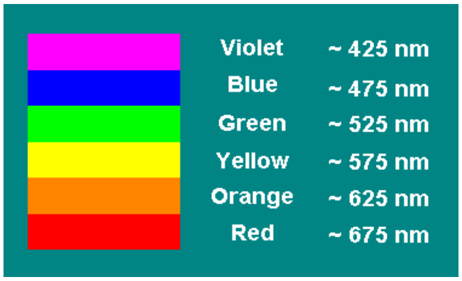
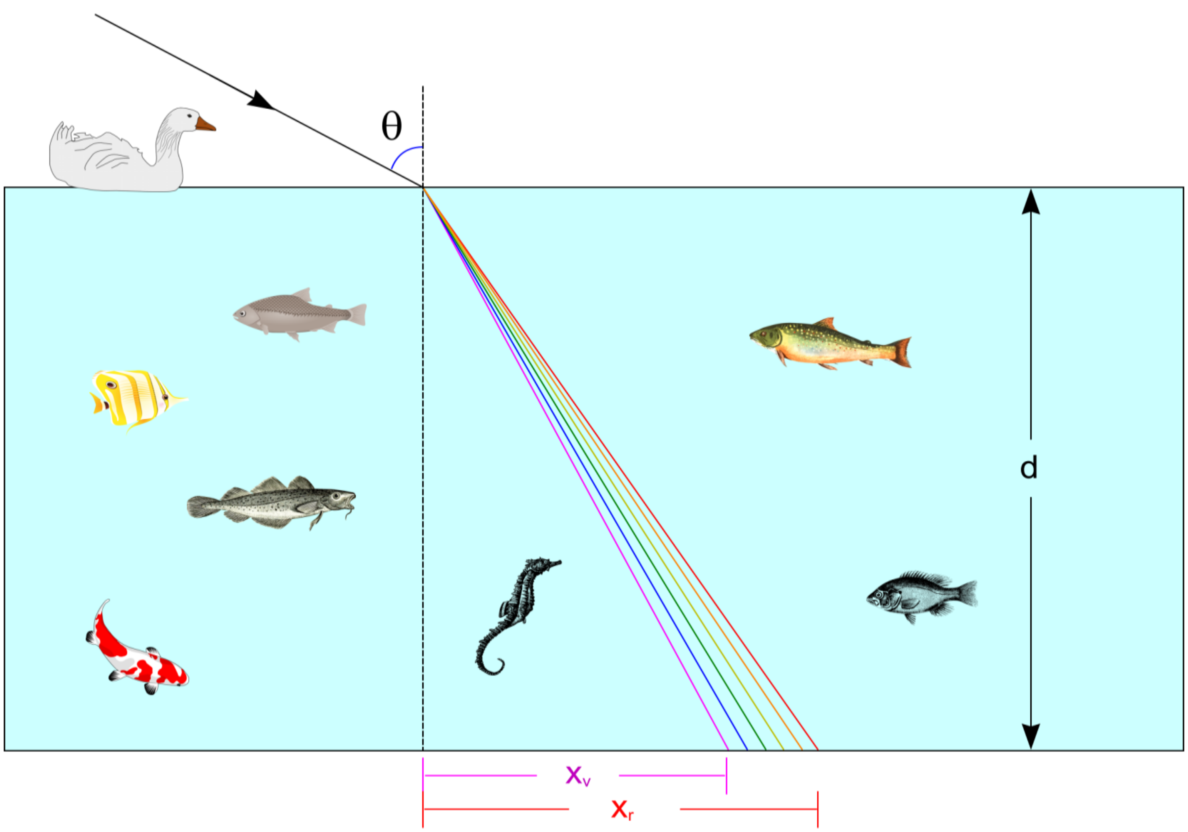
Modern Optics, Prof. Ruiz, UNCA doctorphys.com
Chapter K Homework. Aberrations.



