# H. Pipes

We proceed now to the study of standing waves in pipes. The standing waves in the pipe are actually sound waves. We cannot see sound waves in air. However, we can readily hear the tones. The advantage of our earlier experimentation with ropes (or springs) is that we can see the standing waves. The disadvantage is that we cannot hear the rope waves. With pipes, we can hear the waves but not see them. After studying both strings and pipes, you will have an excellent understanding of standing waves.

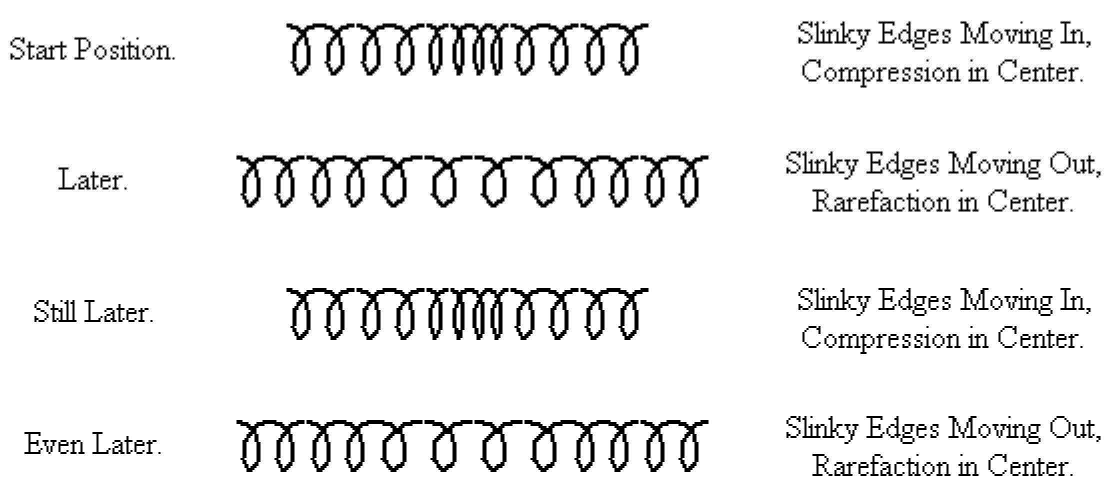
## Open Pipes

The first pipes we will consider are open at each end. These are called *open pipes*. You can look through them. They are cylinders. The air inside vibrates as longitudinal standing waves. We can use the slinky as a model to help us visualize what the air does. We replace the open pipe by a slinky. Note that the ends of the slinky are free. This corresponds to the open ends of the pipe where the air can vibrate freely at the ends. If you are worried about gravity, imagine the slinky in outer space inside the space shuttle.

Fig. H-1 illustrates a slinky with the simplest type of oscillation. The edges of the slinky move in and squeeze the center region, then move out and stretch the middle region. This motion repeats. The movement indicated in Fig. H-1 describes the fundamental standing-wave pattern for a longitudinal wave. Try to think of a simpler vibrational mode for the slinky. You will not be able to. Fig. H-1 then depicts the fundamental for the slinky. **We will see shortly that one-half wavelength is depicted in Fig. H-1.**

The layers of air in an open pipe move in a similar fashion. Due to the difficulty in sketching and analyzing longitudinal waves, we will once again draw analogies with transverse waves.

Fig. H-1. Simplest Standing Wave on a Slinky.

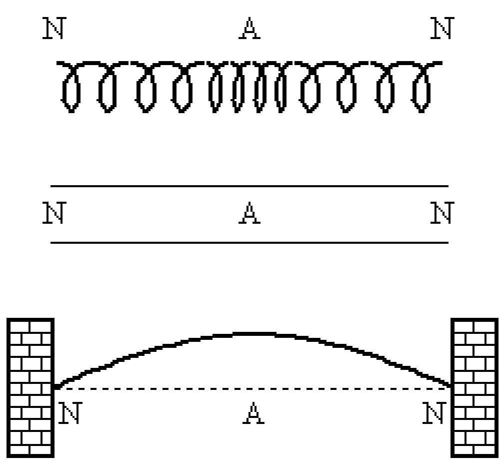


The center of the slinky goes through compressions and rarefactions. These are fluctuations in pressure. Imagine being the slinky ringlet in the center. You get squeezed, then stretched. Here, the maximum changes take place. We have designated the points where the greatest changes from equilibrium occur as antinodes.

