K0. Aberrations. We have been working with the powerful formula and ray-tracing techniques of geometrical optics. In this section we study the lack of sharp focus using optical components. In general, these are slight; however, they are significant when high-quality optics is required. In our simplified ray-tracing model we have a point-to-point correspondence between object an image. That is, for every point on the object, there is a corresponding point on the image. In real life this is not the case. The failure of a reflecting or refracting element to faithfully produce a unique relation between an object point and an image point is called an aberration. In simple terms, an object point spreads out a bit and messes up the image or is imaged in such a way as to distort the image.

We do not care if the size of the object is different such as smaller or larger. Often, this is desired. A change in size is in itself no aberration as long as the image is faithfully reproduced. But in typical situations we do not obtain such properly rendered images.

Why does this happen? If we apply the laws of reflection and refraction with extreme care, we find the lack of perfect image formation. In a sense, it is incorrect to refer to imperfect image formation as an aberration since nature is following the laws of physics precisely. The aim in optical engineering is to design systems where the imperfections, i.e., the aberrations, are not noticed.

The term aberration is useful since it clearly indicates what our goal is in optical systems, namely, to achieve as perfect an image as possible. Aberrations are very complicated to derive from basic principles of optics since the simple paraxial approximations no longer apply.

We used the small-angle approximation for the sine function: $\sin\theta \approx \theta$. But in reality

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots$$

How would you feel about returning to everywhere we used $\sin\theta \approx \theta$ and replace it with

$$\sin\theta \approx \theta - \frac{\theta^3}{3!}.$$

You see what I mean about things getting complicated real fast. Our $\sin\theta \approx \theta$ is a first-order approximation since the power of θ^n is 1, i.e., $n = 1$. Using $\sin\theta \approx \theta - \frac{\theta^3}{3!}$ Is a third-order approximation, much closer to the truth. But also much more complicated.

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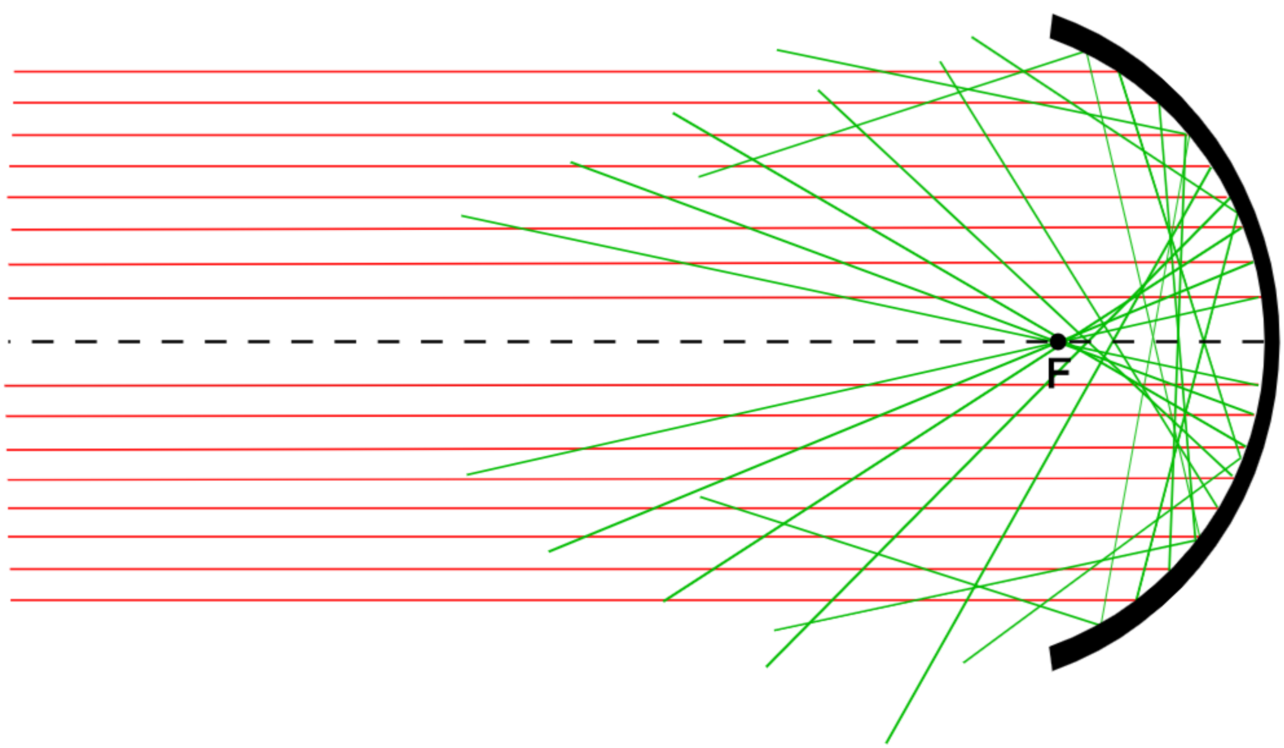
We intend to simply list the aberrations for you and use experimental observation whenever possible to demonstrate their realities. We will not develop a theoretical scheme to understand them since, as we have said above, the theory is too tricky due to the math complications.

Using spherical lenses, and neglecting the spreading out of colors, the basic monochromatic aberrations are five:

- Spherical aberration
- Coma
- Astigmatism
- Field Curvature
- Image distortion.

K1. Spherical Aberration. Spherical aberration occurs in both spherical mirrors and spherically-cut lens surfaces. Below is a figure illustrating spherical aberration in a concave mirror. When the law of reflection is accurately applied, parallel light does not focus at a point. You need a parabolic mirror for that. A spherical mirror falls short in this regard since it is curved too much at distances away from the optic axis. Therefore, the rays that hit the outer sections reflect too much.



Courtesy Wikipedia: **Synkizz.** Creative Commons License

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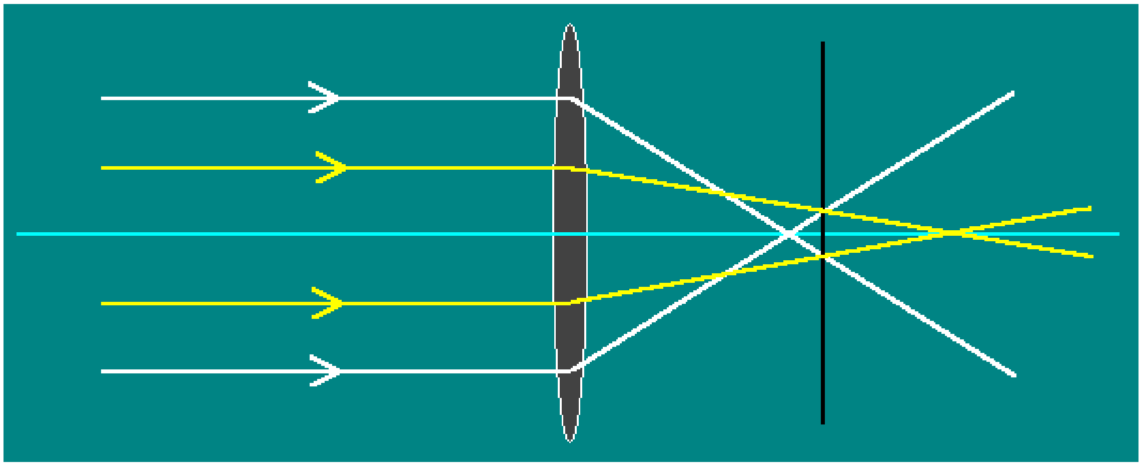
The analogous situation with a lens occurs when light rays hit the margins of the lens and refract too much due to the spherical shape of the glass (see below).

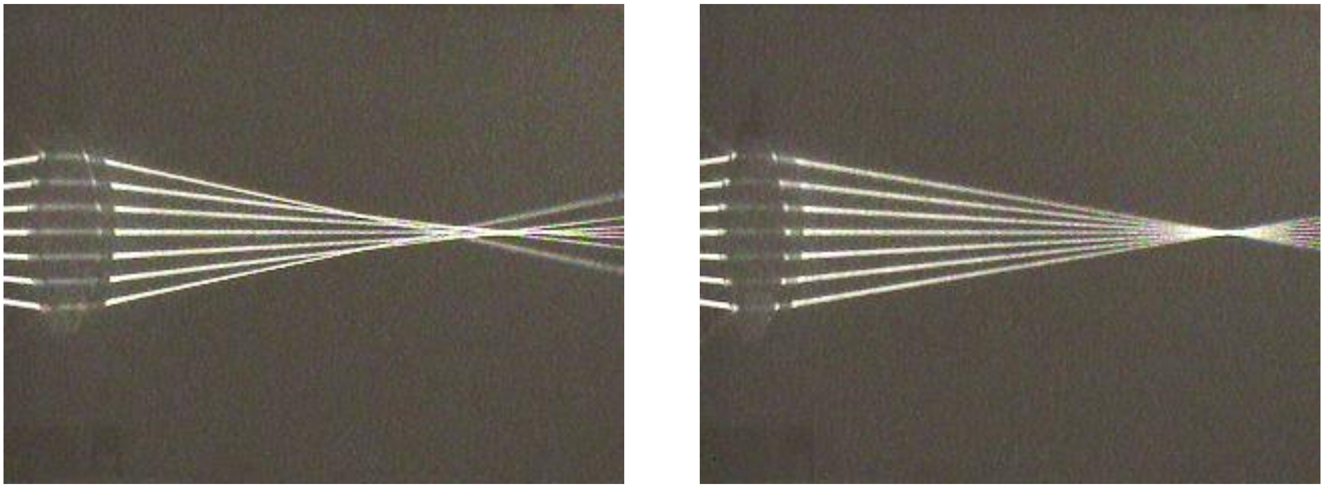
Spherical Aberration (Exaggerated)



The vertical screen has been placed at the least circle of confusion, the smallest circle and thus the best compromise. Refer to the photo on the left below to see rays hitting the outer sections of the lens and refracting too much. You can also detect some chromatic aberration (color spreading). The lens on the right is not spherical, having less spherical curvature near the edges so that the spherical aberration is corrected. A lens that deviates from a spherical shape is called an aspherical lens. However, in typical practice, engineers prefer to introduce more spherical lenses in clever ways to correct for spherical aberration rather than use a more expensive single aspherical piece of glass.

Spherical Aberration (Left) and Corrected (Right)

Spherical Lens | **Aspherical Lens**



Courtesy Richard E. Berg, Lecture Demonstration Facility, Univ. of Maryland

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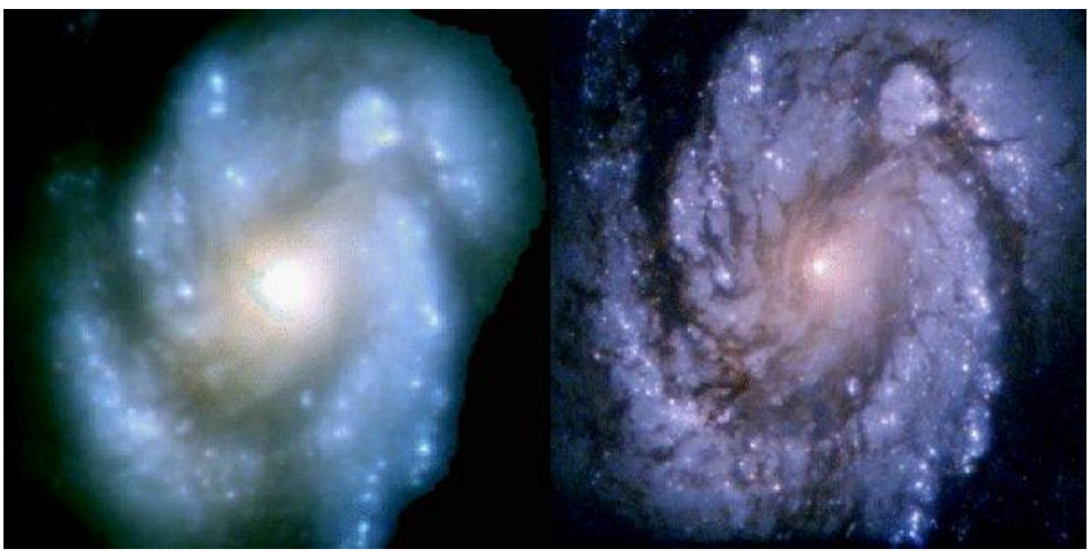
Courtesy NASA/STScI. The masks are to protect the mirror.

