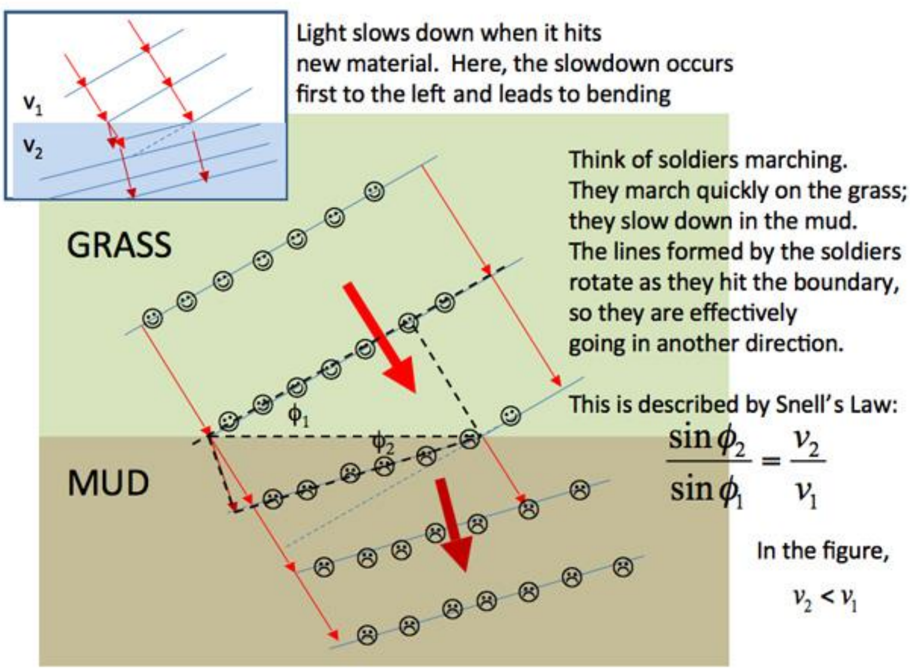
Modern Optics, Prof. Ruiz, UNCA doctorphys.com
Chapter X. Multiple Reflections.

X1. Optical Path Length.

Start with $k = \dfrac{2\pi}{\lambda}$ in general and let $k_o = \dfrac{2\pi}{\lambda_o}$ for light traveling in vacuum. Then $\dfrac{k}{k_o} = \dfrac{\lambda_o}{\lambda}$. Using $c = \lambda_o f_o$ in vacuum and $v = \lambda f$ in some general medium,

$$\frac{k}{k_o} = \frac{c/f_o}{v/f} = \frac{c}{v} = n \text{ since } f = f_o. \qquad \boxed{k = nk_o}$$

Remember the conservation of marching soldiers? The wavelength shortens as the soldiers march from a fast medium to a slow one. Conservation requires that the frequency of passing rows not change as the soldiers march from grass (a fast medium) to mud (a slow medium).



Refraction and Marching Soldiers. Image Courtesy University Corporation for Atmospheric Research (NCAR), Boulder, Colorado, Material Supported by the National Science Foundation (NSF) and NCAR.

When we write general waves as

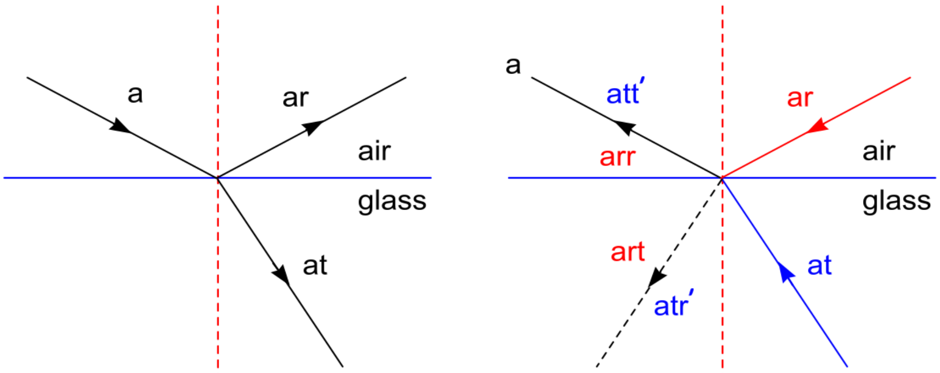
$\sin(kx - \omega t)$ or a phasor $e^{i(kx-\omega t)}$,

an advance in phase for moving a distance x in space is

$$\phi = kx = nk_o x = k_o nx$$, an important result we will use later.

The optical path length OPL = nx is sometimes written as ns.

X2. Stokes and the Interface. In a previous chapter when we worked with the Fresnel equations for reflection and transmission we introduced the reflection coefficient r and transmission coefficient t. Things got fairly complicated. Here is a simpler analysis due to Stokes for our purposes in this chapter.



The left figure introduces the reflection “r” and transmission coefficients “t”. These coefficients work on the amplitude “a” of the incoming wave in the left figure.

These two are defined for the air-to-glass interface.

For the right figure we reverse the directions of the outgoing rays. We then introduce r’ and t’ for the glass-to-air interface, i.e., the reverse situation of air to glass. The top reflected ray in the right figure can be expressed in two ways, one using r for the partial reflection of the “ar” ray and one using t’ for the partial transmission of the at ray. Similarly the dashed ray is represented as a refracted transmission of “ar” and a reflection of the “at” ray. We can make the following two observations:

$$att' + arr = a \quad \text{and} \quad art + atr' = 0 .$$

The first equation leads to $tt' + r^2 = 1$. The second equation leads to $r' = -r$.

