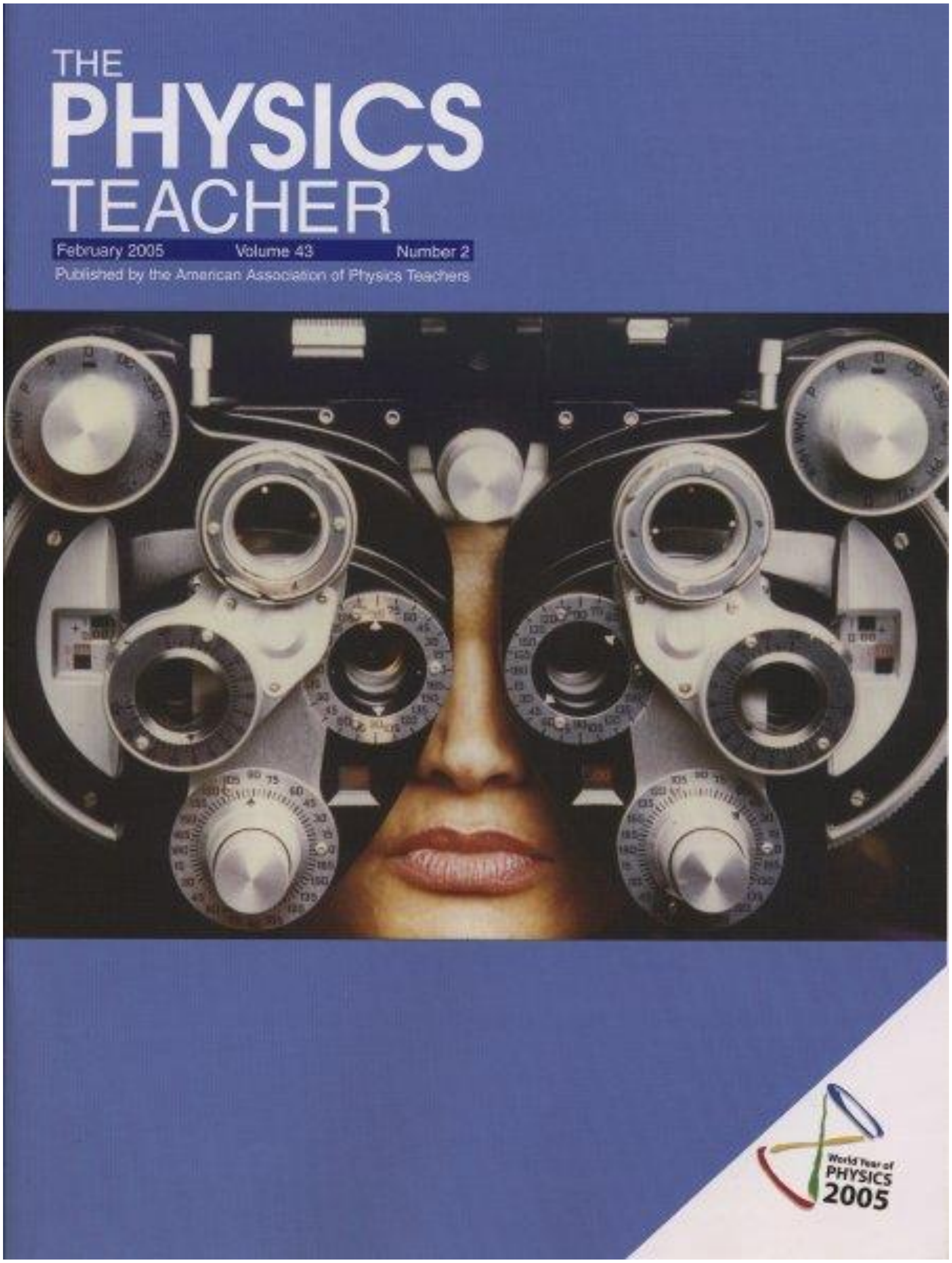
Modern Optics, Prof. Ruiz, UNCA doctorphys.com
Chapter I. Eyeglasses



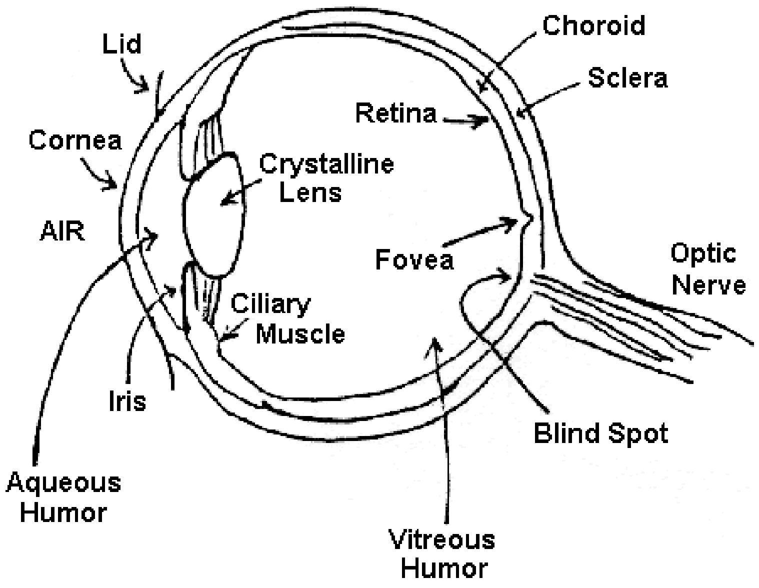
Cover of February 2005 Issue of *The Physics Teacher*

M. J. Ruiz, "Prescribing Eyeglasses for Myopia and Hyperopia," *The Physics Teacher* **43**, 88 (February 2005), featured on the cover of the journal issue. [pdf](pdf)

I1. The Human Eye.

Light entering the eyeball (lighttight enclosure) is focused by the lens system consisting of the ***cornea*** and ***eye lens*** (or ***crystalline lens***). Most of the focusing occurs at the air-cornea interface, where there is a considerable change in indexes of refraction (from air with n = 1 to cornea with n = 1.38). Little refraction occurs at the cornea-aqueous humor interface since there is little change in index of refraction (from cornea with n = 1.38 to aqueous humor with n = 1.33). The index of refraction for the vitreous humor is about the same for the aqueous humor. Light from objects is focused on the light-sensitive region, the ***retina***.

For distant objects, the light rays are essentially parallel when they enter the eye. The ***ciliary muscles*** relax, the ***suspensory ligaments*** (to be discussed more fully later) stretch and the eye lens becomes more flat, offering little additional focusing. For light rays diverging from closer objects, the ciliary muscles become tense, the ligaments become loose, and the eye lens bulges. Greater curvature of the eye lens shortens the overall focal length of the lens system and the images of close objects fall on the retina. This process is called ***accommodation***. The ***near point*** (closest point on which the eye can focus) is about **25 cm** (10 inches) for a young adult.



The ***choroid*** absorbs stray light and serves the same purpose as the antihalation backing in camera film. The ***sclera*** is a tough supporting wall. The ***iris*** (which gives the eye its characteristic color, e.g. blue, brown, hazel) controls the amount of light entering the eye. The opening is referred to as the ***pupil***. The range for the diameter of the pupil is from 2 mm in bright light to about 8 mm in darkness. A small opening (bright days outdoors) minimizes off-axis aberrations. Differences in light levels encountered everyday are far too great for the pupil to adjust for alone. These differences are taken care of by the retina for the most part.

The retina has variable sensitivity (variable "film speed"). It contains receptors which respond to lots of light, the ***cones***, and receptors that only need little light, the ***rods***. The density of cones is greatest in the ***fovea***, where the most precise vision is possible. The rods, which are used when there is little light available, e.g., at night, are distributed more peripherally. The rods do not detect color; at night, "all cats are gray". The eye constantly adjusts to changes in light levels by ***lateral inhibition***, whereby the receptors respond relative to the average illumination of a scene.

The nerves which carry information from the receptors to the brain leave the eye along the ***optic nerve***. There are no receptors in the small region where the optic nerve leaves the eye, the ***blind spot***. It is hardly noticed because it is a small peripheral area, the eye scans slightly which shifts the image around on the retina, and also, because the final vision experienced is a processed image in which the brain plays a role in the processing.

Here we have an experiment to identify the blind spot. Cover your left eye with your hand. Then move to align yourself so that you look straight ahead, directly at the circle. Now move toward and away from the circle, looking directly at it until in your peripheral vision the "X" vanishes. The "X" is then falling on your blind spot. Finally, move farther back and closer to see the "X" appear, disappear, and reappear again. It is only for the proper middle distance that the "X" vanishes.