For the expensive Hubble telescope though, a special aspherical mirror, though not parabolic, was designed for the primary telescope mirror. The primary mirror collects the light and reflects it to a secondary mirror. During the manufacturing phase the mirror was not shaped correctly. The curvature of the mirror was left too close to being spherical. Therefore the Hubble telescope exhibited spherical aberration when it was placed into orbit in 1990. A photo of the 2.4-meter mirror (94 inches) of the Hubble telescope is seen below as workers wear masks to protect the mirror. Sometimes two wrongs can make a right, as seen below.

> "The image on the left is M100 with the original Hubble Wide Field and Planetary Camera (WFPC-1) made shortly before the space shuttle mission to repair Hubble in late 1993. Space shuttle astronauts replaced the optics to correct for a defective curvature in the 2.4-meter mirror (94 inches). The image on the right was made with the new camera system, the Wide Field and Planetary Camera 2 (WFPC-2). Two wrongs made a right in this case as the distorted lens counteracted the distorted mirror shape." NASA/STScI

Hubble Telescope Before and After Spherical-Aberration Correction



Courtesy NASA/STScI.

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The Crew That Repaired Hubble

Courtesy NASA.

“Astronauts included in the STS-61 crew portrait include (standing in rear left to right) Richard O. Covey, commander; and mission specialists Jeffrey A. Hoffman, and Thomas D. Akers. Seated left to right are Kenneth D. Bowersox, pilot; Kathryn C. Thornton, mission specialist; F. Story Musgrave, payload commander; and Claude Nicollier, mission specialist. Launched aboard the Space Shuttle Endeavor on December 2, 1993 at 4:27:00 am (EST), the STS-61 mission was the first Hubble Space Telescope (HST) servicing mission, and the last mission of 1993.” NASA

Physics majors may be familiar with the classic mechanics text *Classical Dynamics of Particles and Systems* by Stephen T. Thornton and Jerry B. Marion. Stephen Thornton is the husband of the astronaut Kathryn Thornton in the above photo. Dr. Kathryn Thornton, astronaut, is also a physicist with a Ph.D. in nuclear physics from the University of Virginia. She is Professor Emeritus of Mechanical and Aerospace Engineering at Virginia; her husband is Professor Emeritus there also, in the Department of Physics.

Regarding the other author Jerry B. Marion (1929 – 1981), I remember him well as he was a Professor at Maryland when I was there. I often saw him in the halls. He was an outstanding author of physics textbooks.

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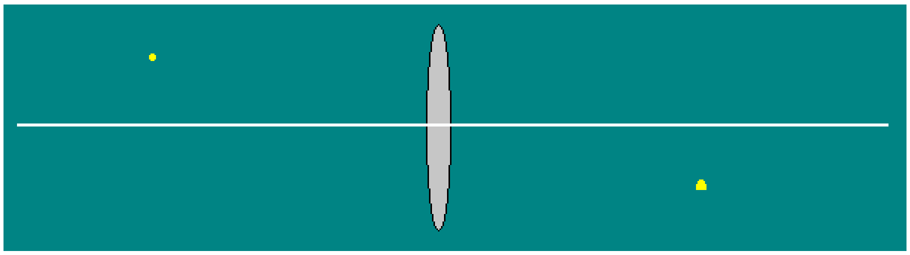
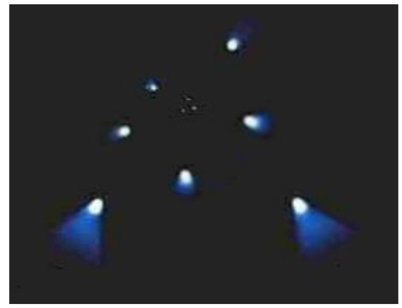
K2. Coma. An off-axis object point has an image that is smeared out in a comet-shape, an aberration called ***coma.*** Notice how the coma aberration is greater for objects farther from the optic axis. The flairs look cute, but they are not wanted.

Coma (Simple Sketch)

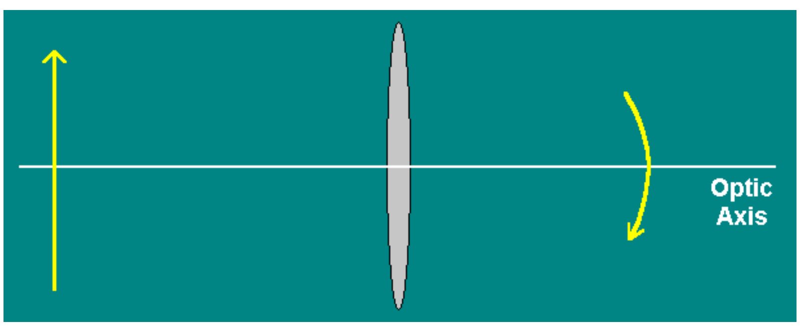


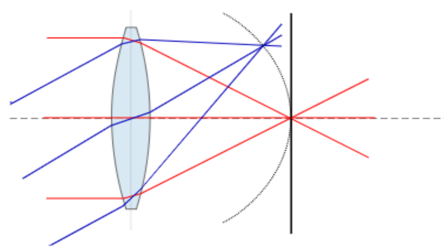
Coma



K3. Field Curvature or Curvature of Field. Below is a sketch of an object arrow in front of a spherical lens. The points along the object that lie away from the optic axis produce a curved image. This phenomenon where a straight or planar object appears curved is called **field curvature** or ***curvature of field***. The effect increases the farther the object points are away from the optic axis, however, it is not as prominent as spherical aberration and coma.

Curvature of Field (Simple Exaggerated Sketches)





Right Image Wikipedia: BenFrantzDale. [Creative Commons License]

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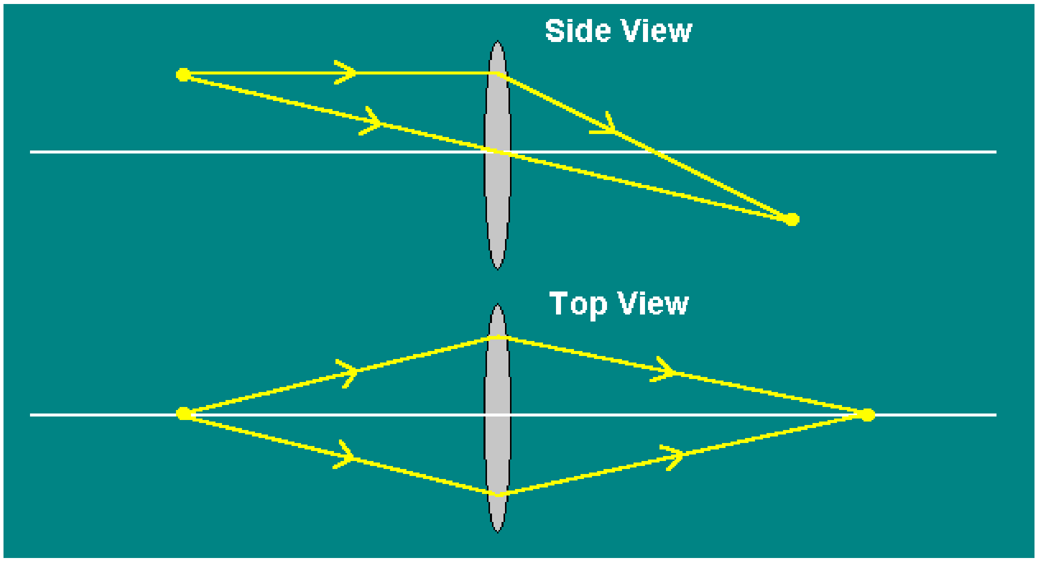
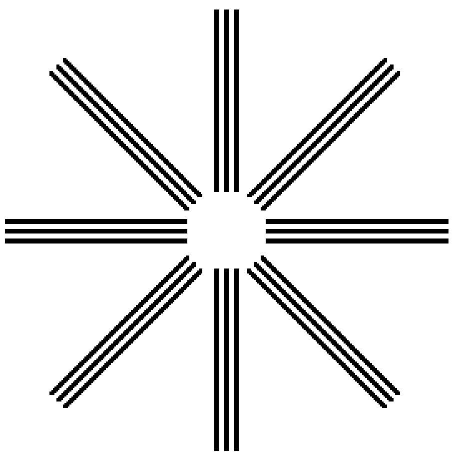
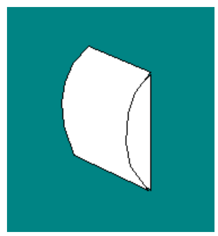
K4. Astigmatism. Consider a point off the optic axis. A sketch from a side view is illustrated in the upper diagram below. The lower diagram is one made from viewing above, looking down at the point. The fan of rays in the vertical plane (looking from the side) focus at a different point than the fan of rays a horizontal plane (observing from above). The different image location for the horizontal and vertical fans of rays is known as ***astigmatism.*** Do not confuse this astigmatism with the astigmatism of vision defects.



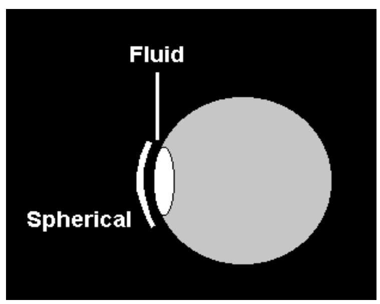
It is impossible to place a white screen at the image plane in order to view a sharp image. If you place the screen where the horizontal fan of rays meet, you see clear horizontal lines, but the vertical are blurred. Similarly, if you choose a screen position to optimize the vertical lines, the horizontal ones are no longer in focus. The best bet is to once again place the screen at the plane where there is least "image confusion." Things get interesting for engineers since the circles of confusion we have encountered for the different aberrations do not necessarily agree.

Astigmatism in vision is different. The shape of the cornea is the typical cause of visual astigmatism. For example, if the cornea is curved too much along one direction, a line in that direction will appear blurry. A cylindrical lens is used to correct for the problem, where the direction of the curved part of the cylinder is chosen to help you focus the troubled line.





A solution to astigmatism without the use of a cylindrical lens is the ***contact lens***. A spherical surface floating on a Fluid tear layer.



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Here is the format for prescriptions written in diopters. First we need the abbreviations used for right and left eyes. Some doctors use R.E. for right eye and L.E. for left. However, the traditional abbreviations are ***O.D.*** (oculus dextrus) and ***O.S.*** (oculus sinister) respectively.

"Sinister" simply means left in Latin. However, through the ages the left eye has been referred to as the evil eye. Left has gotten a bad name. An example is the phrase "left-handed compliment." Note the addition of the axis angle for astigmatism.

Myopia

	Sphere
O.D.	-0.75
O.S.	-1.00

Hyperopia

	Sphere
O.D.	+1.50
O.S.	+2.25

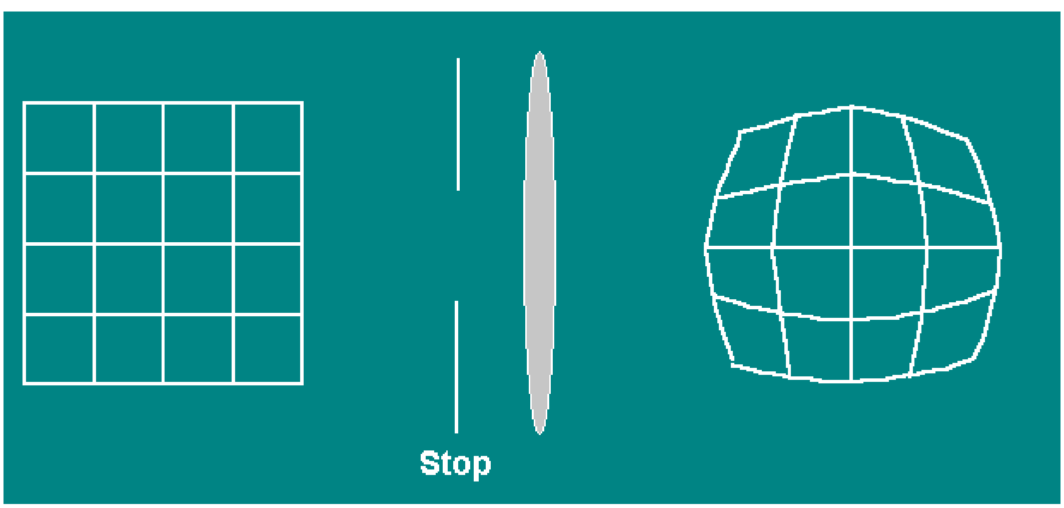
Astigmatism

	Sphere	Cylinder	Axis
O.D.	+2.50	-	-
O.S.	+1.75	+0.5	30

K5. Image Distortion or Distortion. A simple way to correct for the above off-axis aberrations is to block rays that hit the outer sections of the lens. Remember our f-numbers? A ***stop*** is a barrier with a hole in it introduced for the purposes of blocking, i.e., "stopping" some light from reaching the lens. Remember that a pinhole with the proper size produces a one-to-one correspondence between object and image points. Therefore, reducing the aperture size minimizes spherical aberrations where light rays hit the edges of the lens. It also helps to limit coma, curvature of field, and astigmatism since objects have to be closer to the axis to be viewed with the smaller effective "lens size." If much light is available, cutting some out is not a problem.