Caution: Back in the Fresnel interface chapter we found that $t^2 + r^2 \neq 1$. If this result were true, we would have tt’ = t^2, which is not true. So we can safely conclude that reflecting off glass is r and reflecting inside of glass off the air is –r, and that transmission from air to glass (t) is not the same as transmission from glass to air (t’).

Sir George Stokes, 1st Baronet
1819 - 1903

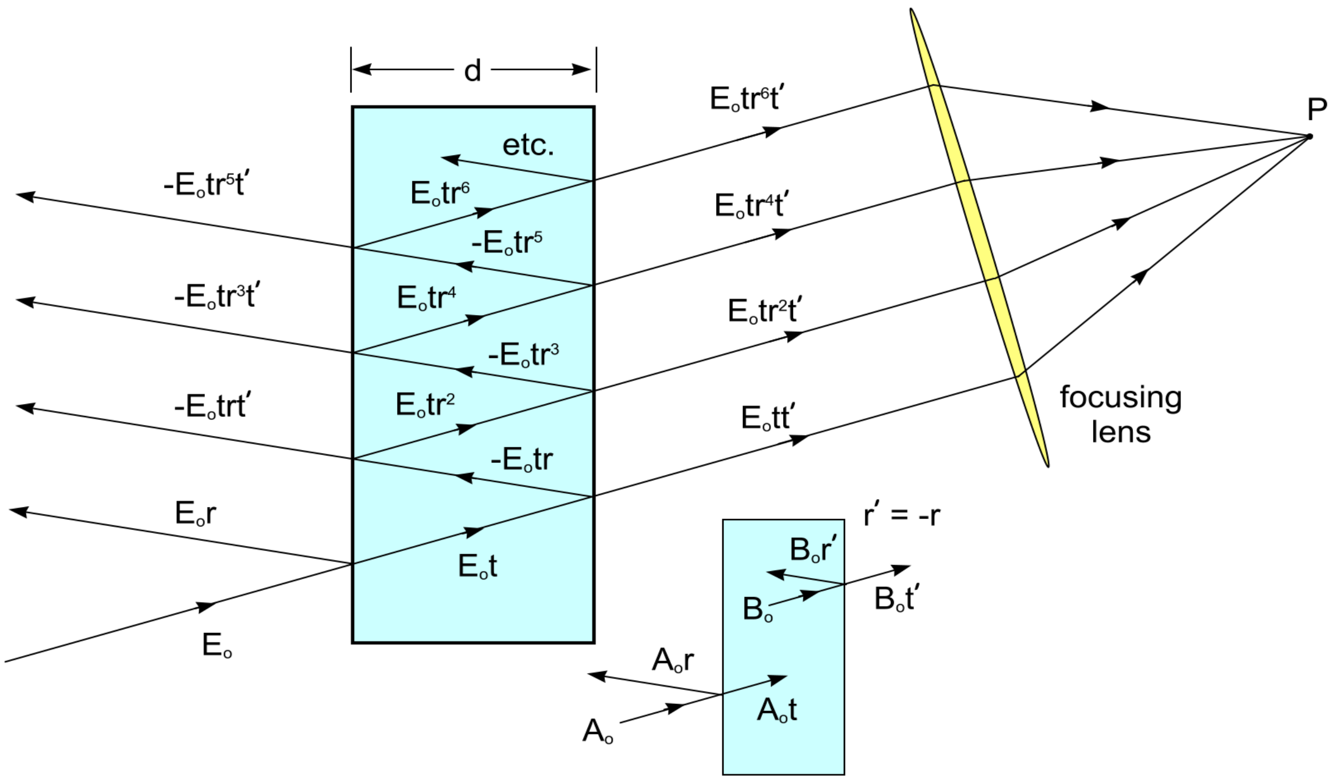
Irish (born in Ireland, English Ancestors)

Physicist and Mathematician

Known Especially for Stokes Theorem in Vector Calculus, which we use in physics to transform two of the Maxwell equations in integral form to the differential form. The other two Maxwell equations are transformed by the divergence theorem (also called Gauss’s theorem), and we did that in class earlier.

X3. Multiple-Beam Interference. Below is a slab with two partially reflecting surfaces. The index of refraction is n. The amplitude coefficients of reflection and transmission are those that we introduced in the previous section X2 and summarized in the lower right inset.

Multiple Reflections



We consider the case where we have normal incidence, i.e., heading directly at the first glass plate with angle of incidence $\theta = 0°$. Let n be the index of refraction of the medium between the two plates. The distance between the two plates is d. The distance traveled of a round trip between the two plates is $2d$. The phase difference for a single round trip between the plates is then

$$\phi = kx = k_o nx = k_o n(2d) \equiv \delta_o \, .$$

The total transmission in the figure is given by

$$E_t = E_o tt' + E_o tr^2 t' e^{i\delta_o} + E_o tr^4 t' e^{2i\delta_o} + E_o tr^6 t' e^{3i\delta_o} + \ldots$$

$$E_t = E_o tt' \left[1 + r^2 e^{i\delta_o} + r^4 e^{2i\delta_o} + r^6 e^{3i\delta_o} + \ldots \right]$$

$$E_t = E_o tt' \left[1 + r^2 e^{i\delta_o} + (r^2 e^{i\delta_o})^2 + (r^2 e^{i\delta_o})^3 + \ldots \right]$$

Remember when we encountered a geometric series in multi-slit diffraction?

$S_n \equiv 1 + x + x^2 + x^3 + ... + x^n$, where we use x now since r is already being used.

$xS_n = x + x^2 + x^3 + x^4 + ... + x^{n+1}$ => $S_n - xS_n = 1 - x^{n+1}$ => $S_n = \dfrac{1 - x^{n+1}}{1 - x}$

For $x < 1$ and $n \to \infty$. Then $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \dfrac{1 - x^{n+1}}{1 - x} = \dfrac{1}{1 - x}$.