The Blind Spot

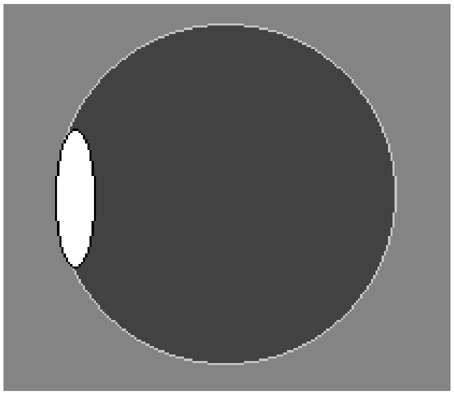
Now try it with the right eye. It will not work because the blind spot is on the nasal side (between the eye and nose) and light entering the eye is reversed both from left to right and up and down. Your brain turns the image around for you. To experience the blind spot in the left eye, align the left eye with the "X" and watch the "O" vanish at the proper distance from the screen. The light from the "O" is then entering the left eye and falling on a region of the retina on the nasal side, its blind spot.

The table below compares the eye to the camera.

Eye Compared to Camera

Camera	Eye
Lighttight Box	Eyeball
Lens	Cornea and Eye lens
Diaphragm and Opening	Iris and Pupil
Film with Antihalation Backing	Retina with Choroid
Shutter	Processing Rate of Eye/Brain

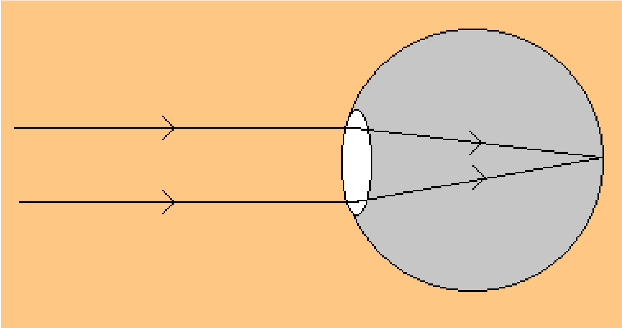
I2. The Eye Lens System.



The lens in the eye acts as an equivalent converging lens much like the compound camera lens. The eye however has two main refractive components, the cornea and eye lens. Here we sketch a simple diagram, assigning the eye an effective lens. The net focal length of the eye focused at infinity is 17 mm. The eyeball itself is about 24 mm in diameter. The effective position of a single lens from the retina depends on the details of the compound lens system, just as it does in a camera.

The dioptric power of the lens system when focused at infinity is obtained from **f = 17 mm**, the equivalent focal length of a lens in air. The result is 1000/17 = 59 diopters, or about **60D**.

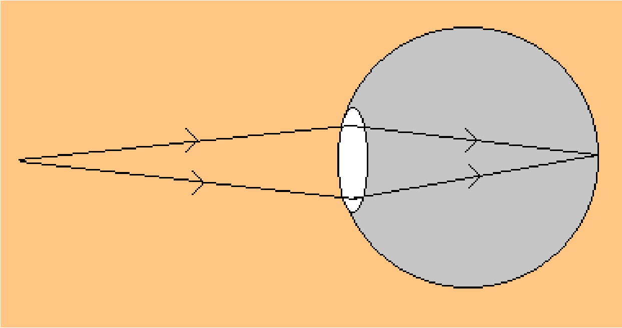
The cornea provides for most of the refraction, about 40D, since the air-cornea interface represents the most dramatic change in index of refraction, from n =1 to n = 1.38. The aqueous humor behind the cornea has an index of refraction of n = 1.34, which is close to that of water (1.33). In fact, the name "aqueous" means like water. The eye lens index of refraction is comparable to the cornea and similarly the vitreous humor index is like that of the aqueous humor. Therefore, there is less refractive power in the eye lens as it is bathed in the eye fluids. Such is necessary because the cornea and eye lens are living tissues that need to be fed. The occasional floater you see as a moving dark shadow in your field of view is material in the vitreous humor.



Eye Focused at Infinity. Total dioptric power =

40D (cornea) + 20D (eye lens) = 60D.

The eye lens gets its dioptric power mostly from its curvature. When the eye is focused at infinity, the curvature is the least and the refractive power is about 20D.



Eye Focused at Nearpoint. Total dioptric power =

40D (cornea) + 24D (eye lens) = 64D.

When viewing an object at the normal 25-cm near point (left diagram, not to scale), the eye lens increases curvature and provides an extra 4D to focus the diverging rays coming from the near object. The total power of 64D corresponds to a focal length of 1000/64 = 16 mm (approximately, rounding off).

The table below summarizes these results.

Dioptric Power of the Eye

Description	Approximate Diopters			Focal Length
	Cornea	Eye lens	Total	
Focusing at Infinity	40	20	60	17 mm
Focusing at Near point (25 cm)	40	24	64	16 mm

I3. The Pupil.

Courtesy Doc's Daughter Christa

The size of the ***pupil***, the aperture, varies from about **2 mm** to roughly **8 mm**.

The effective focal length for the human eye can be taken to be **f = 17 mm**, when the eye is focused at infinity. We will use **f = 16 mm** in the calculations below to simplify the arithmetic. The conclusions are essentially the same either way since 16 is so close to 17.

The ***iris*** acts as the diaphragm in the camera and controls the size of the pupil. We can now arrive at the f-number range for the eye. In bright sunlight, with the pupil diameter of 2 mm, we find

 # = f / d = 16 / 2 = 8 (small pupil size).

At the other extreme, when the pupil is wide open, we obtain

 # = f / d = 16 / 8 = 2 (large pupil size).

Therefore, the f-number range is from **f / 2** to **f / 8**, where the eye can sweep continuously from one limit to the other. Comparing with the typical f-numbers in photography, these are in bold red below.

f/1.4, **f/2**, **f/2.8**, **f/4**, **f/5.6**, **f/8**, f/11, f/16, f/22

The following table indicates the relative light-gathering ability of different f-numbers.

Light-Gathering Capability	Minimum f/number
Low	f / 8
Medium	f / 4
High	f / 1

The f-number span of the eye is not enough to enable us to see such an impressive range of brightness from a star at night to the brilliance of broad daylight. The retina changes sensitivity in order to provide for such impressive variations in available surrounding light. In other words, the "film speed" changes.

The table below compares the human eye to those of nocturnal animals such as the cat, owl, and mouse. Nocturnal animals tend to have f-numbers around f / 1, which has twice the light-gathering capability of the human eye. They need it since they like to hang out at night, while humans like working during daylight.

Eye	Minimum f/number
Human	f / 2
Nocturnal Animal	f / 1

I4. The Retina.

he ***retina***, located at the back of the eye, contains the light-sensitive receptors. These fall into two basic categories, the ***cones*** for day vision and the ***rods*** for night vision or vision in dim light.

The cones can be further broken down into three subtypes. Each of these specializes in one of the three major regions of the spectrum. Loosely, we can refer to these as "blue," "green," and "red" cones, but we will see later that this is a little oversimplified.

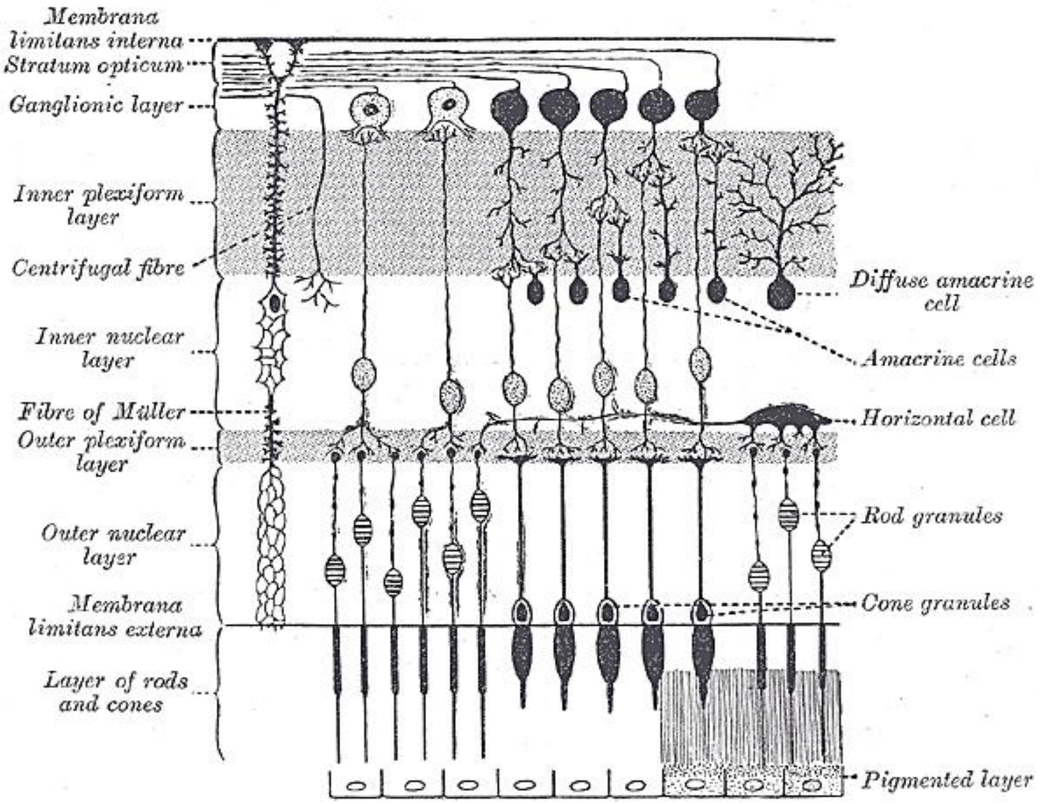
The cones are analogous to fine-grain film. Their light-sensitive regions are smaller than those of the rods. A cone receptor is about 1 micron across and the spacing between them roughly 1.5 microns. However, in our next chapter we will see that retinal-sensitivity varies over a wide range of light levels. The cones and rods can adjust their sensitivities depending on the available amount of light.