See Fig. H-2 for the antinode in the slinky. The edges of the slinky are never stretched or compressed. Therefore, they are nodes. The center diagram in Fig. H-2 represents a pipe, with longitudinal waves in air.

Fig. H-2. Nodes for Slinky, Pipe, and Rope.




The air at the center of the pipe goes through high and low pressures as compressions and rarefactions occupy the center. The air at the edges is free to move and stay at equilibrium pressure. The air layers at the edges never get compressed or rarefied. You can remember this by observing that the air near the edges is in direct contact with the ambient air in the room, which air is at equilibrium pressure.

Similarly, in our example with the rope (lowest diagram in Fig. H-2), it is the center that experiences extremes. At one time, the rope is very high in the center (a crest), later it is at the lowest extreme (a trough). The ends of the rope remain at equilibrium, so the ends are nodes.

Note the excellent correspondence among all three systems in Fig. H-2. We will find the similarities between the pipe and the string most meaningful. As the string or rope wave swings from crest to trough, the pipe wave changes in the central region from one of compression to rarefaction.

There is a one-to-one correspondence for the node and antinode regions. We will be able to use this fact to determine the series of standing-wave patterns for the open pipe.

Our plan is to revisit the string and to further develop the analogies found in Fig. H-2. In this way we will be guided by the string. Due to the correspondences between strings and open pipes, we expect that the open pipe will have the same harmonic series as the string. The ends of the pipe replace the ends of the string. We will copy the node-antinode structure from the string vibrations over to the open pipe for each harmonic in order to find the nodes and antinodes along the pipe.

Fig. H-3 below shows the application of string oscillations to determine the standing waves in an open pipe. The analogy indicates that the same harmonics will be supported on the open pipe. If the lengths of the string and open pipe are the same, then the spacing of the nodes and antinodes will be the same. Such spacing determines the wavelength in each case. Therefore, the wavelengths will be the same. The frequencies will differ since the speeds of the waves are not the same on the string and in the pipe.

But we never committed ourselves to a specific fundamental frequency with strings. We simply called the fundamental frequency "f." Therefore, everything applies here.

The waves on the string are string waves. These transverse string waves shake the air surrounding them and produce sound waves in air. On the other hand, the waves in the pipe are already sound waves, waves vibrating in the air inside the open pipe. Note that the distance between a node and antinode is a quarter-wave. Recall that for the first harmonic, we have one half-wave. See the labeling "N-A-N" for this case in Fig. H-3. Note that from "N" to "A" and from "A" to "N" are quarter-waves.

Fig. H-3. Using the String to Determine the Standing Waves for the Open Pipe.

