Modern Optics, Prof. Ruiz, UNCA **SOLUTIONS** **Fall 2020**

FINAL Exam OPEN Book, OPEN Notes, OPEN Everything Except People.
Timed Exam: 2.5 Hours Plus 1.5 Hour Buffer (includes Preparing the pdf) = 4 Hours.

All Work Must be Shown Clearly for Total Credit. Sig Fig stands for Significant Figures.

[20] P1. An object is 90 cm to the left of a converging lens having focal length f_1 = 60 cm. To the right of the converging lens is a diverging lens with focal length f_2 = -30 cm and the two lenses are a distance d = 90 cm apart. Find the following.

(a) the location of the final image as measured from the first lens to the nearest cm (10 pts),
(b) the magnification of the final image as compared to the original object (6 pts),
(c) the orientation of the image, i.e., inverted or upright (2 pts),
(d) the image type of the final image, i.e., real or virtual (2 pts).

SOLUTIONS

(a) $$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \qquad \frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \qquad s_{o2} = d - s_{i1}$$

$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \quad \Rightarrow \quad \frac{1}{60} = \frac{1}{90} + \frac{1}{s_{i1}}$$ => calculator => $$s_{i1} = 180 \text{ cm}$$

$$s_{o2} = d - s_{i1} = 90 - 180 = -90 \text{ cm}$$

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \quad \Rightarrow \quad \frac{1}{-30} = \frac{1}{-90} + \frac{1}{s_{i2}}$$ => calculator => $$s_{i2} = -45 \text{ cm}$$

The image is located 45 cm to the right of the first lens.

Note that the image is midday between the two lenses.

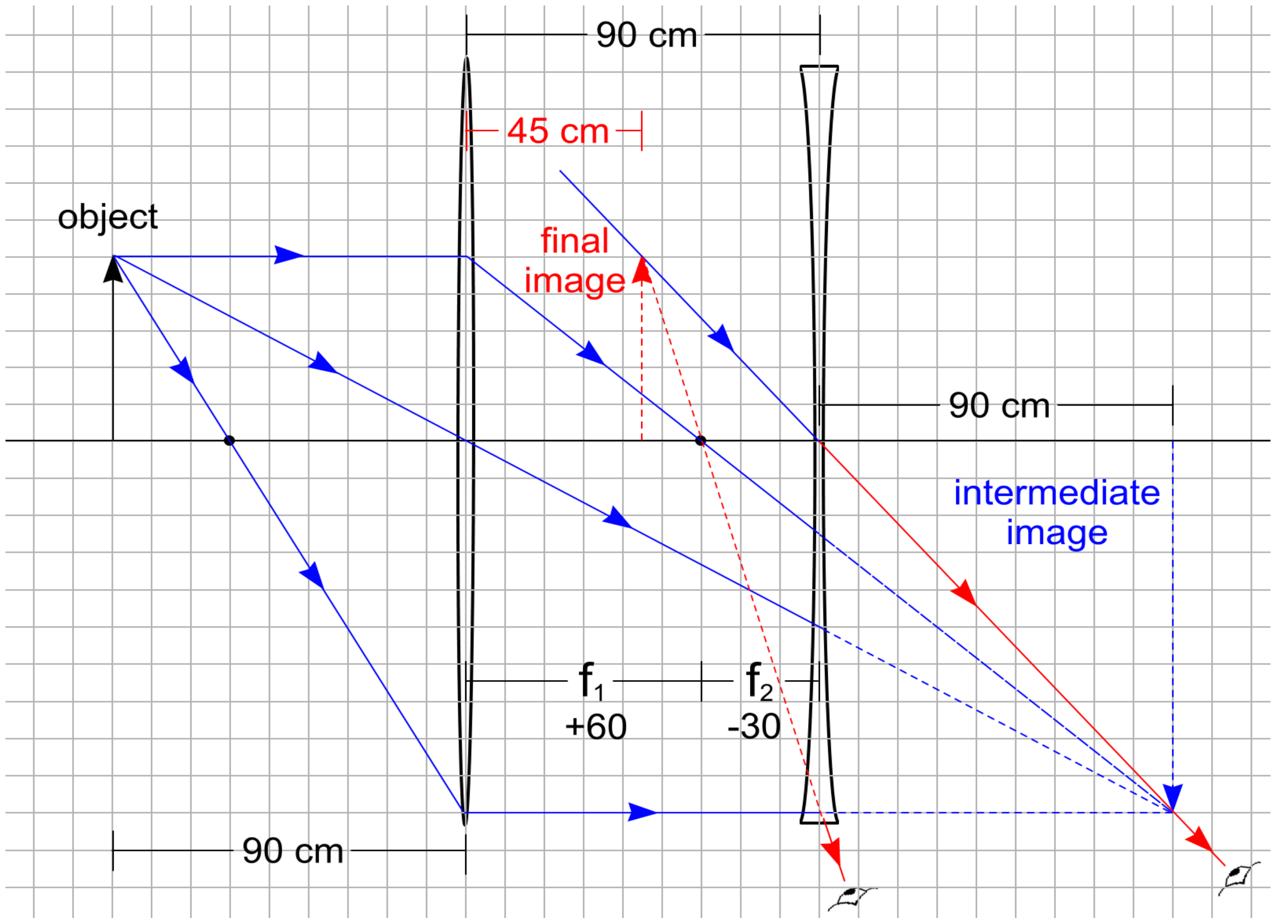
(b) $$M = m_1 m_2 = \left[-\frac{s_{i1}}{s_{o1}}\right]\left[-\frac{s_{i2}}{s_{o2}}\right] = \left[-\frac{180}{90}\right]\left[-\frac{(-45)}{(-90)}\right] = (-2)(-\frac{1}{2}) = 1$$

(c) Orientation: Upright

(d) Final Image Type: Virtual since a diverging lenses never produces a real image.