But we have a complex number $x = r^2 e^{i\delta_o}$. The formula still applies as

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} x^{n+1} = \lim_{n\to\infty} (r^2 e^{i\delta_o})^{n+1} = \lim_{n\to\infty} [(r^2)^{n+1} e^{i\delta_o (n+1)}]$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} [(r^2)^{n+1}] \lim_{n\to\infty} [e^{i\delta_o (n+1)}].$$

The 1st factor $\lim\limits_{n\to\infty} (r^2)^{n+1} = 0$ since $0 < r < 1$ and the 2nd is a finite trig oscillation, i.e.,

$e^{i\delta_o (n+1)}$ oscillates with $\left| e^{i\delta_o (n+1)} \right| = 1$.

The oscillation term will be zapped by the other limit, which is zero. Our series is then

$E_t = E_o tt' \dfrac{1}{1 - x}$ where $x = r^2 e^{i\delta_o}$, giving $E_t = \dfrac{E_o tt'}{1 - r^2 e^{i\delta_o}}$.

Now remember that $tt' + r^2 = 1$, so we can substitute $tt' = 1 - r^2$.

$$E_t = \frac{E_o (1 - r^2)}{1 - r^2 e^{i\delta_o}}$$

Replacing r^2 with the reflectance $R = r^2$, we have $E_t = \dfrac{E_o (1 - R)}{1 - R e^{i\delta_o}}$.

The irradiance is

$$I_T = \frac{1}{2}|E_t|^2 = \frac{1}{2}\left[\frac{E_o(1-R)}{1-Re^{i\delta_o}}\right]\left[\frac{E_o(1-R)}{1-Re^{i\delta_o}}\right]^* .$$

Let the numerator be $\mathrm{A} = \frac{1}{2}E_o^2(1-R)^2$ and

the denominator $\mathrm{D} \equiv (1-Re^{i\delta_o})(1-Re^{i\delta_o})^* = (1-Re^{i\delta_o})(1-Re^{-i\delta_o})$.

$$D = 1 - R(e^{i\delta_o} + e^{-i\delta_o}) + R^2$$

$$D = 1 - 2R\cos\delta_o + R^2$$

Subtract and add $2R$.

$$D = 1 - 2R + R^2 + 2R - 2R\cos\delta_o$$

$$D = (1-R)^2 + 2R(1-\cos\delta_o)$$

$$1-\cos\delta_o = 1 - \frac{e^{i\delta_o}}{2} - \frac{e^{-i\delta_o}}{2} = -\frac{1}{2}(e^{i\delta_o/2} - e^{-i\delta_o/2})^2$$

$$1-\cos\delta_o = -\frac{(2i)^2}{2}\left(\frac{e^{i\delta_o/2} - e^{-i\delta_o/2}}{2i}\right)^2 = 2\sin^2\frac{\delta_o}{2}$$

$$D = (1-R)^2 + 4R\sin^2\frac{\delta_o}{2}$$

Since there is a $(1-R)^2$ factor in the numerator, let's pull this factor out.

$$D = (1-R)^2\left[1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta_o}{2}\right]$$

The coefficient of finesses is defined as

$$\boxed{F \equiv \frac{4R}{(1-R)^2}}$$

and the denominator becomes $D = (1-R)^2 \left[1 + F \sin^2 \frac{\delta_o}{2} \right]$.

Bringing back the numerator $A = \frac{1}{2} E_o^2 (1-R)^2$,

for the irradiance $I_t = \frac{A}{D}$.

$$\boxed{I_T = \frac{I_o}{1 + F \sin^2 \frac{\delta_o}{2}}}$$

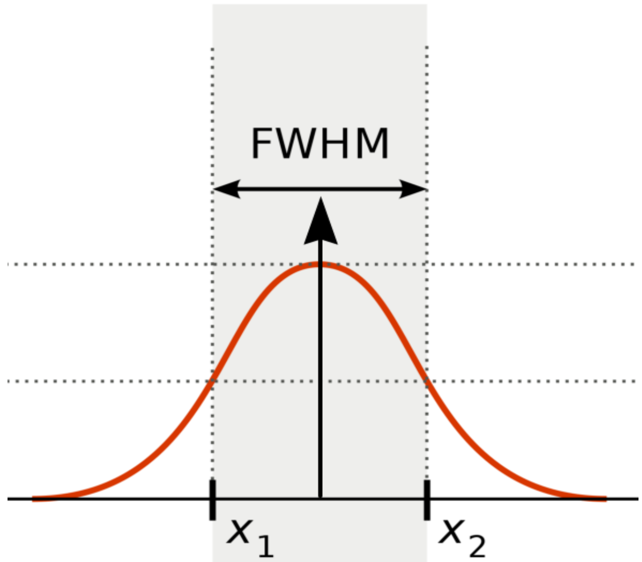
The reflected irradiance, by conservation of energy is $I_R = I_o - I_T$.

Maxima for I_T occur when $\sin^2 \frac{\delta_o}{2} = 0$, i.e., $\frac{\delta_o}{2} = N\pi$.

Recalling that $\delta_o = k_o n(2d)$ for normal incidence, $\frac{\delta_o}{2} = N\pi$ leads to

$k_o nd = N\pi$ for normal incidence.

If the finesse factor F is large, then a plot of I_T as a function of phase δ_o will be narrower because the large F will bring the irradiance down unless you are close to a max. The narrower peaks allow for better spectral resolution of an individual wavelength. A convenient measure is the linewidth defined by the full width at half maximum (FWHM).



Adapted from Wikipedia: Arne Nordm ann (norro). [Creative Commons]

So we want to solve for $\dfrac{I_T}{I_o} = \dfrac{1}{1 + F\sin^2\dfrac{\delta_o}{2}} = \dfrac{1}{2}$.