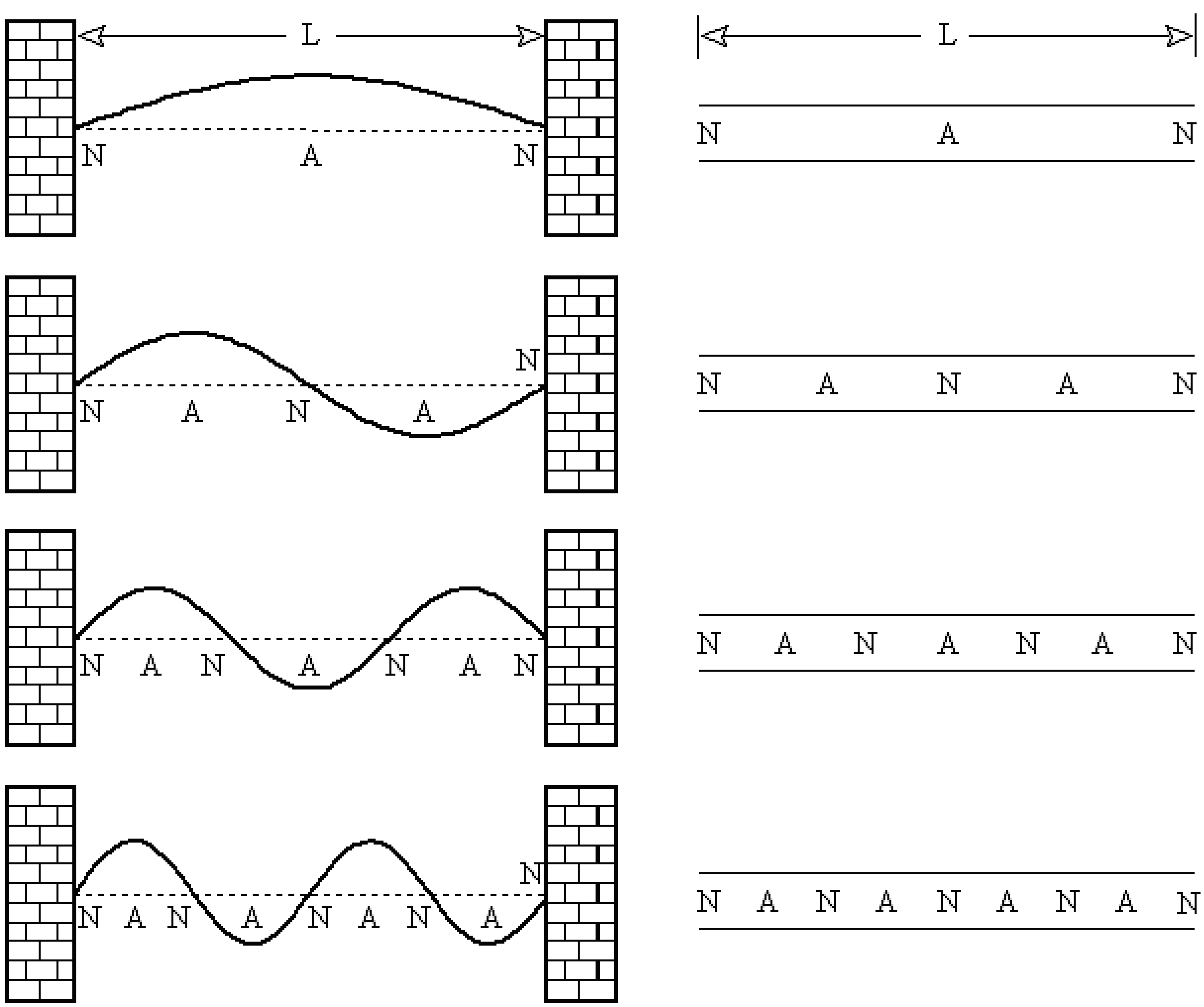


Table H-1 reproduces the first eight harmonics since these also apply to open pipes. The fundamental for the open pipe is determined by the length of the pipe and the speed of sound in air, the medium supporting the standing waves. Whatever this frequency happens to be, we call it "f." Then, the second harmonic (or first overtone) has frequency 2f, the third harmonic (second overtone) has frequency 3f, and so on.

The fundamental for the string is determined by the length of the string (Mersenne's First Law) and the speed of the waves on the string. The wave speed on the string is in turn dependent on the tension in the string (Mersenne's Second Law) and the heaviness (mass) of the string (Mersenne's Third Law).

The fundamental for the open pipe is determined by the length of the pipe, as has been noted above. We can say that the speed of sound inside the pipe is likewise determined by the medium inside. We assume it is air, but it can be some other gas. The speed of sound in the gas is in turn dependent on medium properties such as the temperature, pressure, and the density of the gas. However, to some extent, these properties are dependent on each other.