But this introduces another aberration. A grid is deformed depending on whether you block the rays on the left or right side of the lens. This aberration is known as ***distortion***. There are two types: ***barrel distortion*** for the stop on the left side of the lens and ***pincushion distortion*** for the stop on the right side.

Barrel Distortion (Simple Sketch with Grid Turned to Face You)



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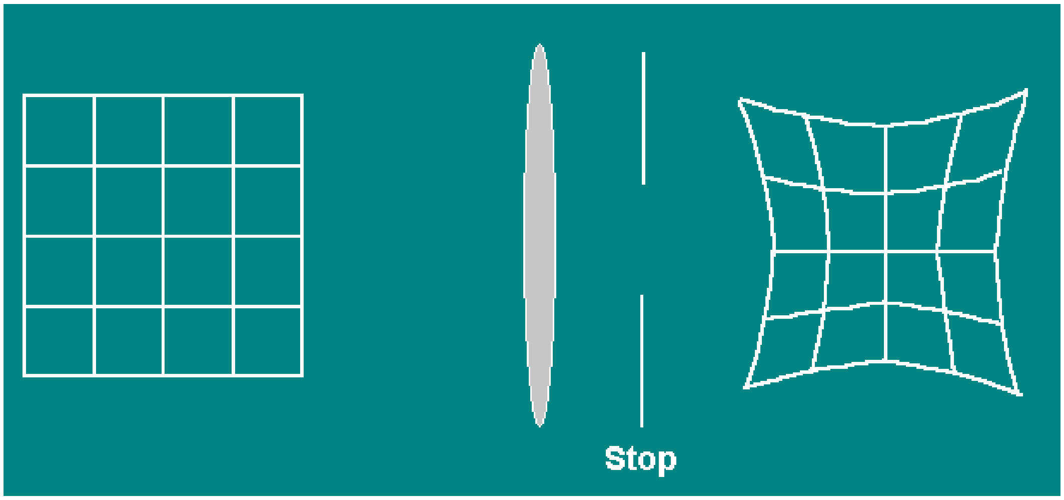
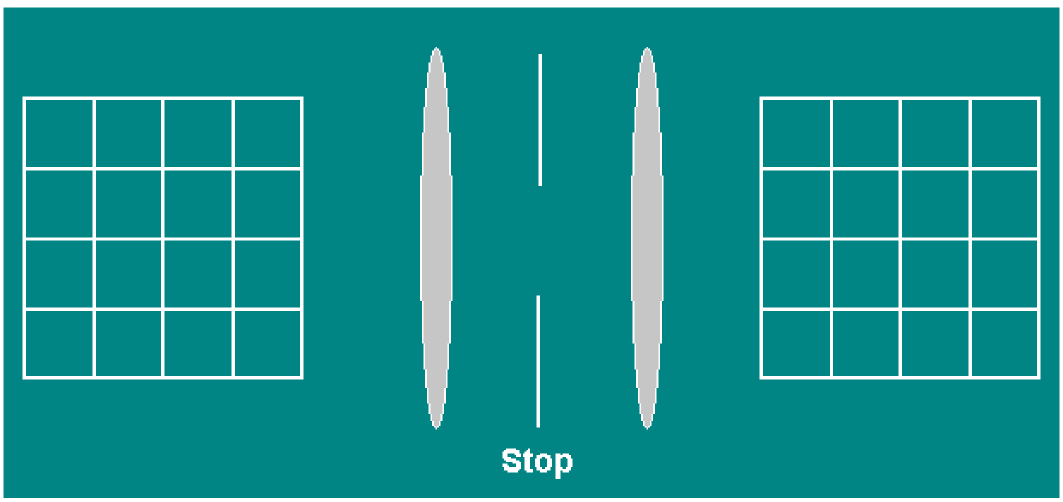
Pincushion Distortion (Simple Sketch with Grid Turned to Face You)



Distortion Corrected (Simple Sketch with Grid Turned to Face You)



Try the informal quiz!

Sph=Spherical, Com=Coma, Cur=Curvature of Field, Ast=Astigmatism, Dis=Distortion

Description	Sph	Com	Cur	Ast	Dis
Pincushion	○	○	○	○	○
Points with Tails	○	○	○	○	○
Marginal Rays Bend Too Much	○	○	○	○	○
Only Vertical Lines in Focus	○	○	○	○	○
A Single Lens with a Single Stop	○	○	○	○	○
The Image of a Cross is Bent Toward Lens	○	○	○	○	○

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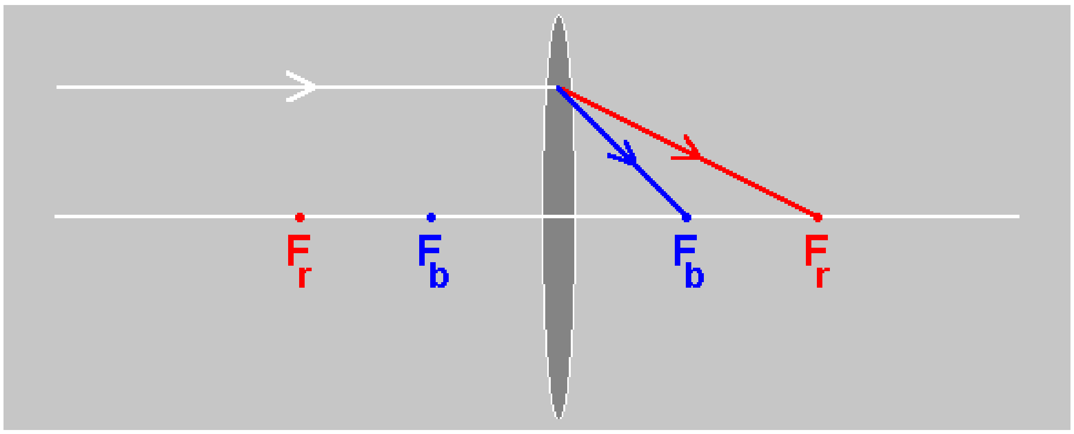
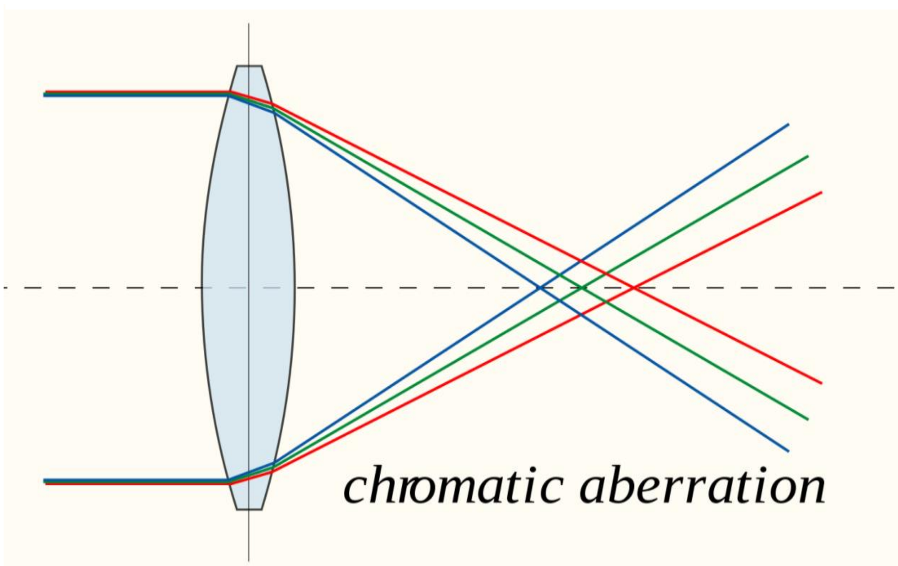
K6. Chromatic Aberration. The spreading out of color by a lens is called ***chromatic aberration***. It is illustrated below with considerable exaggeration to make the point. The blue-end (or violet-end) of the spectrum refracts more than the red end. This phenomenon is called ***dispersion***. Since white contains all the visible wavelengths, these separate during refraction, with red bending less. Remember that the "e" sound in "red" and "less" are the same. Now you can remember: "Red Bends Less."

Chromatic Aberration (Exaggerated)



There are two focal points indicated above, one for our particular blue ray and one for the red. There is actually a spectrum of focal points from the violet (400 nm) to the red (700 nm). The colors spread out.



If you place a screen in the path of the outgoing light rays, you do not find a point focus anywhere. Instead you will see a circle of light, the ***circle of confusion***. The ***least circle of confusion*** is the circle of confusion with the smallest area. It is the best place to put the screen. In a sense, it is a compromise between the extreme blue and extreme red focal points.

Can you find the least circle of confusion in the figure?

Courtesy Wikipedia: **Bob Mellish.** [Creative Commons License]

The basis of dispersion is that the index of refraction depends on wavelength: $n = n(\lambda)$.

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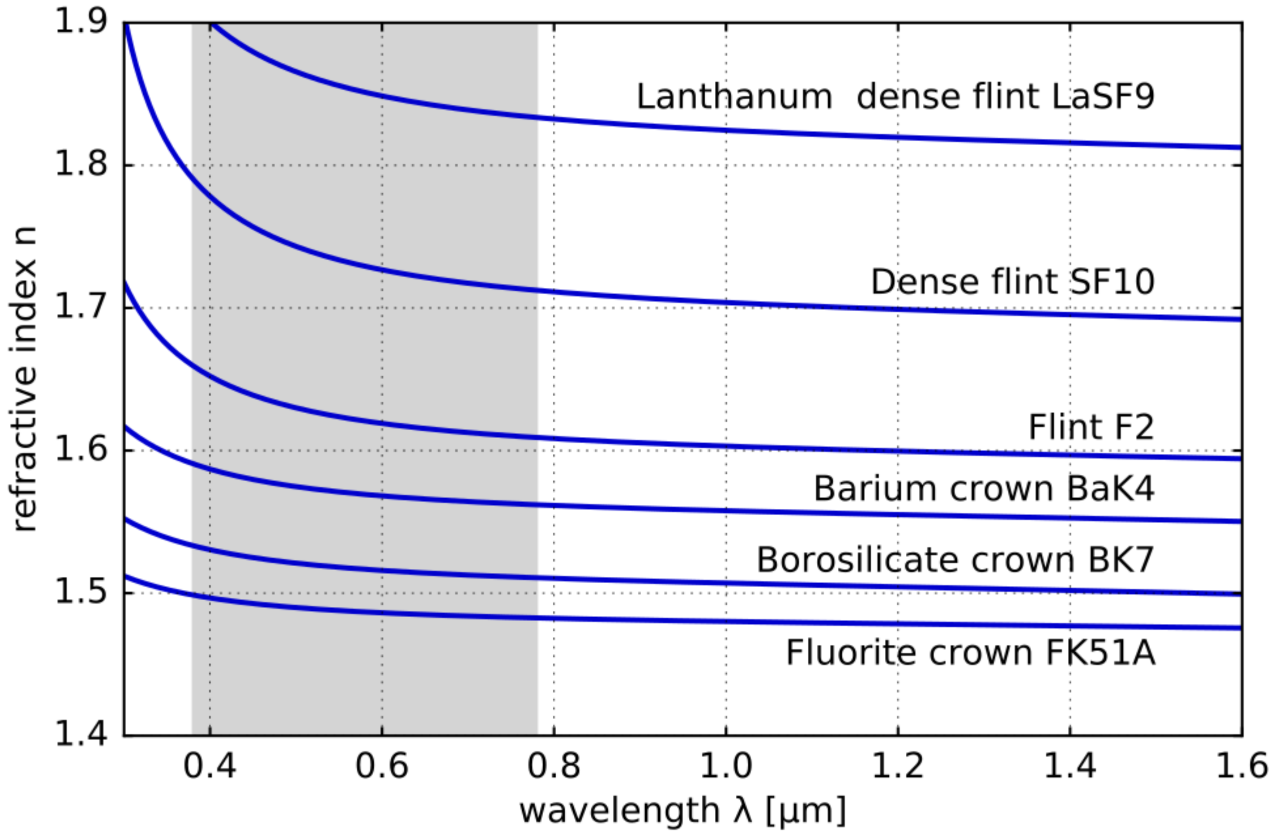
Dispersion n = n(λ) for Some Types of Glass, Gray Section is Visible Spectrum



Courtesy Wikipedia: **Geek3.** [Creative Commons License]

Severe Chromatic Aberration

Courtesy Wikipedia: **freestofrobbies.**
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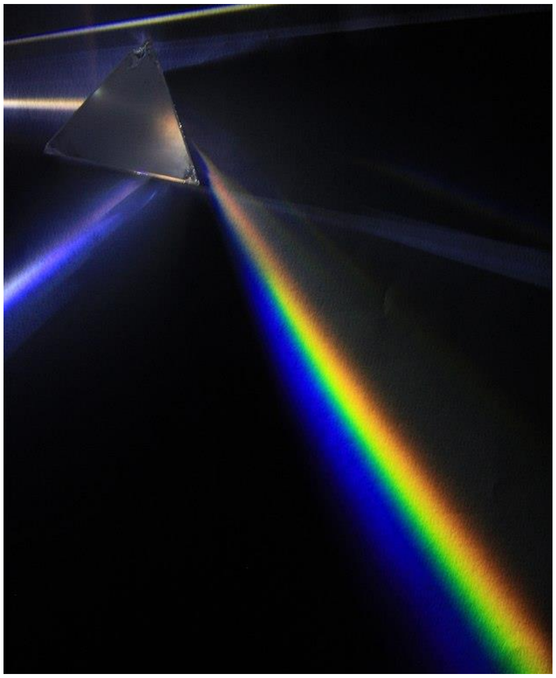
K7. The Prism.