In each eye, there are approximately 6 million cones, distributed more so in the ***fovea***, where detailed imaging takes place. There are 120 million rods located peripherally in the retina of each eye. At night, if you avert your vision by looking a little to the side of an object, you will be able to see it better. Astronomers use this technique in viewing single stars at night. Your instructor discovered it as a kid in observing a white pillow in a dark room (about age 10, Camden, NJ). The pillow could not be seen by looking directly at it, but it was observed when the eyes were directed slightly above or a direction near the pillow. Try it. The trick is called ***averted vision***. Try it in dark room sometime or by looking up at a dark night sky.

The table summarizes information about the visual detectors in each retina.

Cones and Rods

Receptor	Number	Location	Use
Cones	6 million	Fovea and Near Axis	In Bright Light
Rods	120 million	Peripheral	In Dim Light



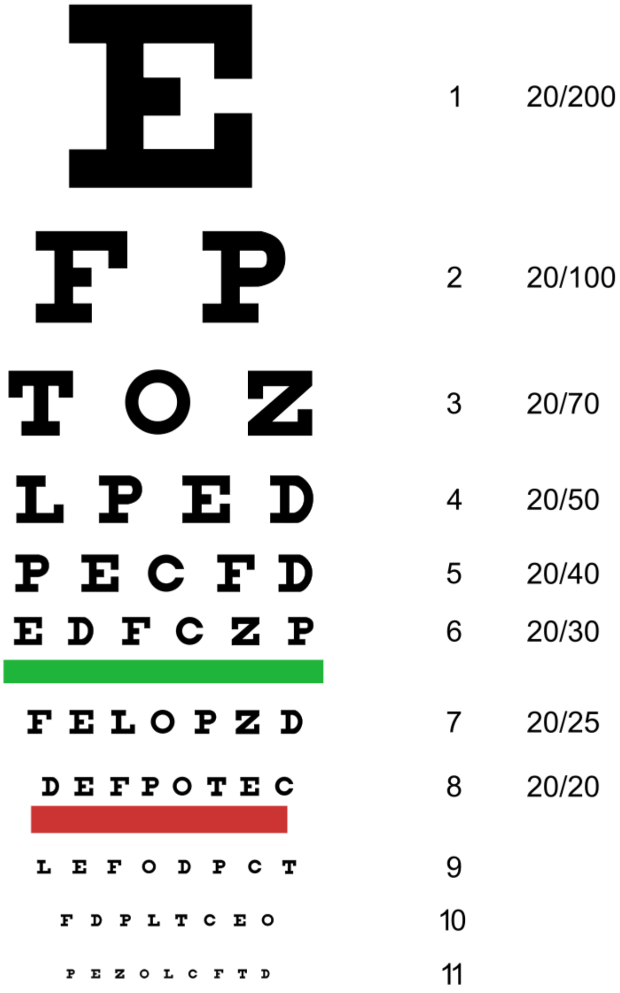
Light Flows in from Top, Henry Gray (1918) Anatomy of the Human Body

The pathways of the neural signals to the brain involve much interaction in the eye itself. Processing of the information begins immediately in the eye. Here is the neural flow:

- receptors - the light-sensitive cones and rods are stimulated;
- horizontal cells - receive information from receptors;
- bipolar cells - receive information from multiple horizontal cells;
- amacrine cells - receive signals from multiple bipolar cells;
- ganglion cells - interface the amacrine cells with the brain.

The processing of information results in effective cross-connections among the receptors. This interconnections play a role in the eye's ability to change its sensitivity. Note also that fewer "neural wires" go to the brain in the "optic cable." Information from 100 million receptors is condensed into about 1 million optic nerve fibers.

I5. The Snellen Eye Chart.



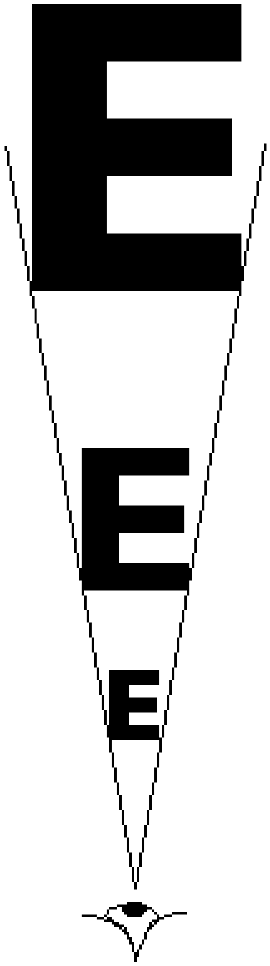
Courtesy Wikipedia: Jeff Dahl

The Dutch ophthalmologist Hermann Snellen (1834-1908) devised his very famous eye chart in 1862 to test vision. The standard version using letters appears here. The Snellen Eye Chart is designed for use at 20 ft. Letters of different sizes are included to determine the visual acuity of the subject.

Using one eye at a time, you start with the bigger letters and work your way down until you cannot discern the smallest letters or until you finish reading the chart. One eye is covered with a card or hand to keep the subject relaxed. Some practitioners may stop the test if one successfully reads the line designated as normal for 20 ft (see line 8).

If an eye can read this normal line, the vision for that eye is said to be 20/20. This means you read at 20 ft (first number appearing in 20/20) what you should read at 20 ft (second number appearing in 20/20). A vision of 20/50 indicates that you read at 20 ft (first number in 20/50) what you should read at 50 ft (second number appearing in 20/50). A vision of 20/15 is superior than the average, where an eye can read at 20 ft what should be read at 15 ft.

For lines 9, 10, and 11 above the distances for the normal eye are 15 ft, 13 ft, and 10 ft respectively. Reading these lines at 20 ft correspond to 20/15, 20/13, and 20/10.



The sizes of the letters that a normal eye can see get larger in a proportional way as distances increase. If you place a letter twice as far away, then you need to double each dimension. The farther letter is then twice as tall and twice as wide. If an eye needs a letter to be twice as tall when compared to what a normal eye requires, then the vision is 20/40. Think of this as follows. The eye should read the proper size for 20 ft at a distance of 20 ft. If the letter needs to be twice as tall, then a normal eye would be able to see this larger letter at 40 ft. Therefore, the poor eye has a vision of 20/40, which is about what you need to pass a driver's test.

Similarly, if an eye can see a letter half the size it needs to be, the eye is 20/10 because the normal eye would need to see a letter half normal size at a distance of one-half as far. A powerful way to pull all this information together so that an eye chart can be used at any distance is described below.

An eye seeing the **20/a** line at **b** ft from the chart has a vision of **20/[20(a/b)]**. Note that the first number in vision assignments is always 20. So, we have a way here to calculate the second number. Let's check the formula for a case where we know the eye is 20/40. Such an eye needs letters brought to half the normal viewing distance. For a large letter, like one on the 20/100 line, this eye needs the letter brought to 50 ft. Let's say we find that this eye reads successfully the 20/100 line at 50 ft. From this we can figure out the acuity. In this case **a = 100 ft** and **b = 50 ft**. Therefore, **a/b = 100/50 = 2**.

Our next step is to figure out the denominator: **20(a/b) = 20(2) = 40**. We can now write the visual acuity as **20/[20(a/b)] = 20/40**. You have just witnessed an important technique in science: checking a formula for a case where you know what the answer should be.

This formula is useful in class when we have contests and let students get as far away from the chart as possible and read lower lines. Suppose the champion is at **50 ft** from a well-lit chart and successfully reads the **20/30 line** with one eye, making her the winner. To find the vision in her best eye note that **a = 30** and **b = 50**. Then **a/b = 30/50 = 3/5**. Next we determine **20(a/b) = 20(3/5) = 12** and we place this number under "20/" to arrive at **20/12** vision.

I6. Best Visual Acuity.