The half maxima occur when

$$F\sin^2\frac{\delta_o}{2} = 1 \text{, i.e., } \sin\frac{\delta_o}{2} = \pm\frac{1}{\sqrt{F}} .$$

For small angles $\sin\dfrac{\delta_o}{2} \approx \dfrac{\delta_o}{2} \approx \pm\dfrac{1}{\sqrt{F}}$ and $\delta_o \approx \pm\dfrac{2}{\sqrt{F}}$.

Indentify $x_2 = +\dfrac{2}{\sqrt{F}}$ and $x_1 = -\dfrac{2}{\sqrt{F}}$ in the above figure to get

$$\text{FWHM} = x_2 - x_1 = \frac{4}{\sqrt{F}} .$$

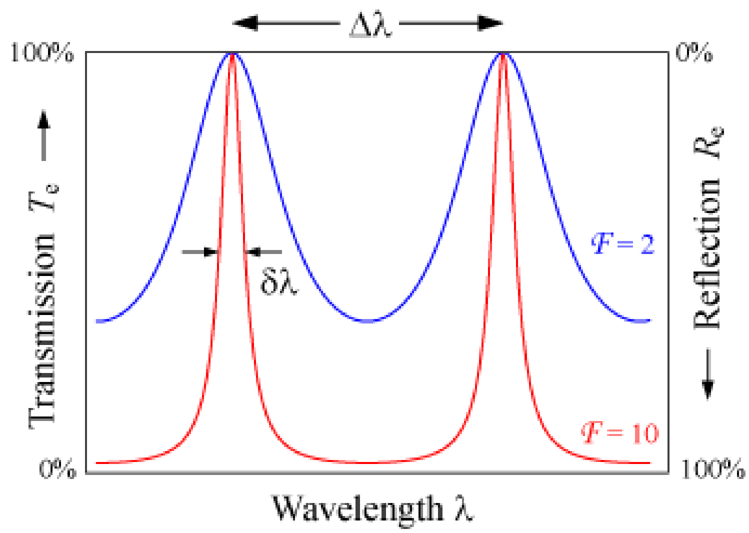
Another finesse is defined with italics: *finesse*, by the definition

$$\Im = \frac{2\pi}{\text{FWHM}} = \frac{2\pi}{4/\sqrt{F}} = \frac{\pi\sqrt{F}}{2} .$$

Remembering that $k = k_o n$, the phase can be expressed in terms of either the vacuum wavelength λ_o or the wavelength λ in the medium:

$$\delta_o = k_o n(2d) = \frac{2\pi}{\lambda_o} n(2d) = \frac{4\pi nd}{\lambda_o} \quad \text{or} \quad \delta_o = k(2d) = \frac{4\pi d}{\lambda} .$$

Plot or Transmission Irradiance Percent as a Function of Wavelength For Two Different Values of *Finesse*



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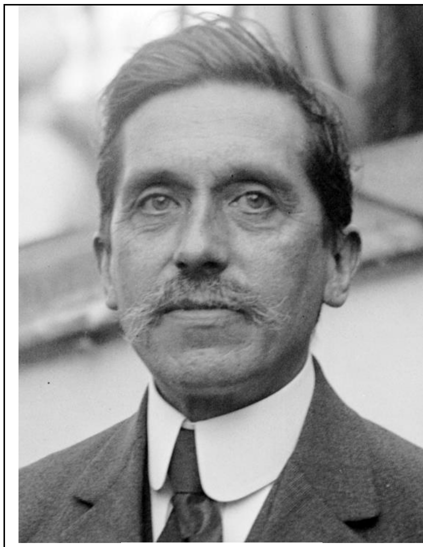
Note the sharper spectral lines for the case $\Im = 10$ compared to $\Im = 2$.

X4. Fabry-Perot Interferometer.

	
Charles Fabry	Alfred Perot
1867 – 1945	1863 - 1925
French Physicist Co-Discoverer: Ozone Layer	French Physicist Published Papers as Pérot

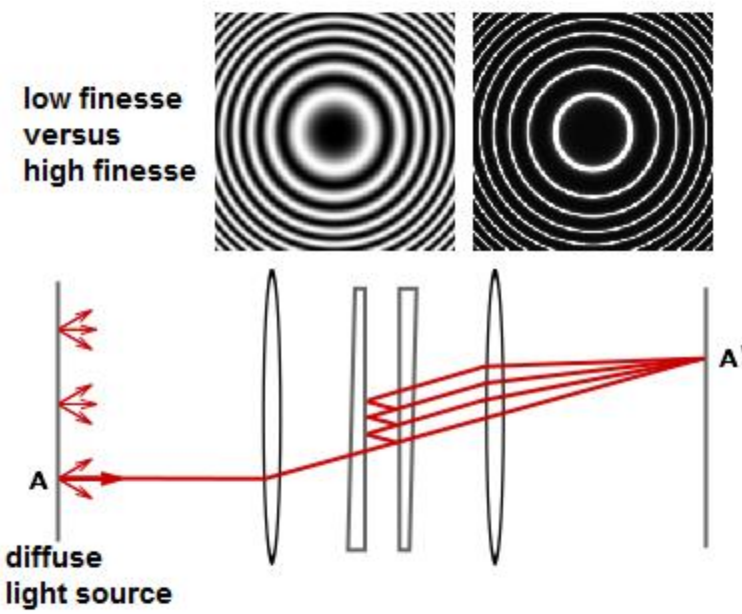
The Fabry-Pérot interferometer is shown below. It employs the general principles we have developed in this chapter.

The design calls for two highly reflecting plates separated by a distance d. The distance can be varied by moving one of the optical elements. The distances are on the order of millimeters or centimeters.