A Diagram was Not Requested.

However, a graphical solution to scale is on the next page.



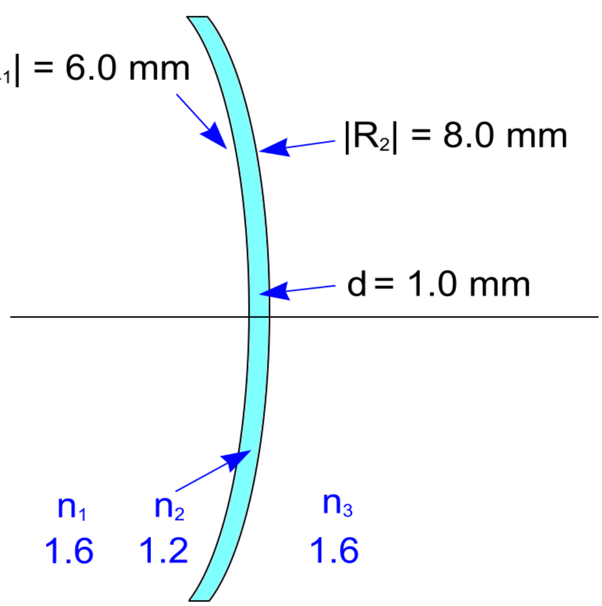
[20] P2. You will be calculating the power in diopters for the following.

(a) the power for the left surface to 4 sig figs (6 pts),

(b) the power for the right surface to 4 sig figs (6 pts),

(c) the power term that depends on the distance d = 1.0 mm to 4 sig figs (6 pts),

(d) the correct value for the total power of the system to 2 sig figs (2 pts).



SOLUTION

$$\frac{1}{f}=\frac{(n_2-n_1)}{R_1}+\frac{(n_3-n_2)}{R_2}-\frac{d}{n_2}\frac{(n_2-n_1)}{R_1}\frac{(n_3-n_2)}{R_2}$$

The above equation is from Exam 2, Problem P2 or Chapter I, Section I9.

The radii have negative curvatures: $R_1=-6.0\text{ mm}$ and $R_2=-8.0\text{ mm}$.

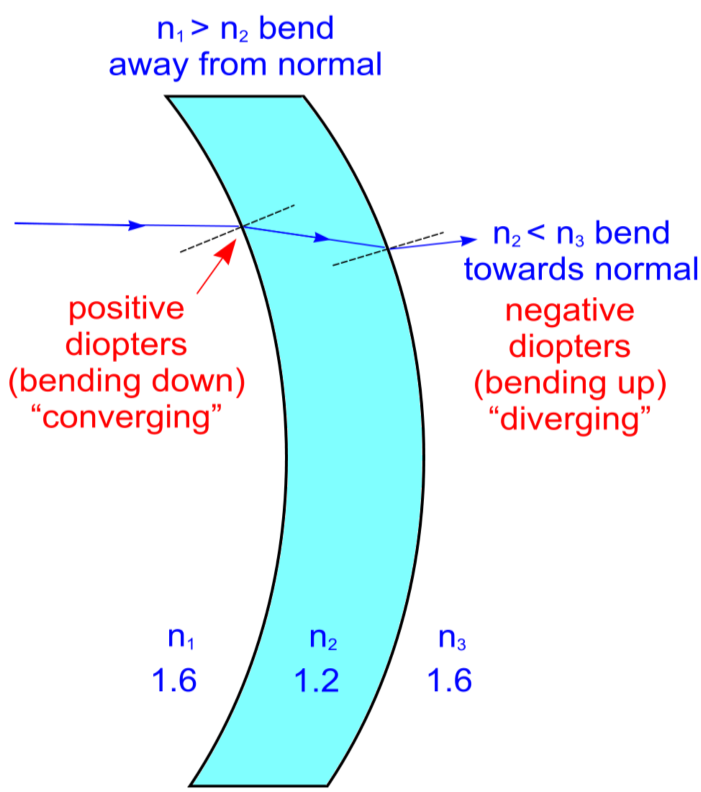
(a) $P_1=\dfrac{n_2-n_1}{R_1}=\dfrac{1.2-1.6}{-0.006\text{ m}}=66.67\text{ D}$

(b) $P_2=\dfrac{n_3-n_2}{R_2}=\dfrac{1.6-1.2}{-0.008\text{ m}}=-50.00\text{ D}$

(c) $-\dfrac{d}{n_2}P_1P_2=-\dfrac{0.001}{1.2}(66.67)(-50.00)=2.778\text{ D}$

(d) $P=P_1+P_1-\dfrac{d}{n_2}P_1P_2=66.67-50.00+2.778=19\text{ D}$

OPTIONAL VERIFICATION OF SIGNS FOR THE CURVED SURFACES



[20] P3. The Fresnel reflection coefficient is $r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$. Find θ_1 to the nearest degree for the specific case where $n_1 = 1$, $n_2 = 2$, and $r_s = -\frac{1}{2}$. Note that the minus sign means there is a phase change on reflection, but this need not concern you. Just use the data as it is given to you with the minus sign for r_s and proceed with the math.