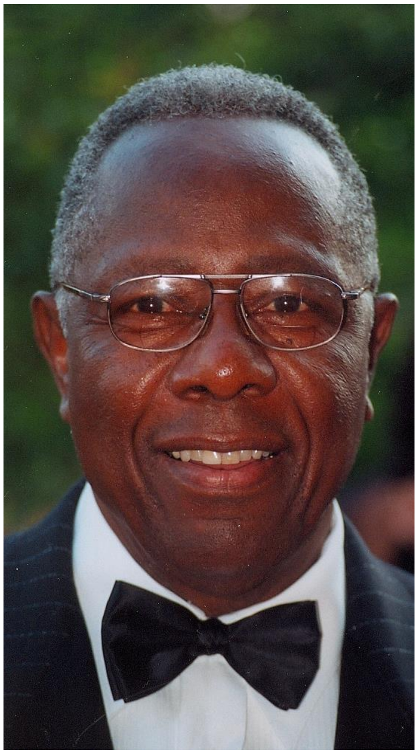
George Herman (Babe) Ruth Jr. (1895-1948).
Courtesy Wikipedia
Photo by Paul Thompson
(1878–1940)

Baseball great Babe Ruth was known to be able to read numbers on distant license plates while his friends had trouble perceiving the color of the plates. He had 20/10 vision in his right eye. Most professional ball players have 20/15 vision. They need this excellent vision in order to see extremely well a 90-mph (140-km/h) fast ball taking less than 0.5 second to reach the plate. You have to decide to swing early during that half-second time frame. You also need fast reflexes and skill.

However, the "Babe" was 20/200 in his left eye, which is considered legally blind! His incredible right eye was able to compensate for the deficiency of his poor eye. The poor vision in the left eye was due to a severe case of ***amblyopia*** (am-blee-OH-pee-uh) or ***lazy eye***, where the eye just refuses to work. About 2 people out of 100 have some form of lazy eye. Cases not treated when young can result in a useless eye as in the case of the Babe.

On April 5th, 1925, Babe Ruth collapsed in Asheville's old railroad station, a station that used to be located in Biltmore Village. He was taken to a hospital in his hometown New York for surgery due to an ulcer. Two seasons later in 1927 he hit his record of 60 homes runs (New York Yankees), a record that stood until 1960 (Roger Marris, New York Yankees, 61). A generation passed before the record was broken in 1998 (Mark McGwire, St. Louis Cardinals, 70) and in 2001 (Barry Bonds, San Francisco Giants, 73).

Babe Ruth hit 714 home runs during his stellar baseball career (1914-1935), a record that stood for decades.



Sharp Visual Acuity and Reflexes
Hank Aaron (755 Home Runs)

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The Babe’s record was broken by Henry L. "Hank" Aaron (b. 1934) on April 8, 1974 when Aaron hit his 715th home run of his career. Hank Aaron went on to finish his career with 755 home runs. Hank Aaron was another star player with excellent vision who played major-league baseball from 1954-1976. As of 2016, Hank Aaron lives in Atlanta, Georgia.

Aaron's record was broken by Barry Bonds (b. 1964) on August 7, 2007. Check out what Dr. Harrison says about Barry Bonds.

"Dr. Bill Harrison, the most renowned visual performance specialist the game of baseball has ever witnessed, has spent nearly 50 years studying how to train the vision of athletes at the highest level possible. ... In almost 50 years of vision testing Major League hitters, Barry Bonds has no equal, according to Dr. Harrison." (baseballnews.com) This measure is not just visual acuity. It includes "hitting specific vision" which means detecting subtle changes in a pitched ball such as speed and location.

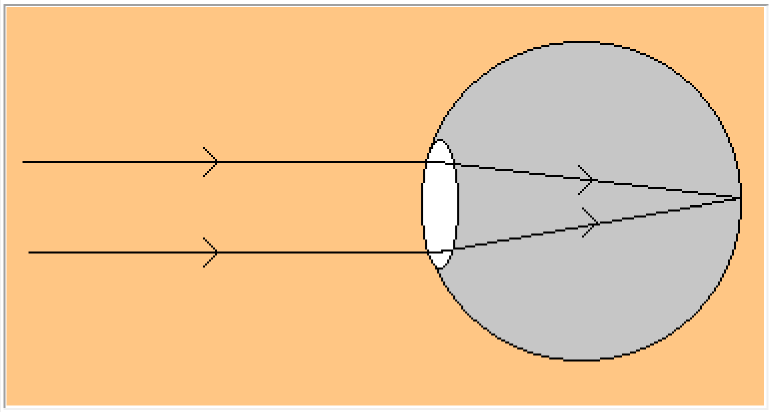
Current Record Holder:
Barry Bonds (762 Home Runs)
Photo of Bonds Courtesy Wikipedia

The best visual acuity for a human eye is 20/10.

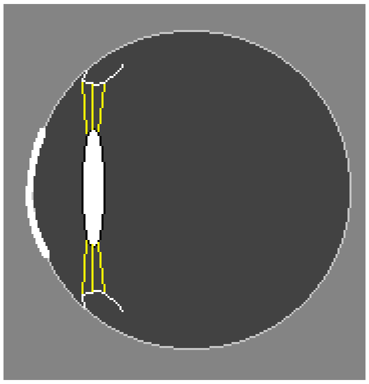
An eagle has 20/5 vision.

I7. Accommodation. The ability of the eye lens to change its curvature and thus its focal length is called accommodation. The eye lens accommodates for far and short subject distances.

Focusing on Far Objects



Focusing Far

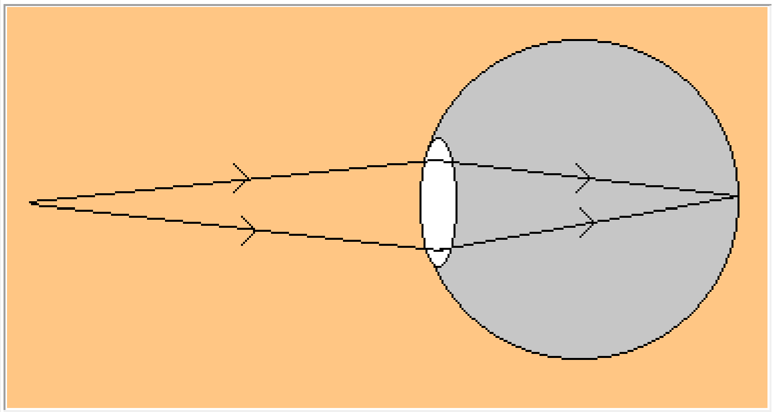


Eye Lens Stretches

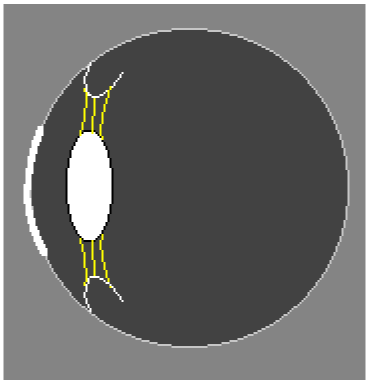
When focusing on distant objects,

- the ciliary muscles relax, pulling away from the lens,
- the suspensory ligaments stretch,
- the eye lens becomes flatter (less focusing power, 20D for eye lens, 60D total).

Focusing on Near Objects



Focusing Near



Eye Lens Bulges

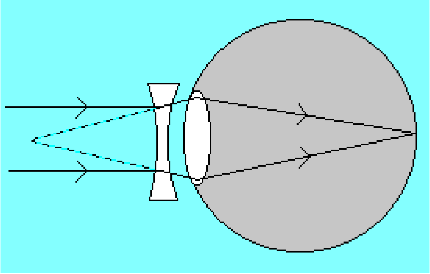
When focusing on nearby objects,

- the ciliary muscles tense (push in),
- the suspensory ligaments become loose,
- the eye lens becomes more curved (more focusing power, 24D for eye lens, 64D total).

I7. Myopia. An eye is said to have ***myopia*** when it cannot focus on distant objects. Such an eye is "sighted to see near" and the condition is also called ***nearsightedness***. The myopic or nearsighted eye has no trouble seeing objects nearby. In fact, it often can see things closer than the average 25-cm near point. The human eye has evolved so that it is relaxed when it focuses at far distances, which has been the usual state of affairs in life over the centuries. Humans have hunted and worked in fields from prehistoric times. Only relatively recently have people began to read.

In many cases, the ciliary muscles just cannot relax enough to stretch the eye lens sufficiently for distant viewing. The eye remains in a constant state focused on near objects. Distant objects appear blurred and out of focus. The 20/20 line on the eye chart cannot be read. The line that the eye can read defines the severity of the myopia. A 20/40 eye does not have as severe a case of myopia as a 20/100 eye.

A diverging lens can be prescribed to correct for myopia. The eye doctor determines the best lens experimentally by placing a series of lenses in front of the patient and receiving feedback from the patient. The score on the Snellen Eye Chart gives an indication as to which lens to start with.