A source sends light to a diffuse screen that then passes through a lens and on to the pair of optical elements, the mirrors or optical flats with highly reflecting surfaces.

Finally the light passes through another lens on its way to the screen.

Schematic of the Fabry-Pérot Interferometer



Wikipedia: Stigmatella aurantiaca. Creative Commons

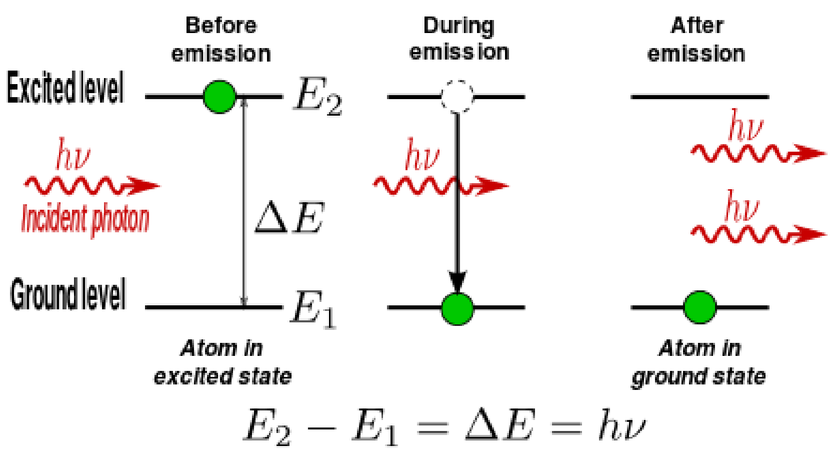
X5. Lasers. We continue with the theme of two reflecting mirrors, but now centimeters apart and even more. Such an arrangement is called an **optical cavity** and is crucial in the laser. The reflecting mirrors allow the light to reflect back and forth building up to an intense beam through stimulated emission, which will be described below. We need to include a description of light as particles: quantum optics. The quanta of light are called **photons**, a name that appeared in a 1926 publication by the physical chemist Gilbert Lewis.

Albert Einstein (1879 – 1955). Albert Einstein, just two years after generalizing his relativity theory, explained that it is possible to stimulate atoms to emit light in step with one another (1917). Such light is called **coherent light**.

Imagine exciting lots of atoms to the same higher-energy state, where they remain briefly in a metastable state. Einstein claimed that sending in a photon with just the right energy can trigger an avalanche effect. All the atoms relax at virtually the same time, giving off a concentrated beam of coherent light.

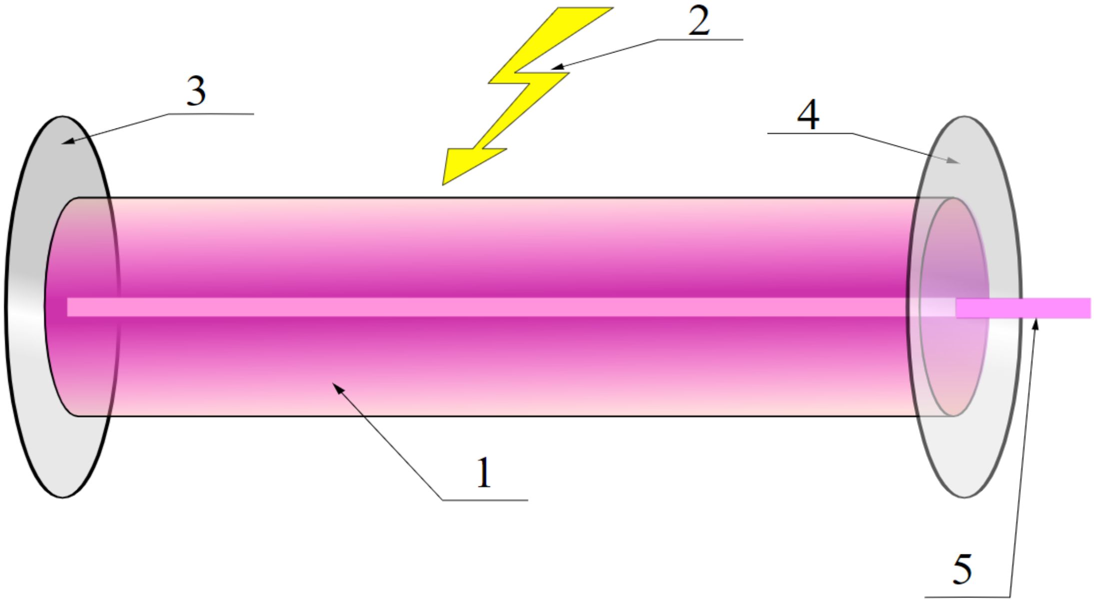
The process is called **Light Amplification by Stimulated Emission of Radiation**, or just **LASER**. If you keep pumping the electrons up to the higher energy levels and use light to trigger the transitions of the electrons back down to the lower level, you have a continuous stream of amplified coherent light, i.e., laser light. Today laser light can be achieved with merely a battery-driven penlight pointer! But the first laser was not invented until more than 40 years after Einstein's theoretical paper on the laser.

The figure below show a photon coming in and stimulating the drop of an electron from a higher energy to a lower one. The new photo joins the initial photon in step with each other, i.e., in phase. So we need both light's particle properties and wave properties to get the total picture.



Wikipedia: V1adis1av. [Creative Commons]

The next figure puts it all together with a lasing material between two parallel reflectors.