Light dispersion of a mercury-vapor lamp with a flint glass prism. Wikipedia: D-Kuru.
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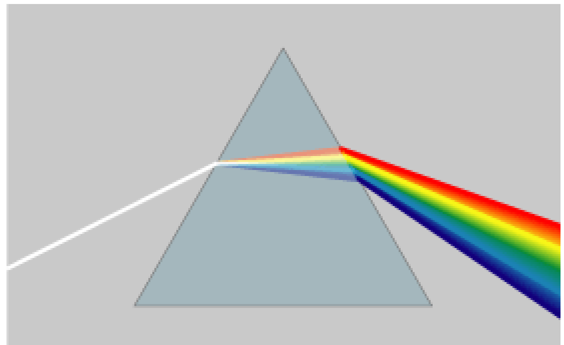
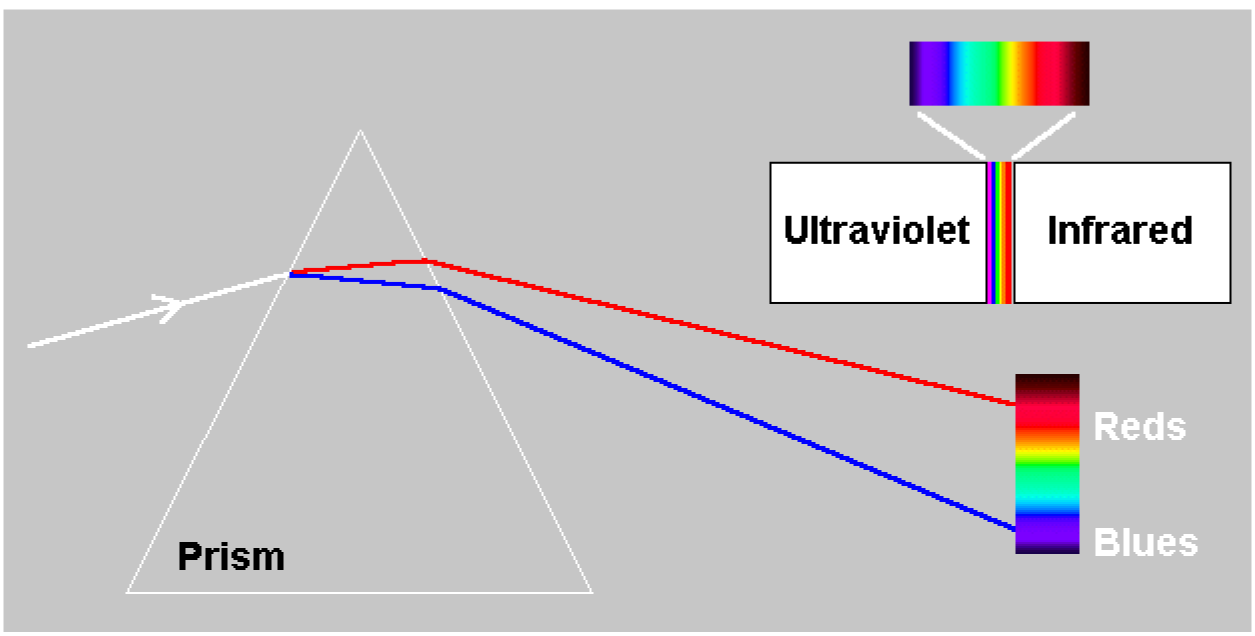
The classic demonstration of dispersion sends white light through a prism.



Wikipedia: Joanjoc~commonswiki.

We have seen with chromatic aberration that a spectrum forms due to the spreading out of colors from white. As noted earlier, this is called ***dispersion***. Dispersion occurs as white light enters a refractive medium.

White Light and Dispersion with a Prism



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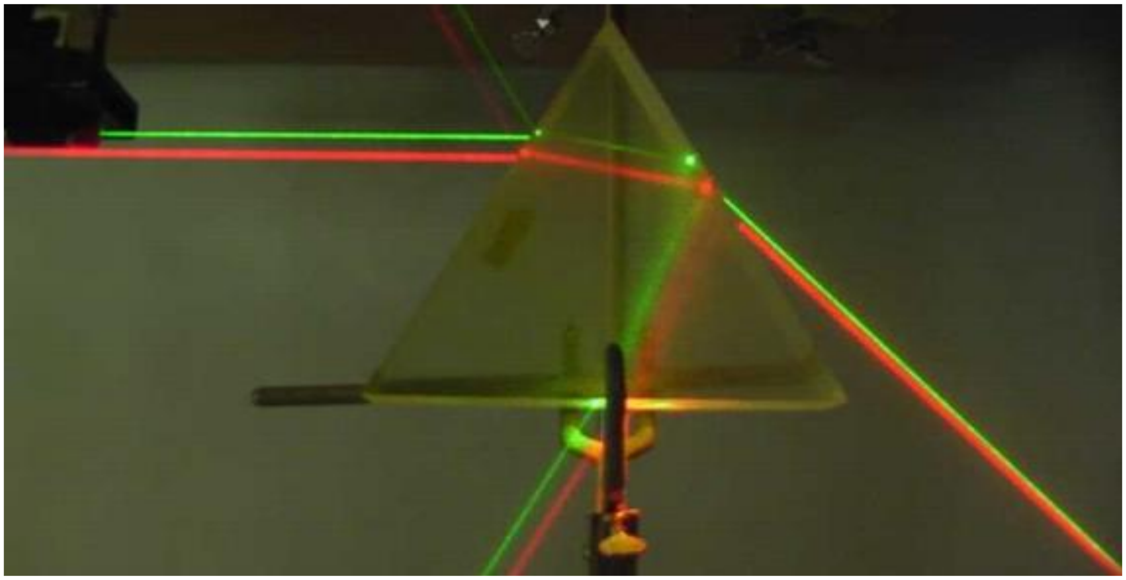
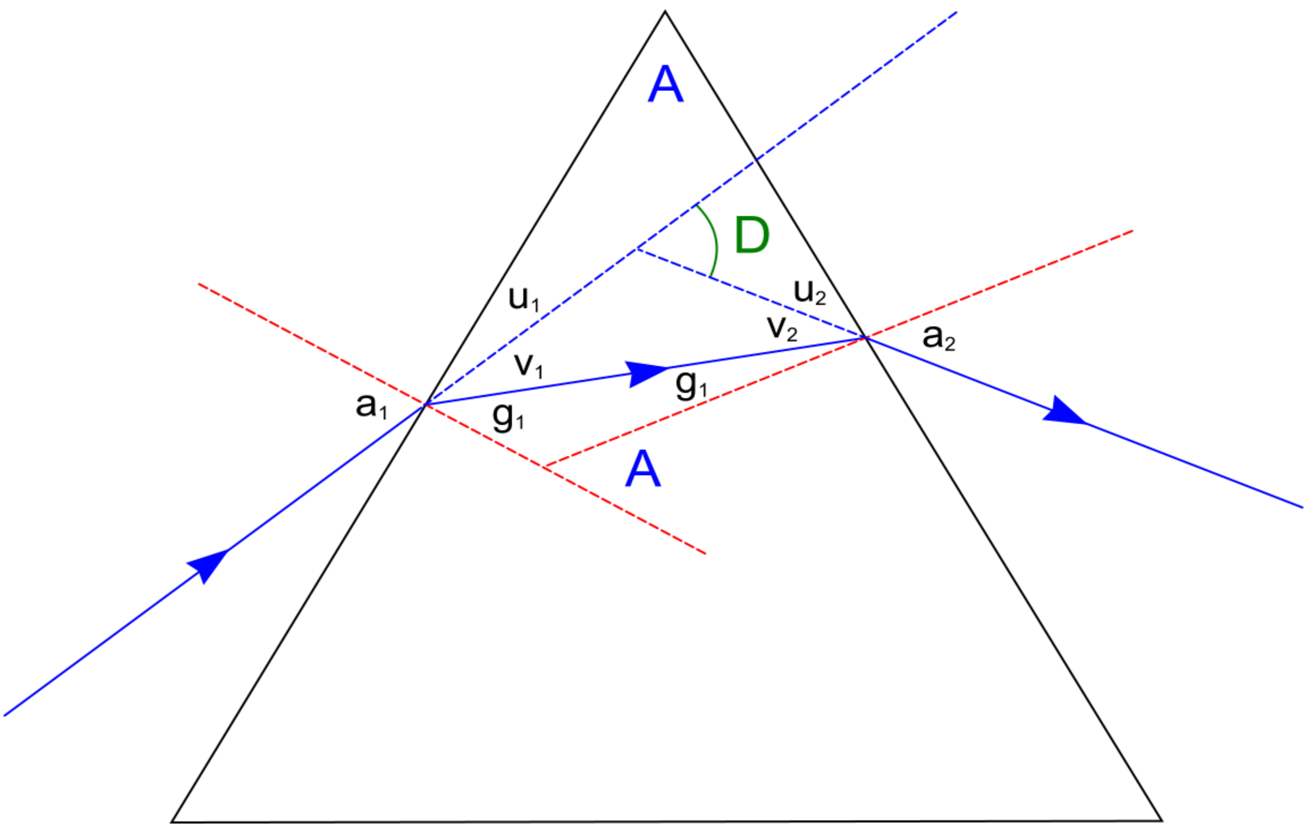
We would like to spend some time on the prism. Consider sending a monochromatic laser beam through a prism.



Courtesy University of Wisconsin-Madison. [The Wonders of Physics Outreach Programs]



Light traveling through prism. The angle D is the deviation angle.

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The angle A is the apex angle. We will prove that the lower angle labeled A is indeed A. For now let A upper be A_{upper} and A lower be A_{lower}. For the ray, there is the angle in air (a_1) entering the glass, the refracted angle in the glass (g_1), the second glass angle (g_2) before the light finally refracts out to the air (a_2).

$A_{upper} + u_1 + v_1 + u_2 + v_2 = 180°$ since these angles form a triangle.

Since the normals make right angles with the glass surfaces,
$u_1 + v_1 + g_1 = 90°$ and $u_2 + v_2 + g_2 = 90°$.

Then $u_1 + v_1 = 90° - g_1$ and $u_2 + v_2 = 90° - g_2$.

$A_{upper} + u_1 + v_1 + u_2 + v_2 = 180°$ => $A_{upper} + (90° - g_1) + (90° - g_2) = 180°$

$$A_{upper} = g_1 + g_2$$

But since A_{lower} is the opposite exterior angle to the triangle with g_1 and g_2,

$$A_{lower} = g_1 + g_2$$

Finally $A_{upper} = A_{lower} = A$, as shown in the figure.

The deviation angle is $D = v_1 + v_2$.

