HW Q1. The Electric Field.

In the second semester of introductory physics with calculus the uniform sphere of charge with total charge Q and radius R is usually addressed. The charge density is

$$\rho = \frac{Q}{(4/3)\pi R^3}.$$

(a) Use Gauss's law to show that the electric field for $r > R$ is

$\vec{E}_{out}(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$, and that it can be written in the form $\vec{E}_{out}(r) = \frac{\rho R^3}{3\varepsilon_0 r^2}\hat{r}$.

(b) Use Gauss's law to show that the electric field for $r < R$ is $\vec{E}_{in}(r) = \frac{\rho}{3\varepsilon_0} r\,\hat{r}$.

Note that $\vec{E}_{in}(R) = \vec{E}_{out}(R) = \frac{\rho R}{3\varepsilon_0}\hat{r}$

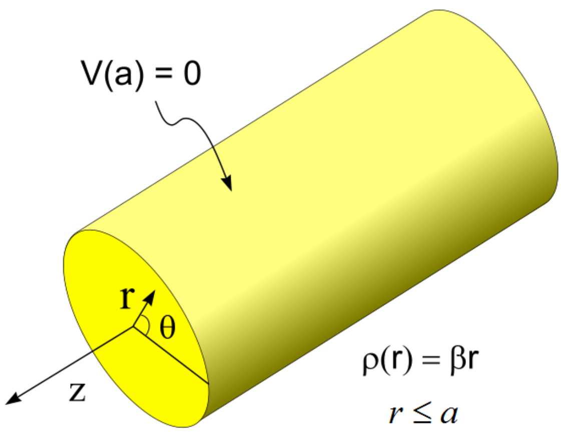
(c) Calculate $\nabla \cdot \vec{E}$ for $r > R$.

(d) Calculate $\nabla \cdot \vec{E}$ for $r < R$.

(e) Explain your answers to (c) and (d) in light of the Maxwell equation $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$.

Note: For full credit all equations must have correct notation, e.g., vector quantities need to have arrows above them or carats for unit vectors, and a vector equation must have vector signs on each side of the equation. Exception: zero does not need a vector sign over it.

HW Q2. Poisson's Equation. Consider cylindrical coordinates defined with the notation as shown in the figure, i.e., (r,ϕ,z). You will see there a long cylinder which you can take to be infinite in length along the z-axis. It has radius $r = a$ and charge density $\rho(r) = \beta r$.



The following equations, one of which includes the Laplacian, describe the physics of the electric potential V and electric field **E**.

$$\vec{E} = -\nabla V$$

$$\nabla \cdot \vec{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad => \quad \nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

(a) Solve Poisson's equation (the one with the Laplacian) in the charge region using cylindrical coordinates to obtain the following with integration constants A and B.

$V_{\text{in}}(r) = -\frac{\beta}{\varepsilon_0}\frac{r^3}{9} + A\ln r + B$, where "in" refers to $r \le a$. Out means $r \ge a$.

(b) Give a physics reason why the integration constant A should be taken to be zero. Then take the negative gradient of your potential to find $\vec{E}_{\text{in}}(r)$. Integrate the charge density to find the total charge Q for a section of length h of the cylinder having the full radius r = a. Then divide by h to find the linear charge density $\lambda = \frac{Q}{h}$ in terms of a and β. You can check your answer by plugging your λ into the classic electric-field formula for a line of charge: $\vec{E}_{\text{out}}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$. Your answer is most likely correct if your $\vec{E}_{\text{in}}(a)$ matches $\vec{E}_{\text{out}}(a)$.