Wikipedia: Tatoute. [Creative Commons]

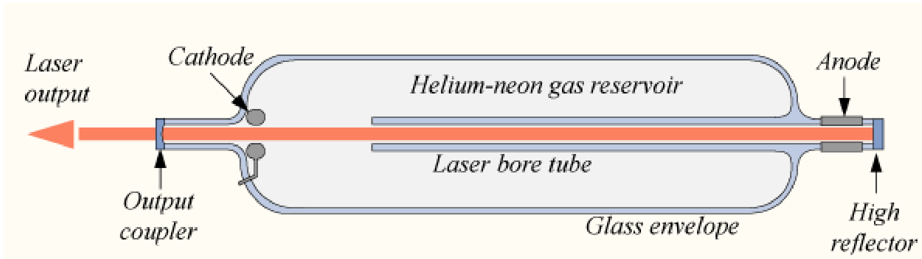
1. Lasing material. The light will amplify due to the stimulated emission.
2. Pumping. Energy is supplied to raise electrons to the higher metastable level.
3. Left Reflector. The light is reflected back.
4. Right Reflector and Output. Light is reflected back but can escape through an opening.
5. Laser Beam. The laser beam emerges through the opening into the room.

We return to a figure from our very first chapter.

Laser Light, Spring 2020. Video: https://youtu.be/Hr5MxFnioow

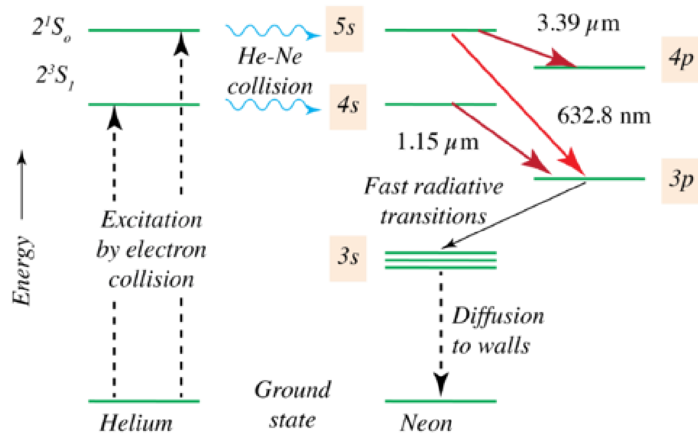
The classic helium-neon (He-Ne) laser is below.

He-Ne Laser Schematic



Wikipedia: DrBob. [Creative Commons]

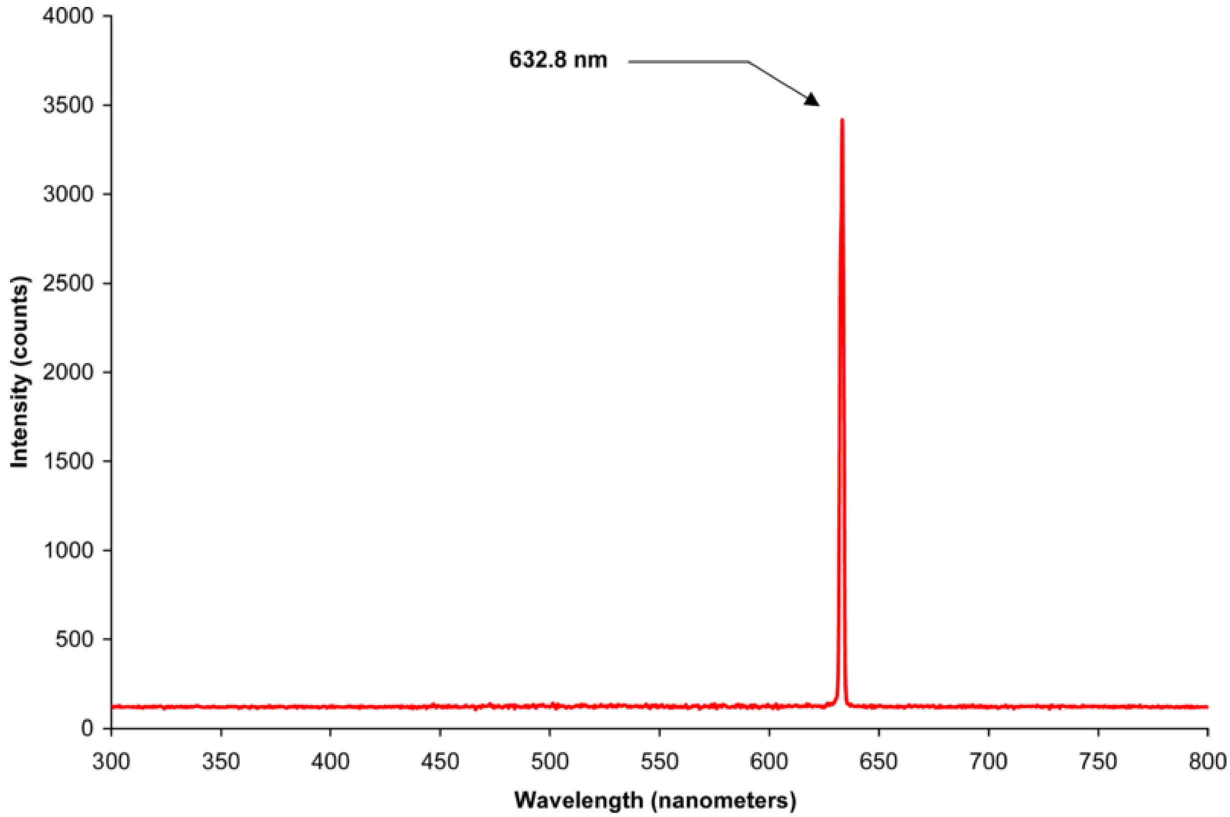
The Laser Light in the He-Ne Laser is from the 632.8 nm Neon Transition.