HW-K1. Dispersion in Water. When Professor Booker and I taught astronomy years ago, we found many schools simplifying the spectrum by assigning 50 nm for each of the basic 6 color regions. So we used that scheme for our UNCA astronomy lab. Though some color regions are narrower and some larger, if you start with 425 nm and keep adding 50 nm you do get sample colors in all 6 regions. The index of refraction for water at these wavelengths appear in the table below

color	Violet	Blue	Green	Yellow	Orange	Red
λ (nm)	425	475	525	575	625	675
n (water)	1.338	1.336	1.334	1.333	1.332	1.331

a) Find a general formula x = x(θ,d,n) for the dispersion shown in the pool below.
b) For θ = 45.00° and d = 4.000 meters, give x_v, x_b, x_g, x_y, x_o, and x_r in cm to 4 significant figures using the corresponding refractive indexes in the above table. Give $\Delta = x_r - x_v$ to the nearest mm.
c) What does your general formula reduce to for small angles.
d) For θ = 5.000° and d = 4.000 m, give x_v, x_r, $\Delta = x_r - x_v$ with your exact and approximate formulas.



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a) $$\sin\theta = n\sin\phi = n\frac{x}{\sqrt{x^2+d^2}} \quad => \quad \sin^2\theta = n^2\frac{x^2}{x^2+d^2}$$

$$(x^2+d^2)\sin^2\theta = n^2x^2 \quad => \quad x^2\sin^2\theta + d^2\sin^2\theta = n^2x^2$$

$$d^2\sin^2\theta = n^2x^2 - x^2\sin^2\theta \quad => \quad d^2\sin^2\theta = (n^2-\sin^2\theta)x^2$$

$$x^2 = \frac{d^2\sin^2\theta}{n^2-\sin^2\theta} \quad => \quad \boxed{x = \frac{d\sin\theta}{\sqrt{n^2-\sin^2\theta}}}$$

b) For θ = 45° and d = 4 meters. $x(\text{cm}) = \dfrac{400\sin 45°}{\sqrt{n^2-\sin^2 45°}} = \dfrac{400\frac{\sqrt{2}}{2}}{\sqrt{n^2-\frac{1}{2}}} = \dfrac{400}{\sqrt{2n^2-1}}$.

color	Violet	Blue	Green	Yellow	Orange	Red
λ (nm)	425	475	525	575	625	675
n (water)	1.338	1.336	1.334	1.333	1.332	1.331
x (cm)	249.0	249.5	250.0	250.3	250.6	250.8

$$\Delta \equiv x_r - x_v = 250.8 - 249.0 = 1.8\text{ cm} = 18\text{ mm}$$

c) Small angles: Small angles mean $\theta << 1$ and $\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} \approx \theta$.

$x = \dfrac{d\sin\theta}{\sqrt{n^2-\sin^2\theta}} \to \dfrac{\theta d}{\sqrt{n^2-\theta^2}}$. But since $n > 1$ and $\theta << 1$, then $\theta << n$ and $\boxed{x \approx \dfrac{\theta d}{n}}$.

$$\boxed{x_{\text{exact}} = \frac{d\sin\theta}{\sqrt{n^2-\sin^2\theta}}} \qquad \boxed{x_{\text{approx}} = \frac{\theta d}{n}}$$

d)θ = 5.000°. Exact $x_v = \dfrac{400\cdot\sin 5°}{\sqrt{1.338^2-\sin^2 5°}} = 26.11\text{ cm}$, $x_r = 26.25\text{ cm}$, $\Delta = 1.4\text{ mm}$

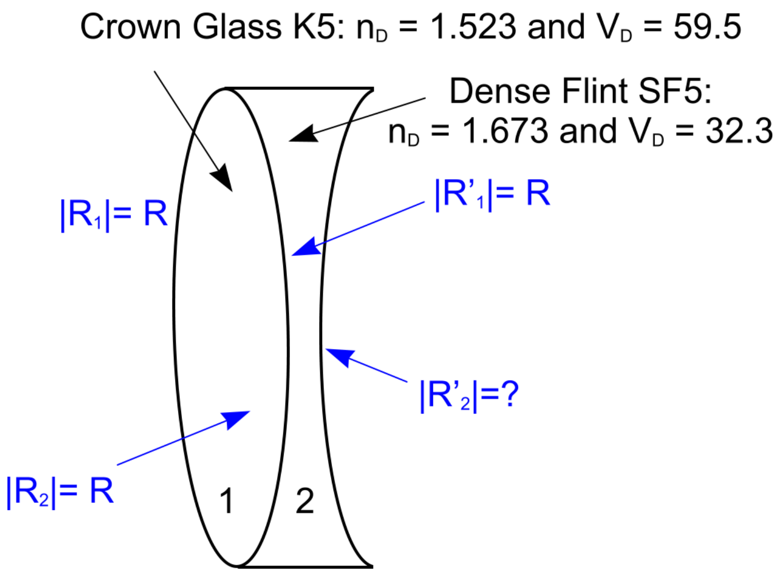
Approx: $5° = \dfrac{5°}{180°}\pi = 0.087266\text{ radians}$ and $x_{\text{approx}} = \dfrac{0.087266\cdot 400}{n} = \dfrac{34.907}{n}$