By opposite angles $a_1 = v_1 + g_1$ and $a_2 = v_2 + g_2$.

Therefore, $D = a_1 - g_1 + a_2 - g_2$ => $D = a_1 + a_2 - (g_1 + g_2)$.

Since $A = g_1 + g_2$ we arrive at the result below.

$$\boxed{D = a_1 + a_2 - A}$$

It would be nice to get rid of that a_2.

$$\sin a_1 = n \sin g_1 \qquad \sin a_2 = n \sin g_2$$

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$$g_1 = \sin^{-1}\left[\frac{\sin a_1}{n}\right] \qquad g_2 = \sin^{-1}\left[\frac{\sin a_2}{n}\right]$$

Since $A = g_1 + g_2$, then $g_2 = A - g_1 = A - \sin^{-1}\left[\frac{\sin a_1}{n}\right]$.

$$\sin a_2 = n \sin g_2 = n \sin\{A - \sin^{-1}(\frac{\sin a_1}{n})\}$$

$$a_2 = \sin^{-1}(n \sin\{A - \sin^{-1}(\frac{\sin a_1}{n})\})$$

Our former result $D = a_1 + a_2 - A$ leads to

$$D = a_1 - A + \sin^{-1}(n \sin\{A - \sin^{-1}(\frac{\sin a_1}{n})\})$$

We would like the incident angle that gives the minimum deviation D.

Setting the derivative to zero looks intimidating: $\frac{dD}{da_1} = 0$.

Remember taking derivative of the arcsine? Here we have an arcsine of a sine that then involves another arcsine and sine. Sometimes, it is most practical to make a plot, or invoke intuition. Our intuition suggests that when the light in the prism is horizontal, we have the minimum deviation.

Imagine decreasing a_1 and watching g_1 decrease. The deviation angle D decreases. But, it is a nice exercise to use a spreadsheet to graph relations. I highly recommend this tool and we will use it now.

Spreadsheets are powerful, but a little messy to program in formulas. Here is where other programming apps like *Mathematica* or *Maple* have strong points. But then you need to learn *Mathematica* or *Maple*. However, those skills are worth acquiring.

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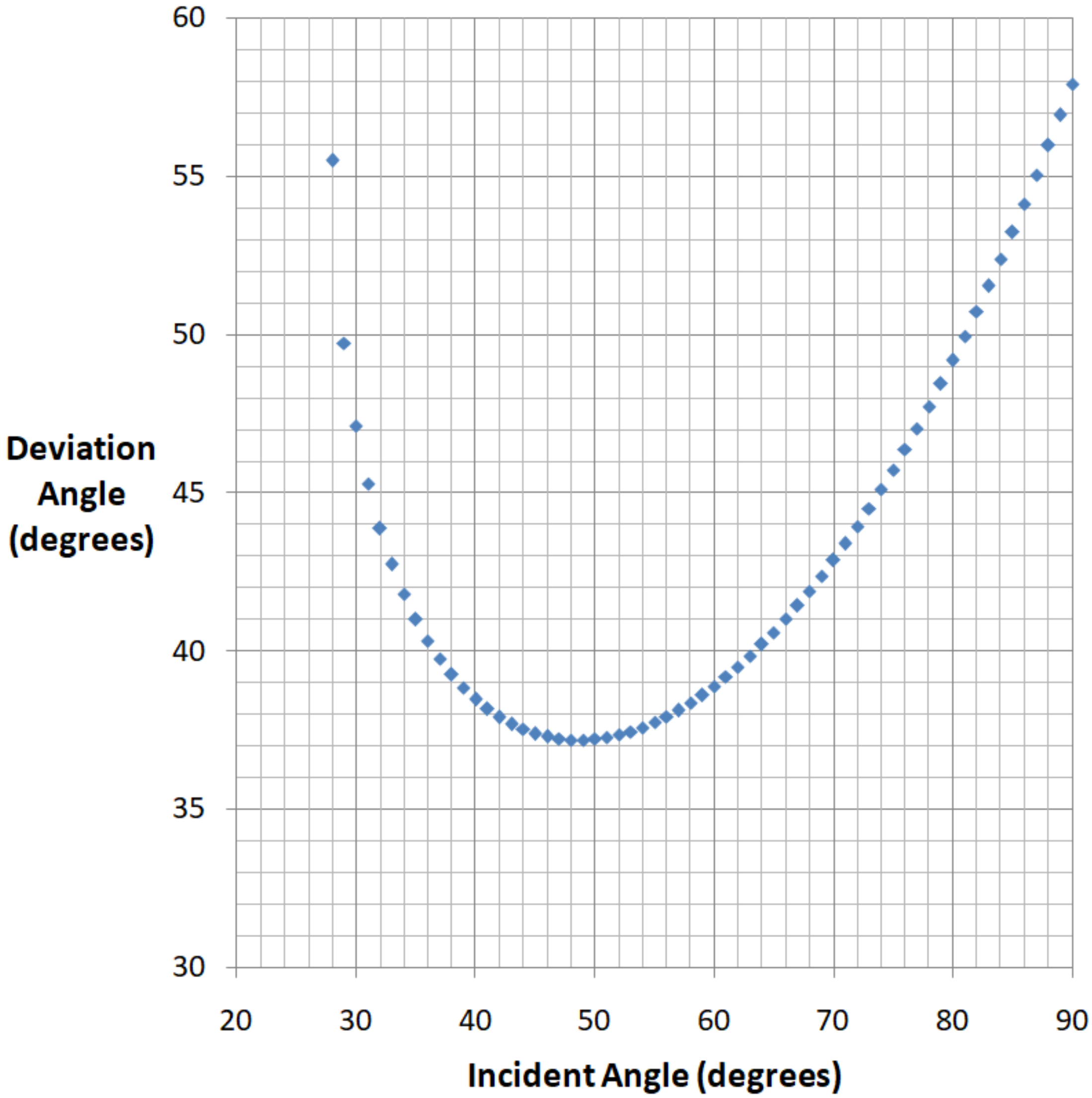
Plot $D = a_1 - A + \sin^{-1}(n \sin\{A - \sin^{-1}(\frac{\sin a_1}{n})\})$ on a spreadsheet,

where $A = 60°$ and $n = 1.5$.

See the plot below and note that whenever I ran into an arcsine of something greater than 1, there is no entry in the plot.



It looks like $a_1 = 49°$ does the trick. Looking at the values in the spreadsheet confirms this value. Then, $g_1 = \sin^{-1}\left[\frac{\sin a_1}{n}\right] = \sin^{-1}\left[\frac{\sin 49°}{1.5}\right] = 30.2°$. But $A = g_1 + g_2$ means

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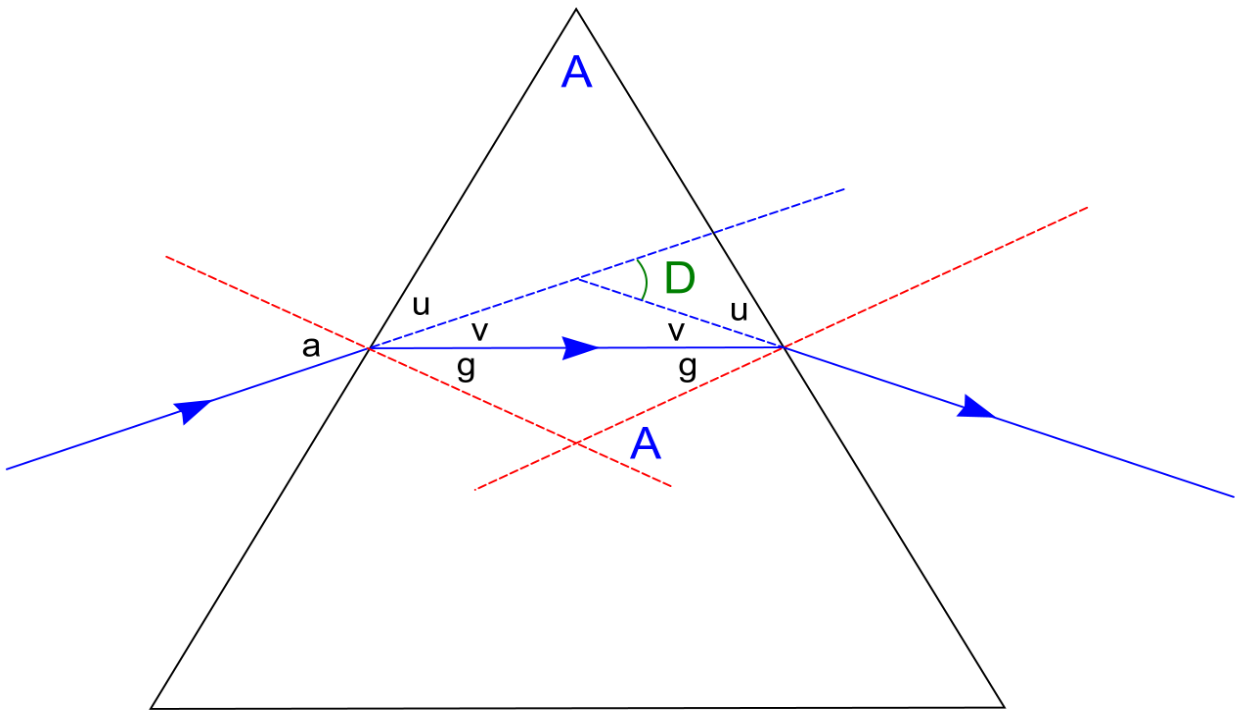
$g_2 = A - g_1 = 60 - 30 = 30°$. Bingo! We have $g_1 = g_2$, an isosceles triangle. We have the situation in the figure below.



Our general formula $D = a_1 + a_2 - A$ now becomes $D_m = 2a - A$, where we include the subscript "m" for minimum, the minimum deviation. Solve for a => $a = \frac{A + D_m}{2}$. Also note that $A = g_1 + g_2$ becomes $A = 2g$ and $g = \frac{A}{2}$. Finally $\sin a = n \sin g$ leads to $n = \frac{\sin a}{\sin g}$ and

$$n = \frac{\sin\left[\frac{A + D_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}.$$

This is an amazing formula. You can shine laser light at the prism to get that horizontal beam in the prism and measure the deviation. From this measurement and the apex angle you can calculate the index of refraction for the glass at the laser wavelength. A beautiful example of an equation with an associated procedure to measure index of refraction.

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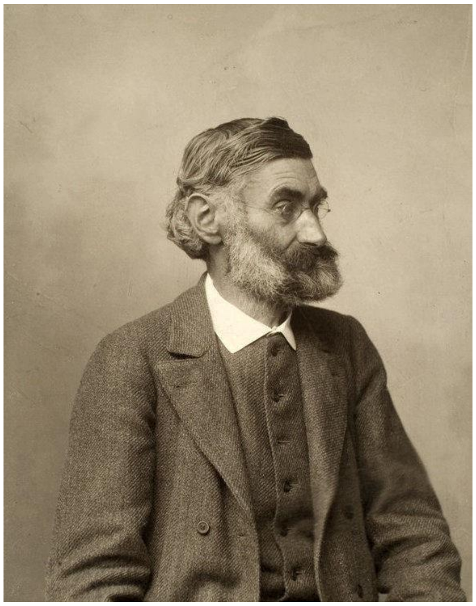
K8. The Abbe Number.



Ernst Karl Abbe (1840 – 1905)
Courtesy Universitätsbibliothek Heidelberg.
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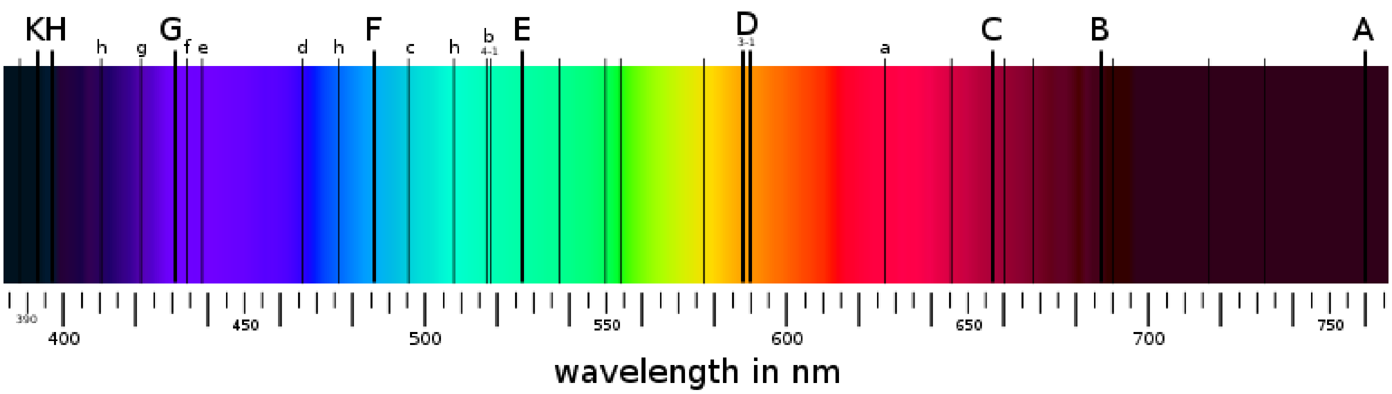
Abbe introduced a measure of dispersion by first subtracting 1 from the index of refraction for an intermediate wavelength and the dividing by the difference for a short and long wavelength.