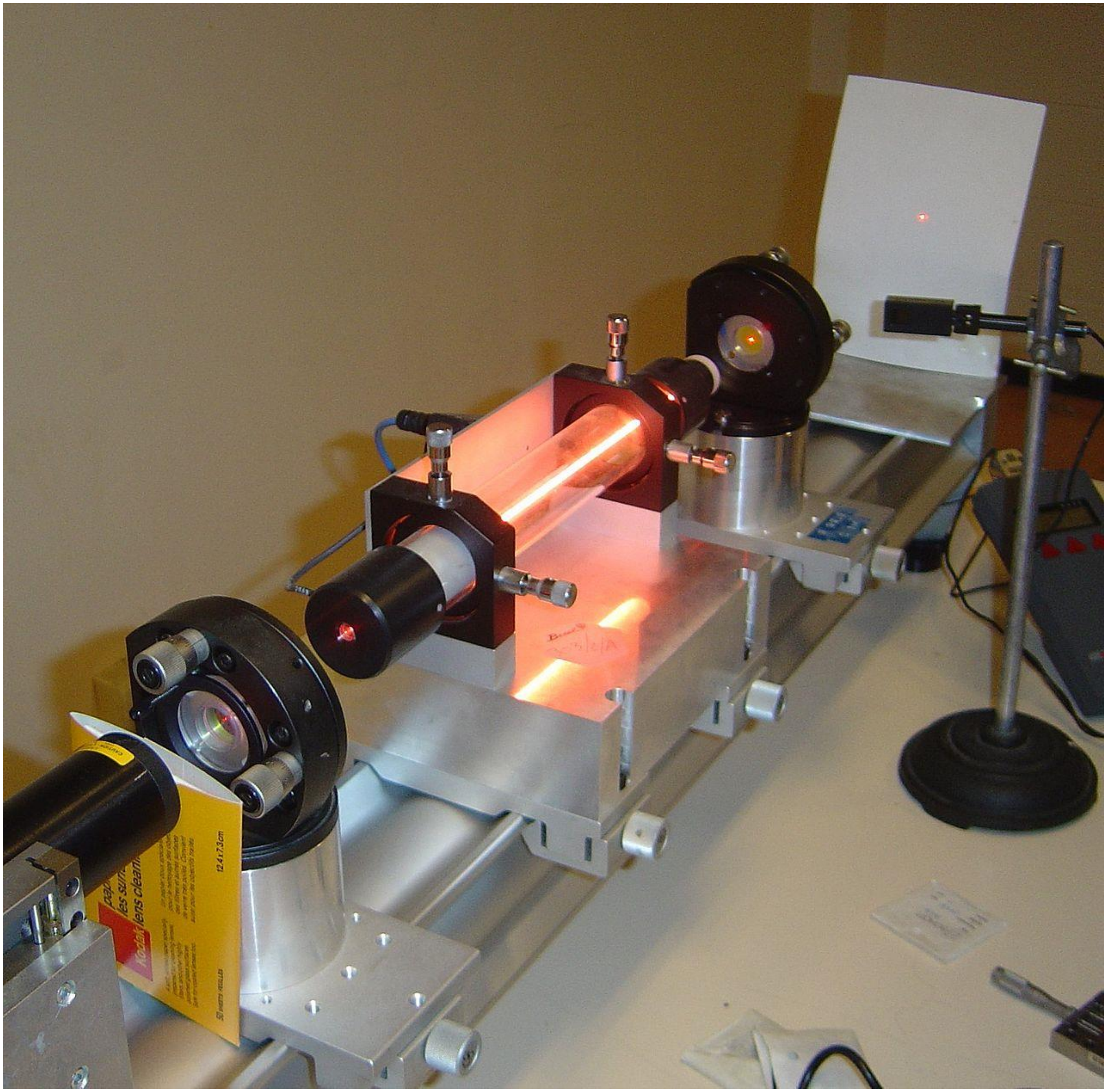
Wikipedia: XuPanda. [Creative Commons]

He-Ne Metrologic ML810 0.8 mW Lab Laser. Courtesy [BMI Surplus]

A Very Monochromatic Source of Light at 632.8 nm



Wikipedia: Deglr6328. [Creative Commons]



“A helium–neon laser demonstration. The glow running through the center of the tube is an electric discharge. This glowing plasma is the gain medium for the laser. The laser produces a tiny, intense spot on the screen to the right. The center of the spot appears white because the image is overexposed there.” Source: Wikipedia from