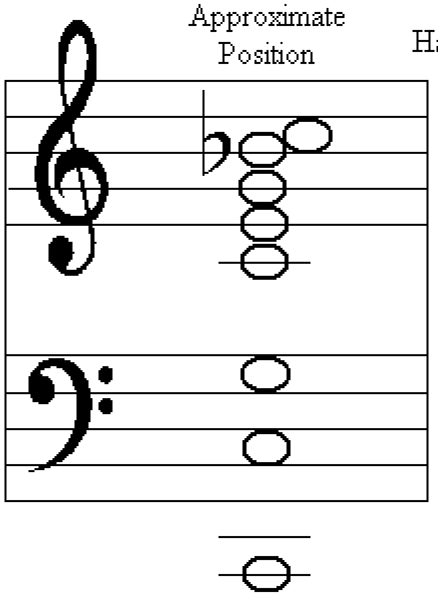
Table H-1. The First Eight Harmonics (Standing Waves on Strings and in Open Pipes).

| Harmonic Number | Wave-Length | Frequency | Interval from First Harmonic |
|---|---|---|---|
| 1 | 2L | f | Unison |
| 2 | 2L / 2 | 2f | Octave |
| 3 | 2L / 3 | 3f | Octave + Fifth |
| 4 | 2L / 4 | 4f | Two Octaves |
| 5 | 2L / 5 | 5f | Two Octaves + Third |
| 6 | 2L / 6 | 6f | Two Octaves + Fifth |
| 7 | 2L / 7 | 7f | (Two Octaves + Minor Seventh) |
| 8 | 2L / 8 | 8f | Three Octaves |

Fig. H-4 lists the first eight harmonics again. We encountered this figure in our study of strings. The significance of harmonics is even more profound now that we find them generated by open pipes as well as strings. The two basic methods of producing sound in nature (strings and pipes) give us the harmonic series. This set of harmonic tones is nature's scale or, what we call in this text, the "physicist's scale."

Fig. H-4. The First Eight Harmonics (Generated by Strings and Open Pipes).

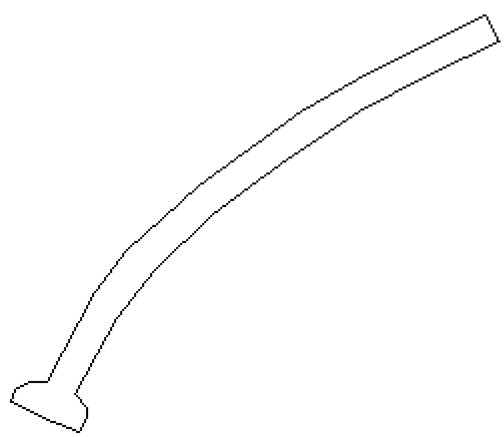



| Harmonic | Frequency | Interval (from H1) |
|---|---|---|
| H8 | 8f | Three Octaves |
| H7 | 7f | (Two Octaves + Minor Seventh) |
| H6 | 6f | Two Octaves + Fifth |
| H5 | 5f | Two Octaves + Third |
| H4 | 4f | Two Octaves |
| H3 | 3f | Octave + Fifth |
| H2 | 2f | Octave |
| H1 | f | Unison |

Notes are approximate since modern tuning does not employ perfect frequency ratios.

A convenient way to demonstrate overtones is to use an inexpensive toy, the *Twirl-a-Tune* or *Whirl-a-Tune* (see Fig. H-5). The Twirl-A-Tune is a corrugated plastic tube with a handle at one end. It produces overtones when whirled around. Air rushes up the tube when twirled. The rush of air against the ridges and valleys of the tube excites the tube into vibrating at special frequencies - the overtone series for the tube. The faster you twirl, the higher the overtones you obtain. The toy is too excitable to get the fundamental. It readily produces the second harmonic or first overtone.