SOLUTION

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{\cos\theta_1 - 2\cos\theta_2}{\cos\theta_1 + 2\cos\theta_2} = -\frac{1}{2}$$

$$2\cos\theta_1 - 4\cos\theta_2 = -\cos\theta_1 - 2\cos\theta_2$$

$$3\cos\theta_1 = 2\cos\theta_2$$

Snell's Law: $\sin\theta_1 = 2\sin\theta_2$

$3\cos\theta_1 = 2\cos\theta_2$ => $9\cos^2\theta_1 = 4\cos^2\theta_2$

$\sin\theta_1 = 2\sin\theta_2$ => $\sin^2\theta_1 = 4\sin^2\theta_2$

Add the equations: $9\cos^2\theta_1 + \sin^2\theta_1 = 4(\cos^2\theta_2 + \sin^2\theta_2)$

$$9\cos^2\theta_1 + \sin^2\theta_1 = 4$$

$$9\cos^2\theta_1 + 1 - \cos^2\theta_1 = 4$$

$$8\cos^2\theta_1 + 1 = 4$$

$8\cos^2\theta_1 = 3$ => $\cos^2\theta_1 = \frac{3}{8}$ => $\cos\theta_1 = \sqrt{\frac{3}{8}}$

$$\boxed{\theta_1 = 52^\circ}$$

[15] P4. Laser light with $\lambda = 632.8 \text{ nm}$ is sent through a small single slit of width b. A screen is 1 meter beyond the slit. The angle of a point on the screen in measured from the center of the slit to the screen from the usual optic axis. What is the width of the slit if the angle between the first minimum to the left of the central maximum to the first minimum on the right side of the central maximum is 1°? Report your answer to the nearest micron.

SOLUTION

$$I(\theta) = I(0)\frac{\sin^2\beta}{\beta^2} \qquad \beta = \frac{1}{2}kb\sin\theta \quad \Rightarrow \quad \beta = \frac{\pi}{\lambda}b\sin\theta$$

For the first minima on one side $\beta = \pi$ and the angle will be one half of the 1° given.

Therefore, $\theta = 0.5°$.

$$\frac{\pi}{\lambda}b\sin\theta = \pi \quad \Rightarrow \quad b = \frac{\lambda}{\sin\theta} \quad \Rightarrow \quad b = \frac{632.8 \text{ nm}}{\sin 0.5°} = 72{,}514 \text{ nm}$$

$$\boxed{b = 73\ \mu m}$$

[15] P5. Calculate $A = \int_{-2}^{\infty} e^{i\pi u^2/2} du$ and the irradiance I

SOLUTION

$$A = \int_{-2}^{\infty} e^{i\pi u^2/2} du = \left[C(u) + iS(u)\right]_{-2}^{\infty}$$

$$A = \left[C(\infty) + iS(\infty)\right] - \left[C(-2) + iS(-2)\right]$$

$$A = \left[C(\infty) - C(-2)\right] + i\left[S(\infty) - S(-2)\right]$$

From the Fresnel Integral Tables in Chapter W, page 6.

$$A = \left[0.5000 - (-0.4883)\right] + i\left[0.5000 - (-0.3434)\right]$$

$$A = \left[0.9883\right] + i\left[0.8434\right]$$

$$\boxed{A = 0.9883 + 0.8434i}$$

$$I = \frac{1}{2}|A|^2 = \frac{1}{2}(0.9883 + 0.8434i)(0.9883 + 0.8434i)^*$$

$$I = \frac{1}{2}(0.9883 + 0.8434i)(0.9883 - 0.8434i)$$

$$I = \frac{1}{2}(0.9883^2 + 0.8434^2)$$

$$\boxed{I = 0.84}$$

[10] P6. Find the photon wavelength in nm to the nearest nm that is emitted in the hydrogen Bohr atom as an electron makes a transition from the 7th orbit to the 1st orbit.

SOLUTION

From Chapter W, page 20. $\frac{1}{\lambda} = R_H\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$ $\quad R_H = \frac{4}{364.5 \text{ nm}}$

$\frac{1}{\lambda} = \frac{4}{364.5 \text{ nm}}\left[\frac{1}{1^2} - \frac{1}{7^2}\right]$ => Calculator => $\boxed{\lambda = 93 \text{ nm}}$

Have a Nice Break!

Squirrel at Grace Episcopal Pumpkin Patch (2010).