en:Kastler-Brossel Laboratory at en: Paris VI: Pierre et Marie Curie

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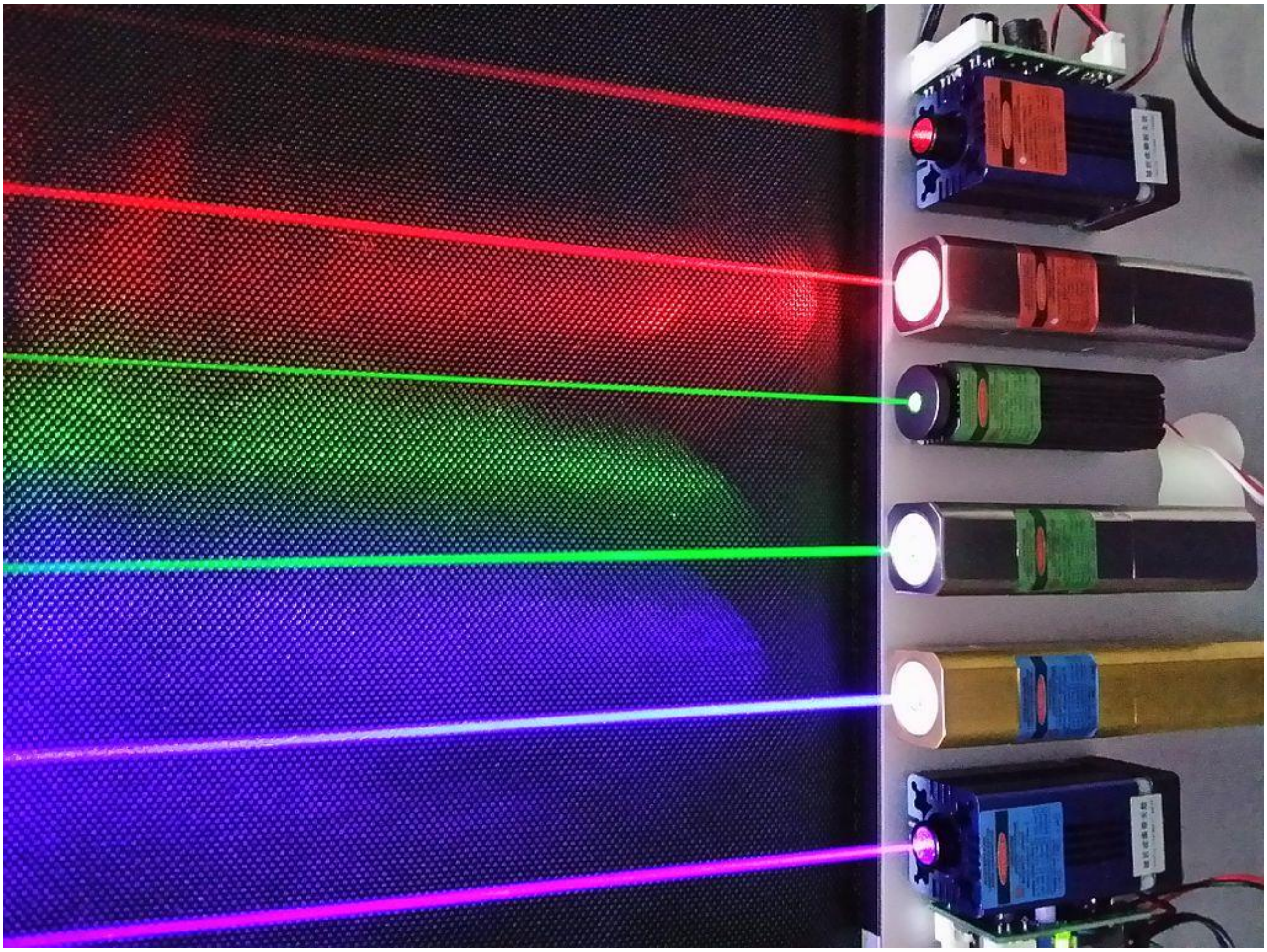
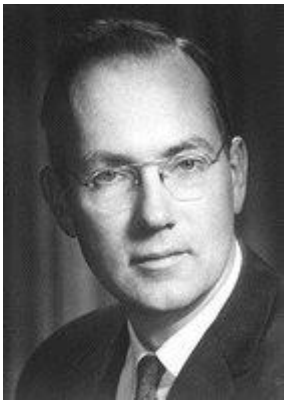


Courtesy Wikipedia: Author 彭家杰. Creative Commons

"English: Six commercial lasers in operation, showing the range of different colored light beams that can be produced, from red to violet. From the top, the wavelengths of light are: 660nm, 635nm, 532nm, 520nm, 445nm, and 405nm. Manufactured by Q-line." Wikipedia

660 nm	Red
635 nm	Red
532 nm	Green
520 nm	Green
445 nm	Blue
405 nm	Violet



Charles H. Townes (1915 – 2015)
Courtesy the Nobel Committee

Born in Greenville, South Carolina

B.S. in Physics and B.A. in Modern Languages from Furman (1935), SC
M.A. in Physics, Duke University (1937), Durham, NC
Ph.D. in Physics, Caltech (1939)

Nobel Prize in Physics (1964)
with Nikolay Basov and Alexander Prokhorov

Charles Hard Townes (1915-2015, living to 99 and a half) one of the inventors of the laser came to visit his sister in Asheville, North Carolina in 1995 at Given's Estates, a retirement community. His sister was kind enough to write your instructor inviting him to these talks, one at the retirement home on a Saturday evening and a second at the First Baptist Church in Asheville the following Sunday morning.

Townes ironically avoided lasers and spoke about astronomy instead; and then on Sunday, the theme was science and religion. However, someone in the audience asked him about his laser invention after the Sunday morning talk. Townes, who shared the Nobel Prize in 1964 for his work on the laser, readily responded. He held up his arm and stated that his suit was black because light was absorbed by it. Then he said the laser is like shining light on the suit with the result of the suit getting brighter and brighter. Of course you need to pump in energy as you can never get energy for free in nature.

Thus he explained Einstein's idea of stimulated emission and the resulting effect of amplification. Townes was born in Greenville, South Carolina and graduated from Furman University in 1935. Two years later he received his masters degree from Duke. Then, in just two more years he was awarded his Ph.D. from Caltech. He invented a device producing stimulated emission in the microwave region of the spectrum: Microwave Amplification by Stimulated Emission of Radiation, i.e., the MASER in 1951.

Later in the 1958 Townes and colleague Arthur L. Schawlow (1921-1999) wrote their famous theoretical paper pushing the stimulated-emission principle into the infrared and visible. Schawlow shared the 1981 Nobel Prize for his work in laser spectroscopy.

Shortly afterward the first laser was built by physicist Theodore Harold Maiman (1927-2007) in 1960. Maiman used a flash quartz tube to send energy to a ruby crystal, the lasing material. Chromium atoms in the ruby crystal reached the excited energy state from which the laser light was generated. A mirror on each end reflected the laser light back and forth to keep the stimulation going. One mirror was made as a half-silvered mirror to allow some of the laser light to escape.