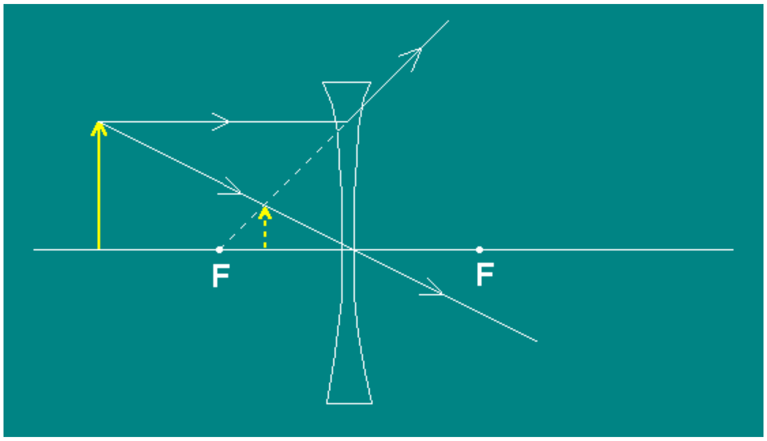


A simple experiment can help us arrive at the correct prescription. We need to know how far the nearsighted eye can see on its own with sharp focus. Suppose the answer is 50 cm. Everything closer than 50 cm is in focus, but things get fuzzy for objects beyond this point. This point is called the ***far point***; i.e., the farthest the myopic eye can see.

For myopia, use a diverging lens with a focal length equal to the negative of the far point, to bring infinity to the far point

In our example the far point is 50 cm. The focal length is then f = -50 cm, with a corresponding diopter value of -100/50 = -2D.

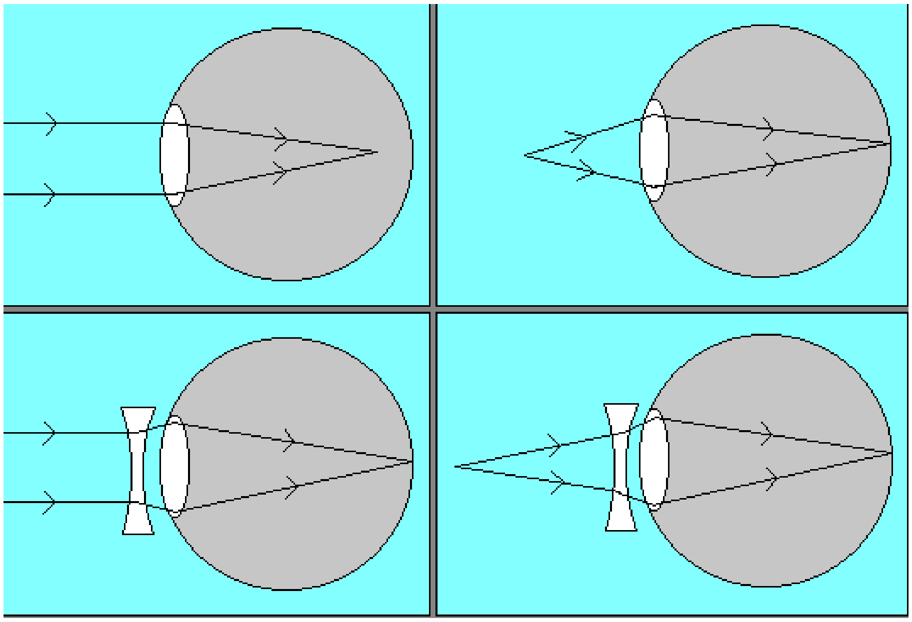
Same Angle from the Horizontal to Top of Image and Object



It is absolutely amazing that the lens reduces the size of the distant objects such as mountain at the same time. The mountains therefore appear the same size as the real ones. Large mountains far away are imaged as small mountains close up. However, the angle from the optic axis to the top of the object is the same as that to the top of the image. The diverging-lens diagram below illustrates this feature. Observe the ray (Ray 2) that goes through the center of the lens. This single ray with one slope includes both the tip of the object and image.

One can leave the glasses on for reading since the additional focusing power of the eye lens can then be used. However, the myopic eye will no longer be able to see super close as before. We have in a sense, pushed the previous super near point out to the normal 25 cm. We have also essentially moved the poor far point out to infinity by the correcting lens.

Accommodation Without and with Glasses for the Myopic Eye



The actual lenses prescribed have an outer convex surface so they look more attractive. The inner surface is concave, with more curvature than the outer convex surface in order to produce an overall negative lens, i.e., diverging.

Individuals have over the years tried various ways to improve their vision without glasses, some of these being dangerous. Here is a list of ways (some not advised) to correct for myopia.

- Squinting - with a small aperture (pinhole camera), there is infinite depth of field (the lens is not needed). Looking through a pinhole made by your fist also works.
- Tearing - tears on the cornea reduce the dioptric power of the cornea since less refraction occurs at a tear-cornea interface when compared to an air-cornea one.
- Pressure on the Eyelid (DO NOT DO THIS) - flattens the cornea temporarily reducing its dioptric power.
- Stretching the Eyebrows Back (DO NOT DO THIS) - to flatten the cornea.
- Flattening the Cornea by Surgery (SEE YOUR DOCTOR) - ***radial keratotomy*** is a surgical procedure where radial incisions are made on the corner to flatten it, thereby reducing its dioptric power.

A few doctors have noted that most causes of myopia are due to the ciliary muscles and have proposed eye exercises to train the muscles. Such exercises are controversial and are not recognized at large as a successful means to correct for myopia.

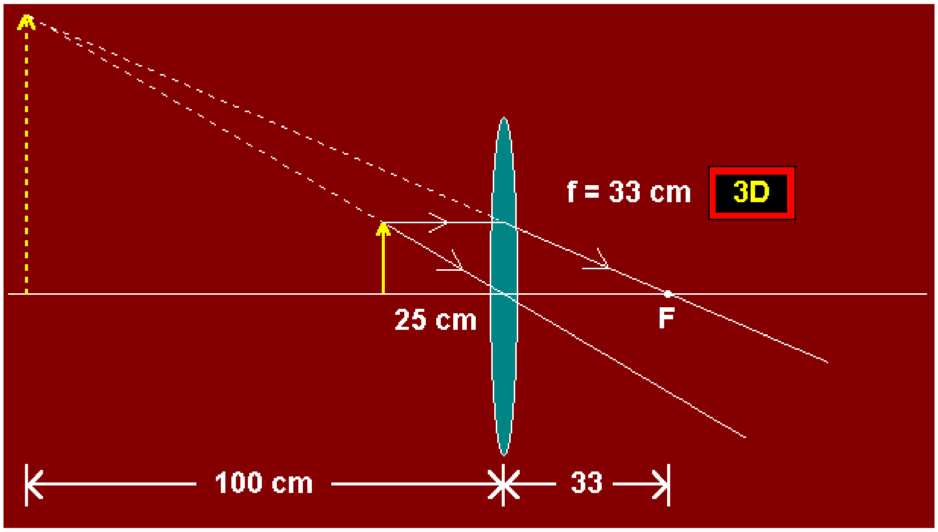
I8. Hyperopia. The inability for an eye to focus on near objects is called ***hyperopia***. Since such an eye is "sighted to see far," the condition also being called ***farsightedness***. The hyperopic or farsighted eye has no trouble seeing objects in the distance. Letters on the Snellen Eye Chart can be read with little trouble. Perhaps, the eye can even see the 20/15 line. However, when a card with fine print is placed in front the hyperopic eye, things look blurred.

Individuals with hyperopia typically extend their hands holding material with small print as far away as they can in order to try to read the writing. Or they may resort to using magnifying glasses for reading. The closest point that a hyperopic eye can see clearly is called the ***near point***. Anything closer than the near point is out of focus.

The solution is to use a converging lens to give the eye more positive dioptric power. With the assistance, the eye can then focus the diverging rays that come from near objects. We design it so that the converging lens places an image of a standard near object at a distance farther away for the hyperopic eye, at the distance of the farsighted eye's near point.

We stressed earlier the importance of knowing more than one way to solve problems. We will provide two methods here for prescribing glasses once we are given the near point. Of course, we will get the same answer by each method.

Prescribing Glasses for Hyperopia. Method 1: Graphical Technique. The case for an eye with a near point of 100 cm is illustrated below. To arrive at the lens prescription, construct a ray diagram following these steps in the order as they are listed. Carefully make such a ray diagram now at this time.



Place a small object at 25 cm from the lens, the normal near point. This is where we want to place the newspaper so the observer can hold it at the normal place.

Sketch a dotted image at the hyperopic eye's near point (in this case 100 cm). This is the image of the newspaper placed where the farsighted eye can see it.