Fig. H-5. *Twirl-a-Tune*.



Up until this point we have considered nodes and anitnodes for pipes in terms of pressure. A pressure node is a place where the pressure remains at the equilibrium pressure. A pressure antinode occurs when the point goes through periods of increased pressure (*compressions*) and times of lowered pressure (*rarefactions*).

Note that the ends of an open pipe are always pressure nodes since the air is free to move and maintain equilibrium. It is never allowed to compress. This is analogous to the nodes at the ends of a string, where the string remains at equilibrium. But note an important distinction. The rope ends are fixed, not allowed to move, while the air layers at the ends of an open pipe do move. This presents no problem because equilibrium is maintained in each case.

However, we like to define another kind of node-antinode for pipes, one that is defined in terms of displacement (motion) instead of pressure equilibrium. If you think in terms of displacement or movement, then a node is a place that doesn't move.

An antinode is a place that undergoes the maximum motion. The nodes and antinodes on a string are of the displacement type. A string node means the string doesn't move there. But with pipes, both pressure and displacement descriptions apply. So we have both kinds of nodes. However we must be careful.

At a pressure node, like at the ends of a pipe, the air layers are free to move and they do, to maintain equilibrium pressure. Because the end layers of air move, these regions are displacement antinodes. On the other hand, at the places where compressions and rarefactions occur, the central air layer does not move. Recall the slinky ringlet that gets pushed on from both sides and then stretched equally from both sides. It doesn't move as it gets squeezed and stretched. Therefore, this ringlet where a pressure antinode occurs, is the place where we have a displacement node.

Remember it this way. A pressure node is a displacement antinode and a pressure antinode is a displacement node. They are opposite of each other. This feature is illustrated in Fig. H-6 below. Note the shorthand notation for displacement nodes and antinodes. A displacement node is a vertical line, indicating no motion. The dashes indicate motion, displacement antinodes. When we draw these near the ends, we extend them slightly since motion overshoots the edges (think of a slinky).

Fig. H-6. Types of Nodes and Antinodes.

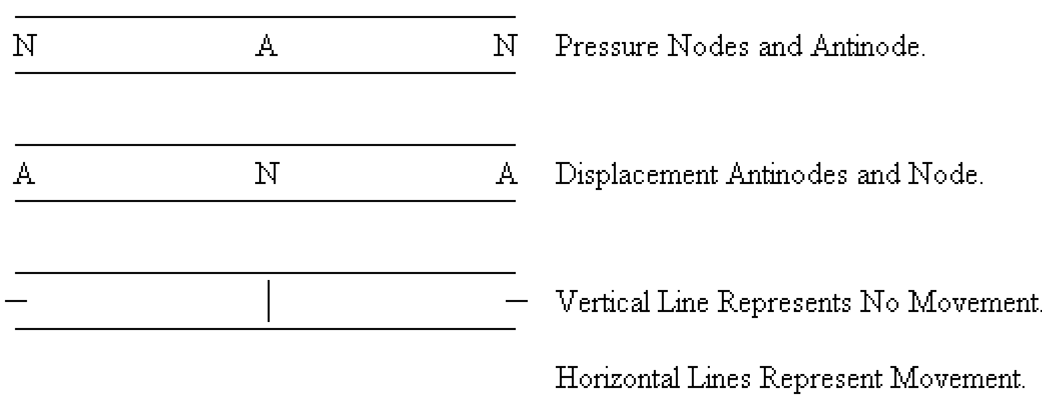


Fig. H-7 illustrates the first four standing waves in an open pipe, where displacement nodes and antinodes are employed instead of pressure nodes and antinodes. The results are opposite the pressure description. Wherever there was a node before, now there's an antinode and vice versa.

The shorthand notation for displacement nodes is included in the diagrams at the right in Fig. H-7. There is a quick way to remember these. Always sketch a dash "-" at each end. You need the number of vertical lines "|" that corresponds to the harmonic number, e.g., the 4th harmonic has 4 vertical lines. Remember you never have two nodes in a row; there is always an antinode in between. So you alternate these.