$x_v = 26.09\text{ cm}$, $x_r = 26.23\text{ cm}$, $\Delta = 1.4\text{ mm}$. Note $\Delta_{\text{approx}} = \Delta_{\text{exact}} = 1.4\text{ mm}$

To the nearest mm you have 1 mm in each case: $\Delta_{\text{approx}} = \Delta_{\text{exact}} = 1\text{ mm}$.

HW-K2. The Achromat.

Design an achromatic doublet shown below that has an effective focal length of f = 50.0 mm. Note that the decimal and following zero indicate 3 significant figures. Calculate R and R'_2 to three significant figures. Is the rear surface convex or concave? Then give the focal lengths for the components f_1 and f_2 to three significant figures. Chemists will be happy with us since our f and the Abbe numbers are to 3 significant figures, while the refractive indexes are to 4 significant figures. Since the least number of significant figures for the input parameters is 3, we should report final answers to 3 significant figures. Chemists keep physicists on the ball when it comes to significant figures. During intermediate steps keep at least 4 significant figures and round off last.



Start with $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ and $f_{1y}V_1 + f_{2y}V_2 = 0$.

I prefer to work with power.

$$P = P_1 + P_2 \quad \text{and} \quad \frac{V_1}{P_1} + \frac{V_2}{P_2} = 0$$

We want $P = \frac{1000}{f_1 \text{ (in mm)}} = \frac{1000}{50} = 20 \text{ D} = P_1 + P_2$

$$\frac{V_1}{P_1} + \frac{V_2}{P_2} = 0 \quad => \quad \frac{V_1}{P_1} = -\frac{V_2}{P_2} \quad => \quad \frac{P_2}{P_1} = -\frac{V_2}{V_1} = -\frac{32.3}{59.5} = -0.5429$$

Summary (2 equations, 2 unknowns): $P_1 + P_2 = 20 \text{ D}$ and $P_2 = -0.5429P_1$.

$$P_1 + (-0.5429P_1) = 20 \text{ D} \quad => \quad 0.4571P_1 = 20 \text{ D}$$

$$P_1 = 43.75 \text{ D} \quad \text{and} \quad P_2 = 20 - 43.75 = -23.75 \text{ D}$$

$$f_1 \text{ (in mm)} = \frac{1000}{P_1} = \frac{1000}{43.75} = 22.9 \text{ mm} \qquad f_2 = \frac{1000}{P_2} = \frac{1000}{-23.75} = -42.1 \text{ mm}$$

For R:

$$P_1 = \frac{1}{f_1} = (n_1 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{2(n_1 - 1)}{R}$$

$$43.75 = P_1 = \frac{1}{f_1} = (n_1 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{2(n_1 - 1)}{R} = \frac{2(1.523 - 1)}{R}$$

$$R = \frac{2(1.523 - 1)}{43.75} = 0.023909 \text{ m} = 23.9 \text{ mm}$$

$$-23.75 = \frac{1}{f_2} = (n_2 - 1)\left[\frac{1}{R_1'} - \frac{1}{R_2'}\right] = (1.673 - 1)\left[\frac{1}{-0.023909} - \frac{1}{R_2'}\right]$$

$$\frac{-23.75}{1.673 - 1} = \frac{1}{-0.023909} - \frac{1}{R_2'}$$

$$-35.290 = -41.825 - \frac{1}{R_2'}$$

$$35.290 = 41.825 + \frac{1}{R_2'}$$

$$\frac{1}{R_2'} = 35.290 - 41.825 = -6.535$$

$$R_2' = -\frac{1}{6.535} = -0.1530 \text{ m} = -153 \text{ mm}$$

This last surface is convex, i.e., it bows outward, but gently.
The first surface of the diverging lens is concave more by a factor of 10.
$R = 23.9 \text{ mm} \approx 10\left|R_2'\right|$, making the second lens overall diverging.