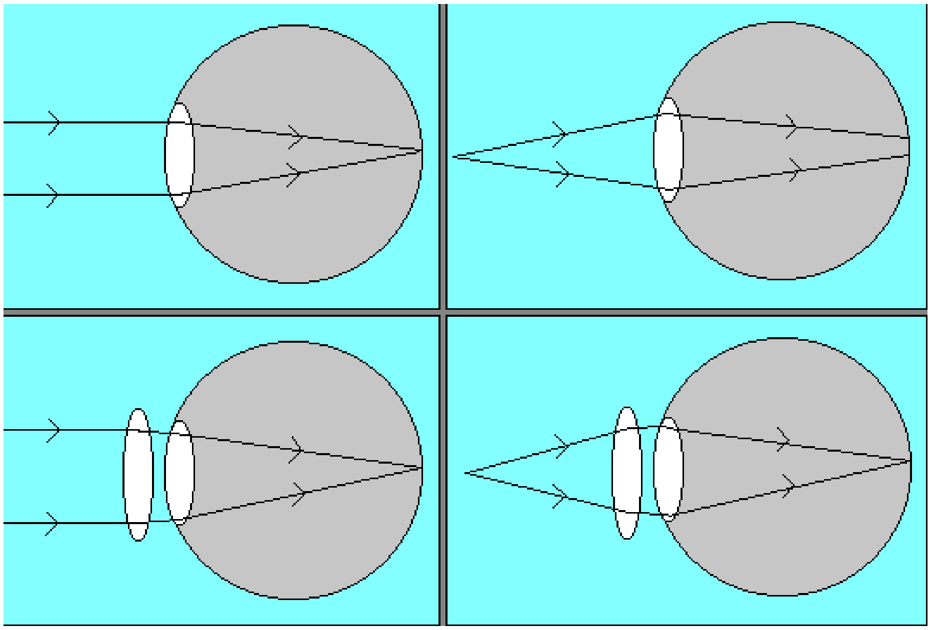
Draw Ray 2 from the object tip through the lens center, and extend the dotted line back to the image tip (the image of the newspaper.

- Draw Ray 1 from the object tip to the lens, and refract as if coming from the image tip.
- Measure the distance from the lens to the point where Ray 1 crosses the optic axis (F).
- You now have the focal length, from which you can determine the diopters.

Note the principle that Ray 2 connects the tips of both the object and image, providing for the same slant, or angle, with respect to the horizontal axis. The image is large, but farther back. Since distant objects appear smaller in general, the net effect is that the image (such as newspaper) appears to the same size as usual. The eye does not perceive the book or newspaper to be a giant.

One can leave the glasses on for distant viewing since the eye lens usually has some ability to accommodate. However, the hyperopic eye seeing at 20/15 without glasses will no longer be able to see 20/15 as before. For this reason, the hyperopic individual may just wear the glasses when reading.

Accommodation Without and with Glasses for the Hyperopic Eye



Squinting, i.e., the pinhole effect, can assist the hyperopic eye. Light then goes through the center of the lens, not really needing the lens since a pinhole produces clear images on its own. Looking through a pinhole essentially is not using the lens. Therefore it doesn't matter whether the lens is either myopic, hyperopic, or for that matter normal. The pinhole improves vision for both the myopic and hyperopic eye.

When we age, the eye lens acquires more layers, like an onion. It loses its resiliency to bulge and therefore dioptric power is lost with age. This normal aging condition is called ***presbyopia*** (prez-bee-OH-pee-uh).

Prescribing Glasses for Hyperopia. Method 2: Mathematical Technique. Remember that the normal variation in the dioptric power of the eye is 4D, going from 60D to 64D. You need that extra 4D to place the newspaper at the normal reading distance of 25 cm. Sketch two lenses close together. Place a dot 25 cm to the left of the first lens. This dot represents the normal distance for reading. Draw rays from this dot to the first lens and make the outgoing light parallel. Then have the second lens focus this light to a point that represents the retina.

Your first lens has a focal length f = 25 cm and a diopter power of 4D. Your diagram shows you that an additional 4D allows you to bring your object in as close as 25 cm. This reasoning gives us the following powerful rule.

The additional dioptric power needed to see at "x" centimeters is given by 100/x.

With this rule, we can figure out the prescription quickly without the ray diagram. In our example the far point is 100 cm. This means that the eye can provide an addition 1D of power (according to the rule). But it needs 4D. So we prescribe 3D so that with the help of glasses it can do 1D + 3D = 4D. The prescription of 3D corresponds to f = 1/3 = 0.33 m = 33 cm and everything checks out.

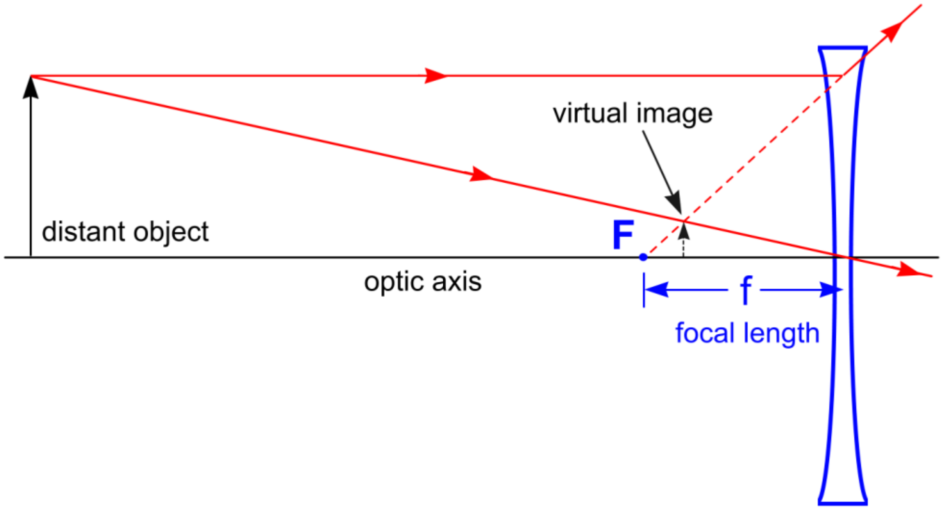
Let's use our above rule to analyze focusing for a young eye. A kid with the capability of increasing the base 60D by 14D of power can focus on objects as close as 100/14 = 7 cm. This is about one-third the normal near point, which is taken to be 25 cm (the average reached around age 42).

I8. Prescribing Glasses from a Photo!.

Photo Credit: UNCA Student Vladislav A Rakhouski (Spring 2019)

The figure shows a scene where some of the light passes through the diverging lens, providing an actual example of figure 2. Note that the components of the virtual image are smaller compared to what is seen outside the eyeglasses. The virtual image is located fairly close to the glasses, as indicated by the ray diagram below.

The power of the diverging lens in figure 3 is -5 dioptres (also written as -5D) with a corresponding focal length of -20 cm. Therefore, images of distant objects will be 20 cm from the lens. Figure 3 includes a hand at the left placed at 20 cm from the lens. The aim of this paper is for students to take photos similar to figure 3, including a subject outside the field of the eyeglasses in the same plane of the virtual image. The photo should have enough clues so that someone viewing the photo can estimate the distance from the lens to the virtual image. The next sections describe such photos in detail and how to use them in a class on lenses.



This section is adapted from my publication:

Michael J. Ruiz, "Dioptres for a Myopic Eye from a Photo" *Physics Education* **54**, 065010 (November 2019) **pdf** and **Video Abstract**

The inspiration for the paper comes from the movie *Memento* (2000). This movie is not a family movie. Be aware that the British Board of Film Classification (BBFC) rates the movie as ‘Passed “15” for strong language and violence’. In the US the movie is ‘Rated R for violence, language and some drug content’.

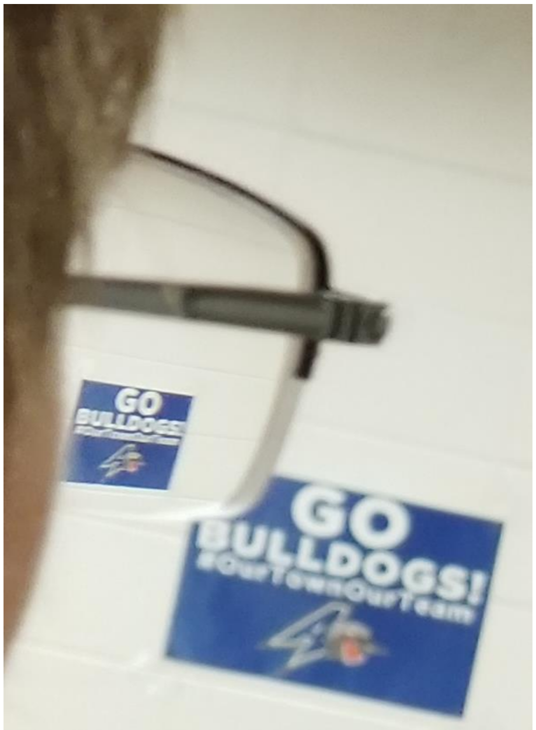
I was watching this movie when a scene appeared with two of the actors, Guy Pearce and Joe Pantoliano having a conversation. The camera was focused on Guy Pearce with the back of Joe Pantoliano’s head partially seen in the left of the scene. The camera captured the virtual image of tattoos from a distant wall through the eyeglasses Pantoliano was wearing. Meanwhile, the wall outside the field of the eyeglasses showed the tattoos as blurred. The effect was very aesthetic and interesting. I wondered if it was an optical accident that the cinematographer liked and kept shooting with the virtual images included.

The figure below a reproduction of the arrangement with two of my students, while a third student took the photograph. A detail of figure is included so that the sharp virtual image of the letters through the lens can be better compared to the out-of-focus image of the poster on the distant wall outside of the lens.

Due to copyright laws teachers can use photos and the video accompanying this paper rather than the film *Memento*. If your students are underage, the images and video from this paper can be used without ever mentioning the movie.