His formula has the form

$$V = \frac{n_{\text{intermediate}} - 1}{n_{\text{short}} - n_{\text{long}}}.$$

He chose specific wavelengths from absorption lines in the solar spectrum. The solar spectrum has missing wavelengths that get absorbed by elements in the Sun's atmosphere.

The Solar Spectrum (Fraunhofer Lines)



Courtesy Wikipedia: Gebruiker:MaureenV. Released to the Public Domain.

Abbe chose $V = \dfrac{n_D - 1}{n_F - n_C}$

$$\lambda_F = \text{H}_\beta \ (486.1 \text{ nm}) \qquad \lambda_D = \text{Na} \ (589.3 \text{ nm}) \qquad \lambda_C = \text{H}_\alpha \ (656.3 \text{ nm})$$

Two of these wavelengths are from the visible Balmer series, where an electron drops to the 2nd orbit in the hydrogen atom. The four visible Balmer lines are

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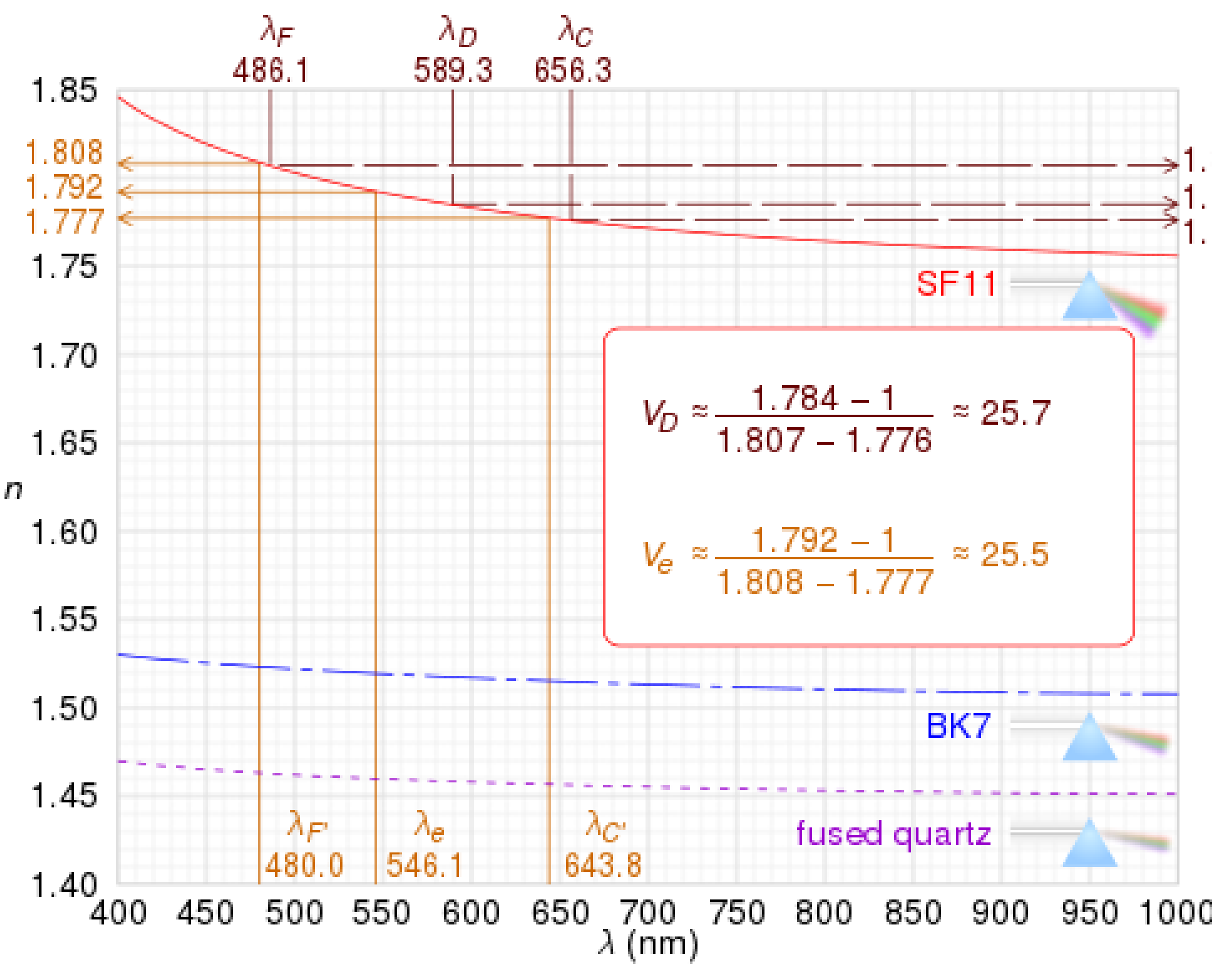
- electron drops from the 3rd orbit to the 2nd orbit: H_α (656.3 nm), RED
- electron drops from 4th orbit to the 2nd orbit: H_β (486.1 nm), CYAN, BLUE-GREEN
- electron drops from the 5th orbit to the 2nd orbit: H_γ (434.0 nm), VIOLET
- electron drops from the 6th orbit to the 2nd orbit: H_δ (410.2 nm), VIOLET

The intermediate wavelength in the Abbe formula is the **average of the sodium lines** from the famous sodium doublet near 590 nm.

- the first line in the sodium 1 doublet $\mathrm{D1} = \mathrm{Na}$ (589.6 nm), YELLOW
- the second line in the sodium 1 doublet $\mathrm{D2} = \mathrm{Na}$ (589.0 nm), YELLOW

Abbe Calculations with Two Different Sets of Wavelengths.



Courtesy Wikimedia: Cmglee. Creative Commons License

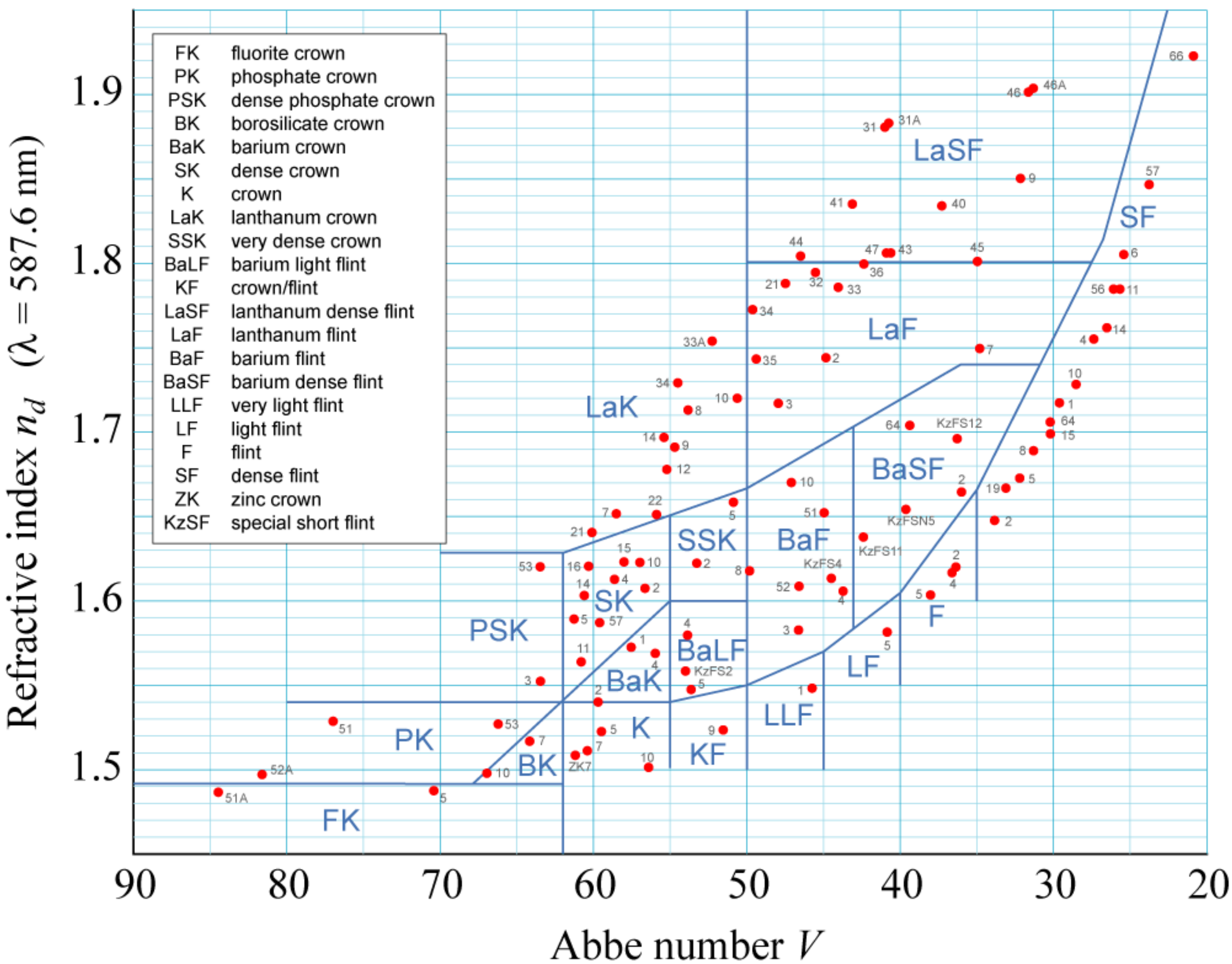
The alternate triple of wavelengths used in the figure give the definition for the Abbe number as

$$V_e = \frac{n_e - 1}{n_{F'} - n_{C'}},$$

where n_e is taken at the mercury e-line. The new lines, easier to obtain in practice, are:

- the mercury e-line $\mathrm{e} = \mathrm{Hg}\ (546.1\ \mathrm{nm})$, GREEN
- the blue cadmium line $\mathrm{F'} = \mathrm{Cd}\ (480.0\ \mathrm{nm})$, BLUE
- the red cadmium line $\mathrm{C'} = \mathrm{Cd}\ (643.8\ \mathrm{nm})$, RED

Abbe Numbers against Refractive Index for Various Glasses



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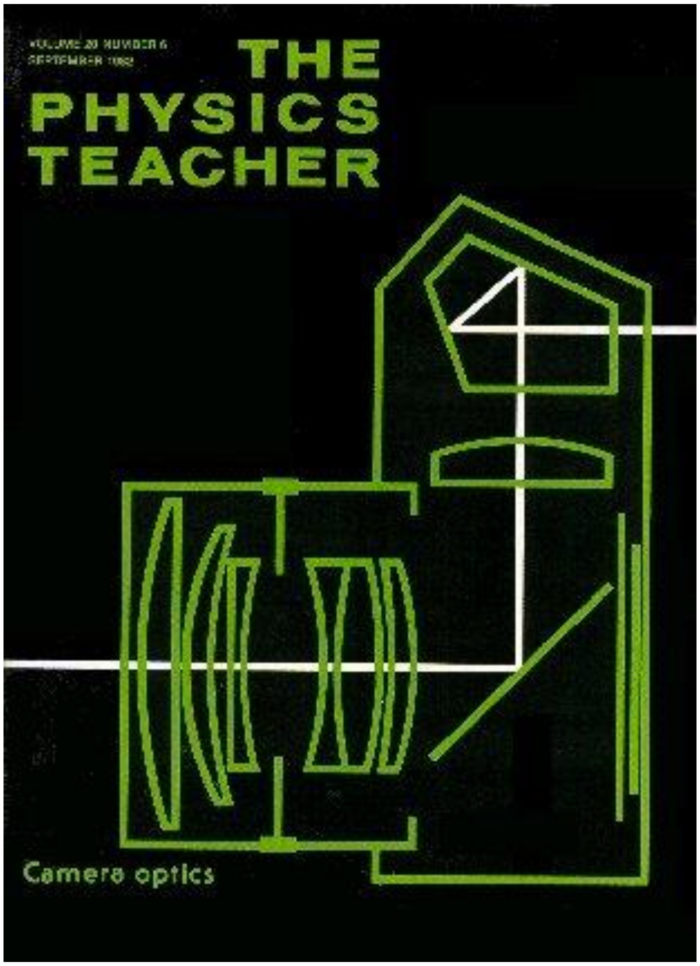
K9. The Achromatic Doublet.



