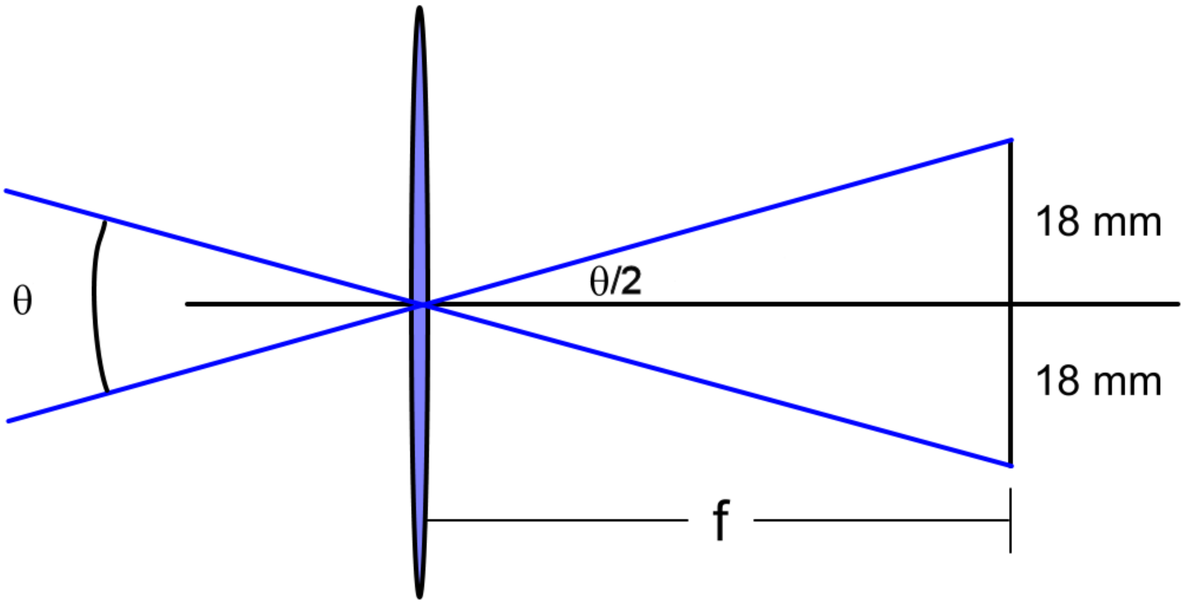
$$\boxed{DoFocus \approx 2c\#}$$

HW-G2. Angle of View. Show that for telephoto focal lengths that the horizontal angle of view for 36 mm by 24 mm film dimension is approximately $\theta = \dfrac{2000^\circ}{f\ (\text{in mm})}$. Then make a table for the focal lengths below, where you give the exact angle of view (show all work for one case) alongside the results of your approximate formula to two significant figures. Include the super wide-angle case to remind ourselves that the approximate formula is not true for wide-angle focal lengths. The approximate formula is restricted to telephoto lenses.

The Biltmore House, Asheville, NC with Different Focal Lengths

		
f = 50 mm (40°)	**f = 100 mm (20°)**	**f = 200 mm (10°)**
		
f = 300 mm (7°)	**f = 400 mm (5°)**	**f = 400 mm (Raising the Camera)**
		
f = 500 mm (4°)	**f = 600 mm (3.4°)**	**f = 800 mm (2.6°)**
		
f = 900 mm (2.3°)	**f = 1200 mm (1.7°)**	**f = 1500 mm (1.7°)**
		
f = 1800 mm (1.1°)	**f = 8 mm (132°)**	

SOLUTION



Note: $\theta = 2\tan^{-1}\dfrac{18}{f}$ or $\tan\dfrac{\theta}{2} = \dfrac{18}{f}$. For small angles, i.e., $f >> 18$,

$$\tan\frac{\theta}{2} \approx \frac{\theta}{2} = \frac{18}{f} \text{ and } \theta \approx \frac{36}{f}\text{ rad} = \frac{36}{f}\text{rad}\cdot\frac{180^\circ}{\pi\text{ rad}} = \frac{6480^\circ}{\pi} = \frac{2063^\circ}{f}$$

For f in mm: $\theta_{\text{approx}} \approx \dfrac{2000^\circ}{f}$ $\qquad \theta_{\text{exact}} = 2\tan^{-1}\dfrac{18}{f}$

Sample Calculation: $\theta_{\text{approx}} \approx \dfrac{2000^\circ}{f} = \dfrac{2000^\circ}{50} = 40^\circ$ (two significant figures)

$\theta_{\text{exact}} = 2\tan^{-1}\dfrac{18}{f} = 2\tan^{-1}\dfrac{18}{50} = 39.6^\circ = 40^\circ$ (two significant figures)

f (mm)	50	100	200	300	400	500	600
θ_{exact}	40°	20°	10°	6.9°	5.2°	4.1°	3.4°
θ_{approx}	40°	20°	10°	6.7°	5.0°	4.0°	3.3°

f (mm)	800	900	1200	1500	1800	8
θ_{exact}	2.6°	2.3°	1.7°	1.4°	1.1°	132° = 130°
θ_{approx}	2.5°	2.2°	1.7°	1.3°	1.1°	**250°**