Photo of two students having a conversation where the virtual image due of the poster on the distant wall is included. Photo Credit: Vladislav A Rakhouski.

Figure 4b. Detail of figure 4a in order to better compare the sharpness of the virtual image to the blurred distant wall.



Posed challenge

Estimate the eyeglass prescription using your physics detective skills and the photos here alone!

You are not even in the same room as the guy wearing the glasses but you can estimate his prescription in diopters from the photo!

How?

But first, some classroom warm-up activity. The focal length needs to be equal to the far point of the myopic eye so that it can see distant objects clearly. In class, I first ask a couple of nearsighted volunteers with strong glasses to come to the front of the class. I have them stand close to the computer screen on the desk at the front, take off their glasses, and give them a moment to adjust. Then I have each in turn cover one eye with a hand and look at the computer monitor as far away as possible where the computer text is still clear. When this distance is reached, another student can measure the distance from the computer monitor to the observing eye. It is not uncommon to find someone that cannot see clearly farther than 20 cm from the computer screen.

Now it is time for mathematics. The 20 cm measured is the focal length labeled f in figure 2 and I remind them that they need to insert the minus sign for a diverging lens. I then ask them to use the equation to arrive at the prescription in dioptres. The power in dioptres is

$$P(\text{dioptres}) = \frac{100}{f\ (\text{in cm})} = \frac{100}{-20} = -5 \text{ dioptres} = -5\text{D}.$$

Of course the experimental method used by the eye doctor for the precise prescription is the gold standard. However, the physics demonstration with just the meter stick shows the power of physics in making an estimate of a prescription with no specialized equipment.

Now comes the magic. I ask the students if it is possible to estimate a prescription from a photo such as the previous figure using solely the photo as the only source of information. At first, this feat seems impossible. The subject wearing the glasses is not present in the room for us to make any measurements. We just have the photo.

I ask the students first to identify the main subject that the photographer has captured. The students readily answer the lady. She is the main subject and she is in sharp focus. But the virtual image through the eyeglass lens is also in sharp focus. Therefore, the virtual image through the eyeglasses and the female subject must both be at a similar distance from the camera. Since the wall is fairly far away, the virtual image seen through the eyeglass lens should be close to the focal plane of the eyeglass lens. If we know the focal length, i.e. the distance from the eyeglass lens to the focal plane, we can find the prescription. Can the distance from the eyeglass lens to the virtual image be determined? Yes, because the photographer focused on the female student, who is at a similar distance from the eyeglasses. How far is she from the guy with whom she is having a private conversation?

To make the students feel more at ease, I remind them we are going for an estimate, so is the distance more like 25 cm, 50 cm, 75 cm, 100 cm, etc.? The estimate they will give is 50 cm, which leads to a prescription of -2D for the eyeglasses in the above figure. For fun and to involve the students more, I sometimes ask two students to come up front and stand close facing in each other like the two people in the photo. I tell them the distance doesn’t have to be exact. Just look at the photo and approximate things. Then I ask the class to give an estimate. Once again, 50 cm is the typical answer. Sometimes I ask a third student to come up and use a meter stick to measure the distance between the two students facing each other.

In summary, the prescription discussion started with expensive equipment in the doctor's office. Then, the power of physics led to experimental measurements in class with a meter stick, where individuals stood from a computer or poster at the front of class to determine far points. Finally, a prescription was estimated from just a photo! For this last case the students can go through the reasoning mentally as the mathematics requires merely the calculation -100/50 = -2D.

We went outside to shoot a photo where a distant school banner was also in focus.

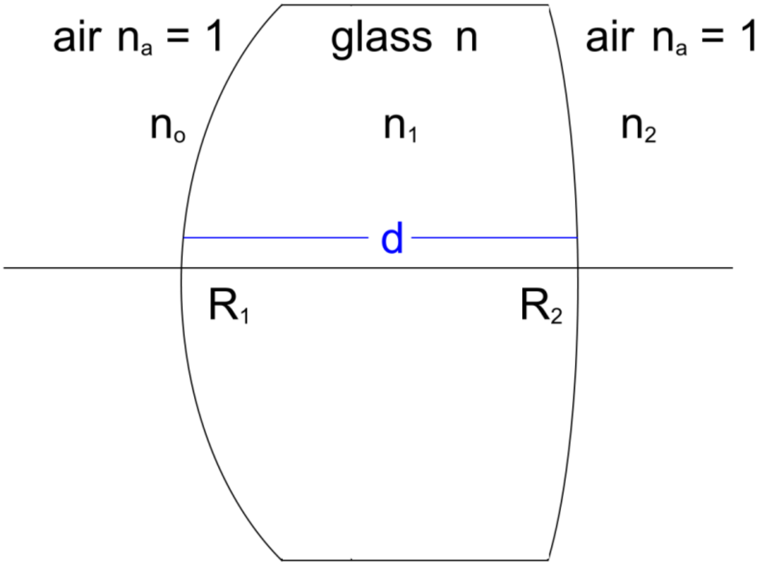
Photo similar to the previous one but with the poster farther away so that the far banner is relatively more blurred compared to the virtual image of the banner through the eyeglasses.

Photo Credit: Vladislav A Rakhouski.

Detail of figure showing virtual image in focus (left) and distant flag blurred (right) as the camera is focused on the virtual image.

I9. Dioptric Power of Eye.

We need a formula with three media. Let's be clever and generalize our former formula.



$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

$$\frac{1}{f} = \frac{(n-1)}{R_1} - \frac{(n-1)}{R_2} + \frac{(n-1)(n-1)d}{nR_1R_2}$$

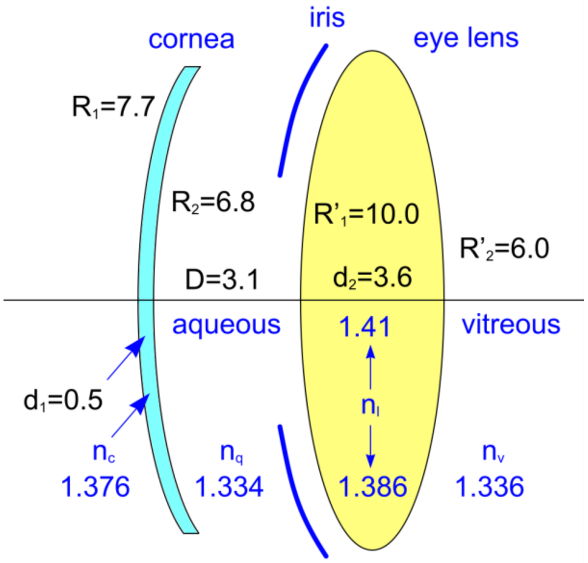
$$\frac{1}{f} = \frac{(n-1)}{R_1} + \frac{(1-n)}{R_2} - \frac{(n-1)}{nR_1}\frac{(1-n)d}{R_2}$$

$$\frac{1}{f} = \frac{(n_1-n_o)}{R_1} + \frac{(n_2-n_1)}{R_2} - \frac{(n_1-n_o)}{n_1R_1}\frac{(n_2-n_1)d}{R_2}$$

$$\boxed{\frac{1}{f} = \frac{(n_1-n_o)}{R_1} + \frac{(n_2-n_1)}{R_2} - \frac{d}{n_1}\frac{(n_1-n_o)}{R_1}\frac{(n_2-n_1)}{R_2}}$$

Gullstrand's Eye Model

dimensions in mm



$$\frac{1}{f}=\frac{(n_1-n_o)}{R_1}+\frac{(n_2-n_1)}{R_2}-\frac{d}{n_1}\frac{(n_1-n_o)}{R_1}\frac{(n_2-n_1)}{R_2}$$

$$\frac{1}{f_1}=\frac{(n_1-n_o)}{R_1} \qquad \frac{1}{f_2}=\frac{(n_2-n_1)}{R_2}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{n_1}\frac{1}{f_1}\frac{1}{f_2} \qquad P=P_1+P_2-\frac{d}{n_1}P_1P_2$$