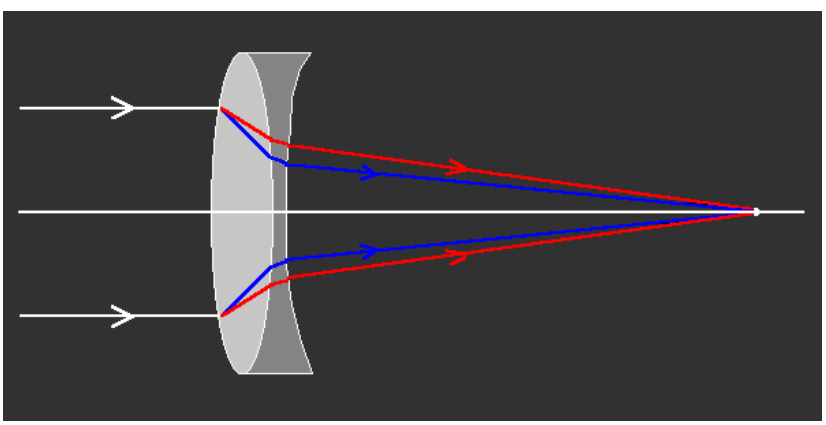
See the doublet cemented together to the right of the aperture. That pair is an achromatic doublet to reduce chromatic aberration. We are going to design one.

But first note the stop in the middle of the two lens-component groups. The center stop balances the barrel distortion with the pincushion distortion.

All 6 lens components work together to minimize the aberrations we have discussed in this chapter.

The 6 pieces of glasses have a net focal length of f = 50 mm for the standard camera lens of the classic SLR camera.

Mine, pictured at the left, actually had a net focal length of 55 mm. It had a bayonet mount that made it easy to pop off. You pressed a special part of the lens that released it so you could turn it and get it off easily. You could then swap the lens for another such as a telephoto or zoom. Or you could insert a teleconverter.



See how the blue and red ray are made to come much closer together in the left sketch? Engineers study the detailed manner in which the colors spread out in the different types of glass in order to find the best materials to work with.

We will design an achromatic doublet. Time for optical engineering!

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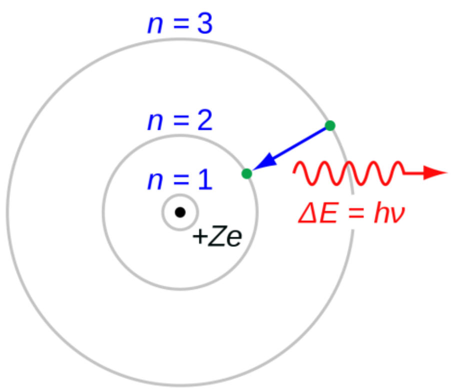

We will be using the reference wavelengths 486 nm, 589 nm, and 656 nm. As we noted earlier, the red 656 nm comes from the electron dropping from orbit 3 to 2.

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Clouds of beautiful red are common in the universe since hydrogen gas abounds in the universe. When nearby starlight excites the gas, the red glow of electrons making the 3-2 Balmer transition gives us spectacular photos. Here the ***Pelican Nebula*** is captured along with the ***North America Nebula*** to its left. Both are in the constellation ***Cygnus***, the Swan, where the major stars of Cygnus are outside the photo to the right and below. The pelican is "looking at the "east coast of North America," – us, in North Carolina.

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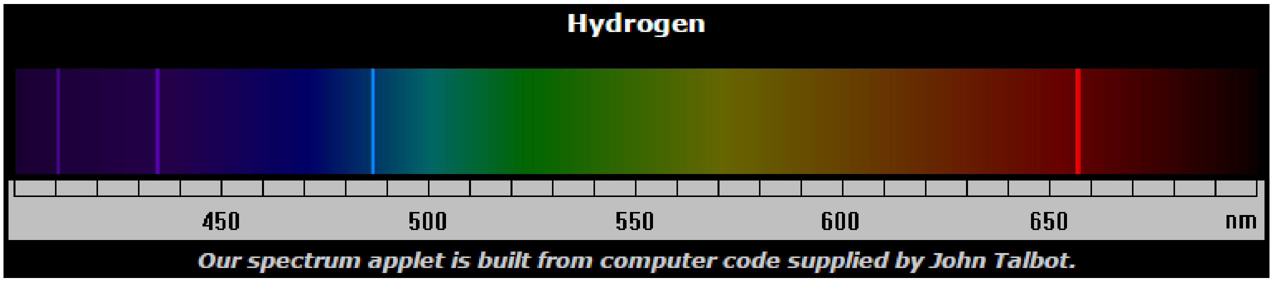
The cyan or blue-green 486 nm comes from the electron transition from orbit 4 to 2 in the hydrogen atom. Here are the four visible Balmer lines. Read right to left: 3-2. 4-2, 5-2, 6-2. The more energy when the electron drops down from higher levels is given by the energy formula $E = hf = \dfrac{hc}{\lambda}$, where h is the Planck constant.



The 589.3 yellow comes from the average of the famous sodium doublet at 589.0 nm and 589.6 nm.

Sodium Vapor Street Lamp

Courtesy Wikimedia: Magnus Manske. Creative Commons License

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Back to the design of an achromat, the achromatic doublet. We start the design by pulling out our thin-lens formula and power equation when two lenses are touching, i.e., the distance between them is zero.

$$\frac{1}{f_1}=(n_1-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \qquad \frac{1}{f_2}=(n_2-1)\left[\frac{1}{R_1^{'}}-\frac{1}{R_2^{'}}\right] \qquad \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

Define k and k' so that

$$\frac{1}{f_1}=(n_1-1)k_1 \qquad \frac{1}{f_2}=(n_2-1)k_2 \qquad \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}\ .$$

Also, let's write $V=\dfrac{n_D-1}{n_F-n_C}$ as $V=\dfrac{n_y-1}{n_b-n_r}$, where y = yellow, b = blue, and r = red.

Let's require in our design specs that f_1 and f_2 be chosen so that

$$\frac{1}{f_b}=\frac{1}{f_r}\ \text{, i.e.,}$$

$$\frac{1}{f_{1b}}+\frac{1}{f_{2b}}=\frac{1}{f_{1r}}+\frac{1}{f_{2r}}$$

Since: $\dfrac{1}{f_{1b}}=(n_{1b}-1)k_1$, $\dfrac{1}{f_{2b}}=(n_{2b}-1)k_2$, $\dfrac{1}{f_{1r}}=(n_{1r}-1)k_1$, $\dfrac{1}{f_{2r}}=(n_{2r}-1)k_2$;

then $(n_{1b}-1)k_1+(n_{2b}-1)k_2=(n_{1r}-1)k_1+(n_{2r}-1)k_2$.

$$(k_1n_{1b}-k_1)+(k_2n_{2b}-k_2)=(k_1n_{1r}-k_1)+(k_2n_{2r}-k_2)$$

$$k_1n_{1b}+k_2n_{2b}=k_1n_{1r}+k_2n_{2r}$$

$$k_1n_{1b}-k_1n_{1r}=-k_2n_{2b}+k_2n_{2r}$$

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$$k_1(n_{1b}-n_{1r})=-k_2(n_{2b}-n_{2r})$$

$$\frac{k_1}{k_2}=-\frac{(n_{2b}-n_{2r})}{(n_{1b}-n_{1r})}$$

Now we lead up to a very beautiful step. We can write for the yellow line

$\frac{1}{f_{1y}}=(n_{1y}-1)k_1$ and $\frac{1}{f_{2y}}=(n_{2y}-1)k_2$, which gives us the pair

$k_1=\frac{1}{f_{1y}(n_{1y}-1)}$ and $k_2=\frac{1}{f_{2y}(n_{2y}-1)}$.

The ratio $\frac{k_1}{k_2}$ is also equal to $\frac{f_{2y}(n_{2y}-1)}{f_{1y}(n_{1y}-1)}$, leading to

$$\frac{k_1}{k_2}=-\frac{(n_{2b}-n_{2r})}{(n_{1b}-n_{1r})}=\frac{f_{2y}(n_{2y}-1)}{f_{1y}(n_{1y}-1)}.$$

$$\frac{f_{2y}}{f_{1y}}=-\frac{(n_{2b}-n_{2r})/(n_{2y}-1)}{(n_{1b}-n_{1r})/(n_{1y}-1)}$$

Here comes the beautiful step, We note that $V=\frac{n_y-1}{n_b-n_r}$. Then

$$\frac{f_{2y}}{f_{1y}}=-\frac{V_1}{V_2} \qquad \text{and} \qquad f_{1y}V_1+f_{2y}V_2=0.$$

The sum of the products of the yellow focal lengths with their respective Abbe numbers is a constant. You can play with the index of refraction and curvatures to obtain this requirement. Note that if $f_{1y}>0$ (converging), then $f_{2y}<0$ (diverging), or vice versa. Our doublet has a converging lens and a diverging lens. You can pick two types of glass and play with the radii of curvature.

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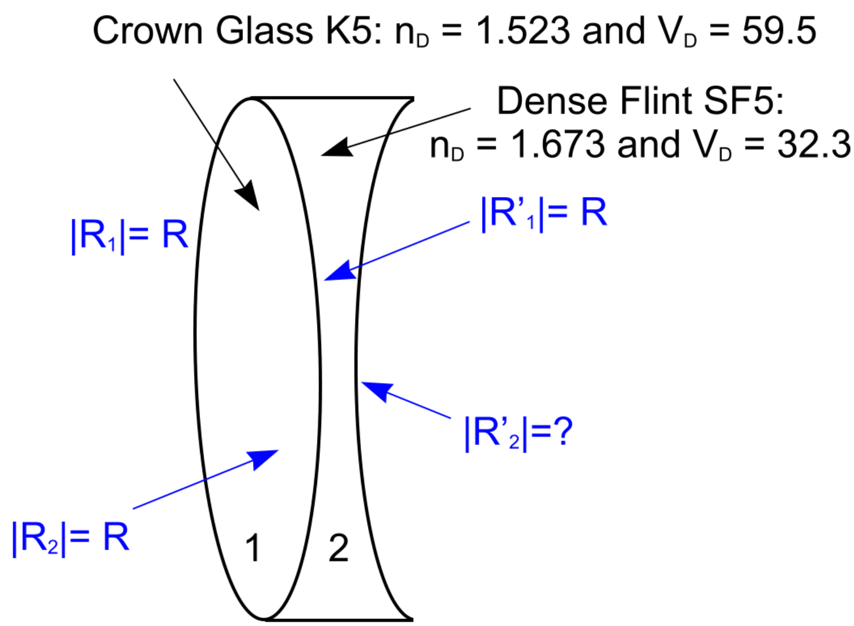
To complete our design for a specific double, let's pick a biconvex converging lens and a concave-plano diverging lens according to the figure below with $f_1 = 20.0 \text{ mm}$.



The math in $\frac{1}{f_1} = (n_1 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ tells us that for a converging situation $R_1 > 0$ and we must have $R_2 < 0$. If both radii were positive we would get nonsense. Therefore,

$$\frac{1}{f_1} = (n_1 - 1)\left[\frac{2}{R}\right],$$

$$\frac{1}{20.0 \text{ mm}} = (1.523 - 1)\left[\frac{2}{R}\right],$$

$$\frac{1}{20 \text{ mm}} = 0.523\left[\frac{2}{R}\right]$$

$$\frac{1}{20.0 \text{ mm}} = 1.046\left[\frac{1}{R}\right]$$

$$R = 1.046 \cdot 20.0\ \text{mm} = 20.92\ \text{mm}$$

For the second lens we must have $\dfrac{f_{2y}}{f_{1y}} = -\dfrac{V_1}{V_2}$ satisfied.

Therefore $f_{2y} = -f_{1y}\dfrac{V_1}{V_2} = -20\dfrac{59.5}{32.3} = -36.84\ \text{mm}$.

Remember, a large focal length means weak lens.

$$\frac{1}{f_2} = (n_2 - 1)\left[\frac{1}{R_1'} - \frac{1}{R_2'}\right]$$, where $|R_1'| = R$ and $|R_2'| = ?$.

Since $n_1 < n_2$ the light will diverge at the interface of the two lenses: we must have $R_1' = -R$. And the last surface will cause a divergent ray too; so the sign as we have is right for R_2'. Now, should it turn out that $R_2' < 0$ from the math, that will mean the curvature will be opposite to what we thought. In that case it will be convex, bowing outward, instead of concave as we have it in the figure.

$$\frac{1}{f_2} = (n_2 - 1)\left[\frac{1}{R_1'} - \frac{1}{R_2'}\right] = (n_2 - 1)\left[\frac{1}{-R} - \frac{1}{R_2}\right]$$

$$\frac{1}{-36.84} = (1.673 - 1)\left[\frac{1}{-20.92} - \frac{1}{R_2}\right]$$

$$\frac{1}{36.84(1.673 - 1)} = \left[\frac{1}{20.92} + \frac{1}{R_2}\right]$$

$$0.040333 = 0.047801 + \frac{1}{R_2} \quad \Rightarrow \quad R_2 = -133.9\ \text{mm}$$

THE LAST SURFACE IS CONVEX!

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The focal length of the system is found from $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$ or

$$f = \frac{f_1 f_2}{f_1 + f_2} = \frac{(20.0)(-36.84)}{20.0 + (-36.84)} = \frac{-736.8}{-16.84} = 43.75 \text{ mm} = 43.8 \text{ mm}.$$

We expect $f > f_1$ as the diverging lens makes it weaker.

The first lens has a power of $P_1 = \dfrac{1000}{f_1(\text{in mm})} = \dfrac{1000}{20} = 50.0 \text{ D}$.

The second lens has a power of $P_2 = \dfrac{1000}{f_2(\text{in mm})} = \dfrac{1000}{-36.84} = -27.1 \text{ D}$.

The total $P = \dfrac{1000}{f\,(\text{in mm})} = \dfrac{1000}{43.75} = 22.9 \text{ D} = 50.0 - 27.1 \text{ D}$

K10. Gems. Diamond has a high index of refraction and much dispersion.

Diamond Engagement Ring Courtesy Wikipedia: YippeeD. Creative Commons License

The above "elegant diamond engagement ring holds an emerald cut centre diamond, flanked by small round diamonds and a pair of rare, colour-change round alexandrites. There are six family birthstones hidden inside the band." Lizunova Fine Jewels

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There are numerous gems with diverse physical characteristics. Four of the physical properties related to optics and important in assessing the worth of a gem are luster, transparency, color, and hardness. The ***luster*** refers to the gem's ability to reflect light at its surface. Strong surface reflections, such as those that occur with diamond, give a gem its ***brilliance***.

The ***transparency*** is a measure of the transmitted light. Considering color, we need to be careful. Remember our spectral-distribution transmission curves for filters. The wavelengths transmitted consists of white minus the wavelengths absorbed by a filter or gem. **The transmitted color is the natural color of the filter or gem and is dependant on the thickness of the filter or gem.** For example, when light passes through more many stacked blue filters, the light that finally leaves is a very dark saturated blue. Finally, the dispersive color, or ***fire*** is due to the variation in the index of refraction of the gem, its dispersion.

The ***hardness*** is important since we desire surfaces resistant to scratches. The German mineralogist Friedrich Mohs (1773-1839) developed the hardness scale (1812) named after him. The scale ranges from very soft at 1 to hardest at 10. Below is a table giving examples for each rating of hardness in the ***Mohs' Scale***.

The Mohs Scale for Hardness (KNOW that Diamond is a 10

Hardness	Mineral	Common Example
1	**Talc**	Pencil lead 1.0 - 2.0
2	**Gypsum**	Fingernail 2.5
3	**Calcite**	Copper Penny 3.5, Brass
4	**Fluorite**	Iron
5	**Apatite**	Tooth Enamel, Knife Blade, Glass 5.5 - 6.0
6	**Orthoclase**	Steel File 6.5
7	**Quartz**	Scratches Glass
8	**Topaz**	Topaz
9	**Corundum**	Sapphire, Ruby
10	**Diamond**	Diamond

Courtesy Department of Geological Sciences, University of Saskatchewan

A piece of talc is so soft that a fingernail scratches it easily. The higher-rated substances can scratch the lower-rated ones. However, you cannot scratch a hard substance, such as diamond, with a softer one such as glass. Only diamond can scratch diamond. In the old days of vinyl records, the "diamond needle" was a long-lasting choice for playing records. If you could not afford one, you might spring for a sapphire.

The cut of the gem is important for its optical characteristics. The best cut depends, in part, on the average index of refraction, which in turn determines the average critical angle. Each small

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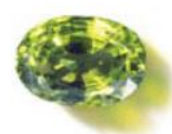
flat surface of a cut gemstone, called a ***facet***, is made with the intent of maximizing the beauty of the gem. Light that undergoes total internal reflection should emerge at an angle for the observer to see.

January	February	March	April
Garnet	**Amethyst**	**Aquamarine**	**Diamond**
			
Dark Red	Purple	Pale Blue	Clear

May	June	July	August
Emerald	**Pearl**	**Ruby**	**Peridot**
			
Green	Cream	Red	Pale Green

September	October	November	December
Sapphire	**Opal**	**Topaz**	**Turquoise**
			
Blue	Iridescent	Yellow	Sky Blue

Constructed years ago from images from these jewelers below. You can visit their websites.

John Anthony Jewelers, 133 Montgomery Avenue, Bala-Cynwyd, PA
Chatham, and Longnecker Jewelry, 314 Norris Ave, McCook, NE.

K11. The Eye and Aberrations. The eye corrects for aberrations in a variety of ingenious ways. We will consider each of the main categories of aberrations and indicate what the eye does to minimize each.

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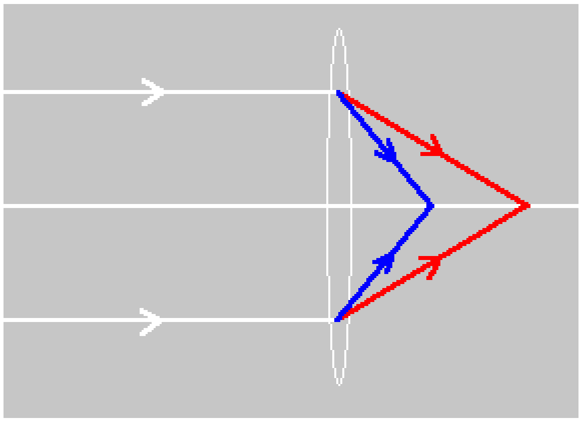


Chromatic Aberration. There are several ways the eye corrects for chromatic aberration, the aberration where blue bends more than red. The optical elements in the lens system for the eye have more chromatic aberration in the blue and ultraviolet that borders the blue (the UVA). So the eye downplays the short-wavelength end of the spectrum.

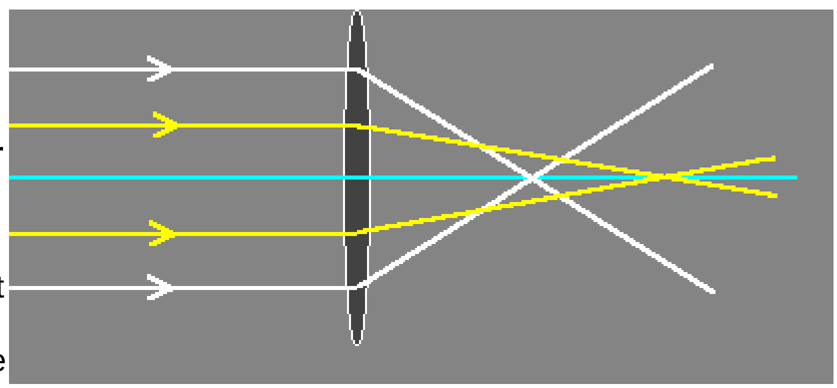
First, the eye lens absorbs ultraviolet. However, it is not good to tax your eyes with UVA since exposure increases the risk of ***cataracts***, cloudy areas, forming on the eye lens.

Second, the light-sensitive cones have their peak sensitivity at 555 nm, which is the yellow-green. Third, the fovea, used for seeing details, has less blue-sensitive cones than green and red. Finally, the fovea is covered by a delicate yellow-pigmented layer called the ***macula lutea***. Some blue is absorbed as light passes through this fine layer to reach the fovea.

The features minimizing chromatic aberration are summarized below.

- The eye lens absorbs UV and some blue (discarding these shorter wavelengths).
- The cones are most sensitive in the yellow-green.
- The fovea has a deficiency of blue-pigment cones.
- The fovea includes a yellow pigment layer to absorb blue light.

Spherical Aberration. There are two main ways the eye corrects for spherical aberration, the aberration where marginal rays refract too much. Spherical aberration occurs due to the spherical shape of lenses. So to optimize the optical system against spherical aberration, the cornea is not spherical. It is an ***aspherical*** optical element where the outer regions have less curvature. This reduced curvature lessens the refraction of marginal rays that occur with strictly spherical surfaces.



Second, the eye lens has a nonuniform index of refraction. The index of refraction near the center of the eye lens is 1.406, decreasing as you go toward the margins, where it is only 1.386. Such a gradual change in a quantity is called a ***gradient***.

These two main ways in dealing with spherical aberration are summarized below.

- An aspherical cornea - the outer regions are less curved.
- A nonuniform eye lens - gradient in the index of refraction, lessening toward the margins.

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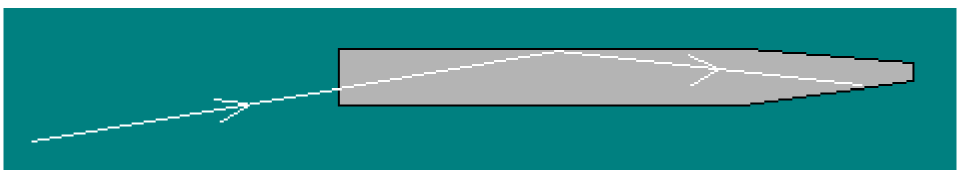
Off-Axis Aberrations. Off-axis aberrations are those that occur when light enters an optical system at large angles from sources a considerable distance away from the optic axis. Think of off-axis rays as rays that are very slanted and think of on-axis rays as rays that are nearly parallel to the optic axis. Off-axis aberrations include curvature of field, astigmatism, coma, and distortion. Off-axis aberration is corrected by discarding off-axis rays for the detailed vision in the fovea. The fovea is nearly on axis.

Light from sources far from the axis falls on the peripheral regions of the retina, where vision is not sharp anyway. If you want to look at something to see it clearly, you need to look directly at it.

The light-sensitive cones have added built-in protection against rays slanted at large angles. The ray in the diagram below enters the rear of the cone first and then reaches the light-sensitive tip in a manner similar to light going through a light pipe. This is an advantage of having the cones facing away from the incoming light. Light rays close to being on axis undergo total internal reflection. This is called the ***Stiles-Crawford Effect***, after its discovers who first observed the phenomenon in 1933. If the incoming ray is very slanted, total internal reflection will not occur and the light will not reach the light-sensitive tip of the cone.

Total Internal Reflection and the Stiles-Crawford Effect



Independent of the previous discussion, a general way to minimize off-axis aberrations is to simply stop down the aperture. The iris does this when plenty of light is available. This provides additional help so that in well-lit environments, such as bright sunny days, vision is very sharp.

There is more. The eye also scans each scene without our conscious awareness. The types of eye motion are:

- drifts - smooth movements of 1 minute of arc (1/60 degree) at 1 Hz (one per second),
- tremors - small rapid motion of 0.25 minute of arc at 50 Hz,
- saccades - large rapid jumps of 5 to 10 minutes of arc at 4 Hz.

The brain then interprets all this sensory data, playing a major role in correcting for all aberrations. The eye motions and interpreting brain make it difficult to see the blind spot during regular use of our eyes. The brain knows there is no "hole in the universe." Only with careful experimentation and attention can you recognize that a blind spot is indeed present. Though it is said that British King Charles II (Reign: 1660-1685) of the House of Stuarts became good at placing his blind spot over the heads of members of his royal court, making heads vanish. Charles II may have been preoccupied with vanishing heads since his dad Charles I (Reign: 1625-1649) was beheaded during the Puritan Revolution led by Oliver Cromwell, member of Parliament.

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