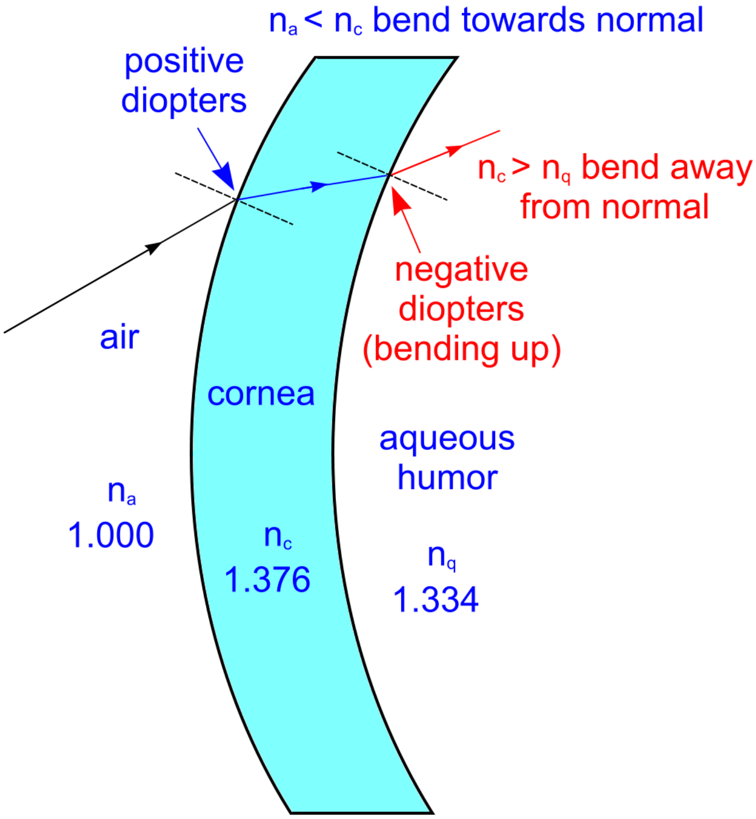
$$P_{\text{front}}=\frac{1}{f_1}=\frac{(n_1-n_o)}{R_1}=\frac{(n_c-n_{air})}{R_1}=\frac{1.376-1.000}{0.0077}=48.8312\text{ D}$$

$$P_{\text{back}}=\frac{1}{f_2}=\frac{(n_2-n_o)}{R_2}=\frac{(n_q-n_c)}{R_2}=\frac{1.334-1.376}{0.0068}=-6.1765\text{ D}$$

$$-\frac{d}{n_1}P_1P_2=-\frac{0.0005}{1.376}(48.8312)(-6.1765)=0.10960\text{ D}$$

But wait! How can be sure we got the signs right?

The best way to check that $P_{\text{front}} > 0$ and P_{back} should be negative is to resort to a sketch.



$$P = P_1 + P_2 - \frac{d}{n_1} P_1 P_2 = 48.8312 - 6.1765 + 0.10960 = 42.7643 \text{ D} = 40 \text{ D}$$

Summary: $P_{\text{front}} = 48.8 \text{ D}$, $P_{\text{back}} = -6.12 \text{ D}$, $P_{cornea} = 43 \text{ D} = 40 \text{ D}$

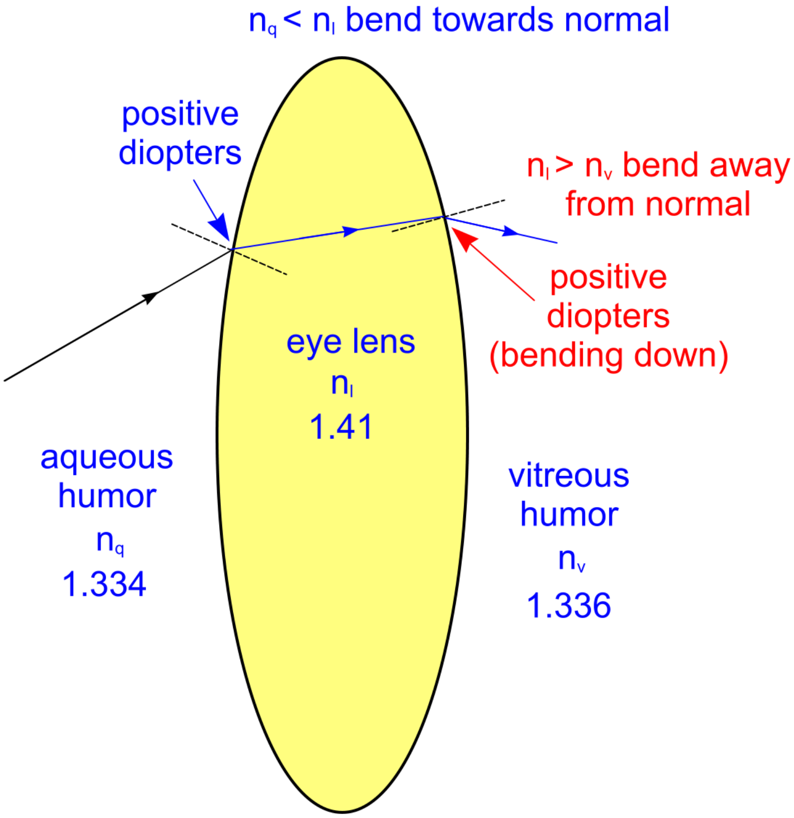
Eye Lens (Crystalline Lens)

$$P_{\text{front}} = \frac{1}{f_1} = \frac{(n_1 - n_o)}{R_1} = \frac{(n_l - n_q)}{R_1^{'}} = \frac{1.41 - 1.334}{0.01} = 7.6000 \text{ D}$$

$$P_{\text{back}} = \frac{1}{f_2} = \frac{(n_2 - n_o)}{R_2} = \frac{(n_v - n_l)}{R_2^{'}} = \frac{1.336 - 1.41}{-0.006} = 12.333 \text{ D}$$

$$-\frac{d}{n_1} P_1 P_2 = -\frac{0.0036}{1.41}(7.6000)(12.333) = -0.239 \text{ D}$$

Check the signs for front and back.



$$P = P_1 + P_2 - \frac{d}{n_1} P_1 P_2 = 7.6000 + 12.333 - 0.239 = 19.67 \text{ D}$$

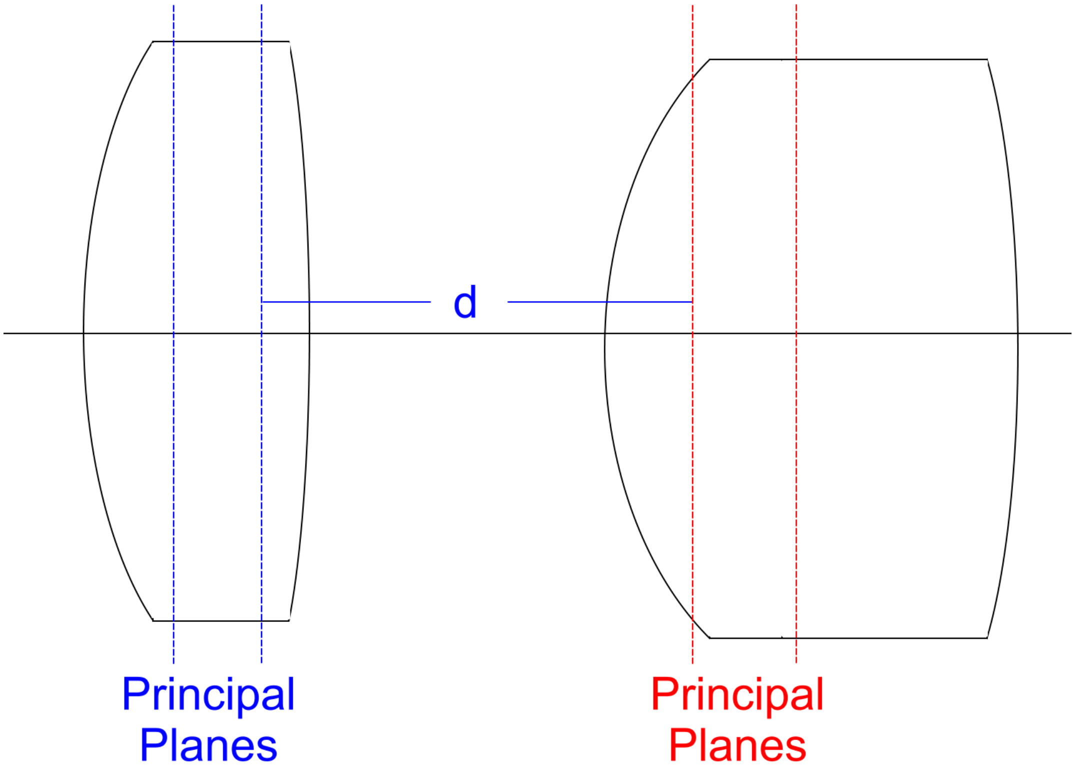
Summary: $P_{\text{front}} = 7.6 \text{ D}$, $P_{\text{back}} = 12.3 \text{ D}$, $P_{lens} = 20 \text{ D}$

Can we use

$$P = P_{cornea} + P_{lens} - \frac{d}{n_q} P_{cornea} P_{lens}$$

to find the total power of the system? Not really.

Technically the d should be measured from the back principle plane of the first lens (cornea) to the front principal plane of the second lens (eye lens). We will not worry about this fact in estimating the last term in the above expression.



This subtlety is beyond the scope of course. So we will estimate using our d.

$$-\frac{d}{n_q}P_{cornea}P_{lens} = -\frac{0.0031}{1.334}(40)(20) < 2 \text{ D}$$

Summary

$$P_{cornea} = 40 \text{ D} \qquad P_{lens} = 20 \text{ D} \qquad P_{total} \approx 60 \text{ D}$$

Due to Accommodation: $20 \text{ D} \le P_{lens} \le 24 \text{ D}$

For a 10-year old: $20 \text{ D} \le P_{lens} \le 30 \text{ D}$