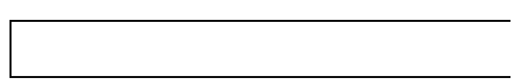
The only problem is that it's a little hard to space them evenly the first time you try it. It helps if you remember one more thing. For odd harmonics, there is a vertical line in the center; for even, there is a dash in the center. The shorthand notation for displacement nodes will be very helpful when we take up the study of closed pipes next.

Fig. H-7. The First Four Standing Waves in an Open Pipe Defined by Displacement.


## Closed Pipes

A *closed pipe* is a pipe closed at one end. See Fig. H-8 below. We see immediately that the standing-wave patterns will be different. The closed end forces a displacement node there, a place where there is no motion because "you are up against the wall." The other end is open, free, a displacement antinode. Sketch a little dash at this end. You have the first standing wave!

Fig. H-8. Closed Pipe.



We will use an important observation we made earlier concerning quarter-waves. The fundamental for the closed pipe is compared to that of the open pipe in Fig. H-9 below. For the open pipe we find one half-wave fitted to the pipe length L. The half-wave is spanned by going from an antinode to node and then from a node to an antinode (see open pipe in Fig. H-9). Going from the antinode to node takes you one half the distance across the half-wave. Therefore, an antinode to a node gives us a quarter-wave. Likewise, going from the center node to an antinode is also a quarter-wave. We state these important observations below.

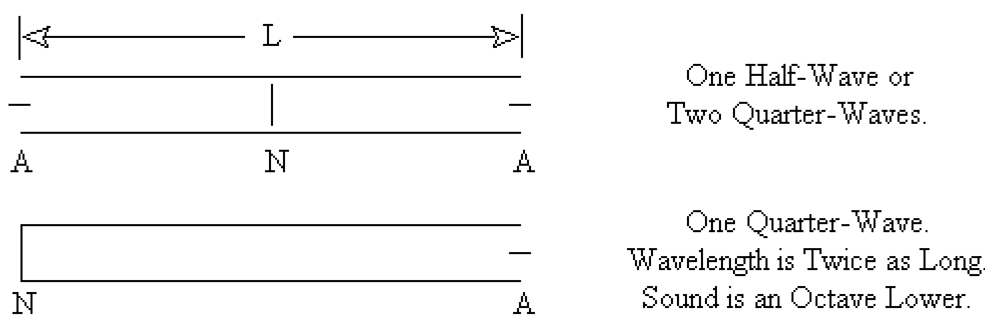
Between a node and antinode is a quarter-wave (quarter of the wavelength).

Between an antinode and node is a quarter-wave (quarter of the wavelength).

Now look at the closed pipe in Fig. H-9. We surely have a displacement node at the closed end. There is no motion at the wall. We also have the usual displacement antinode at the open end. But a node is always followed by an antinode and vice versa. Therefore, the standing wave in Fig. H-9 is the simplest standing wave for a closed pipe.

Imagine a slinky with the left end glued to a wall and the right end free to move. Now pull the right end of the slinky and let go. The right end oscillates in and out at the right end. The left end stays fixed to the wall. You have the slinky version of the fundamental for a closed pipe.

Fig. H-9. Fundamentals for Open and Closed Pipes (Displacement Nodes).




The single quarter-wave in the closed pipe in Fig. H-9 above is twice the length of each of the two smaller quarter-waves found in the open pipe above it. The wavelength of the fundamental for the closed pipe is therefore twice as long as the fundamental wavelength for the same-size open pipe. If we increase the wavelength, we lower the frequency. Since the wavelength is doubled for the closed pipe, the frequency is halved.

The open-pipe fundamental has twice the frequency of the closed-pipe fundamental and is therefore an octave higher. Alternatively, we can say that the fundamental frequency of the closed pipe is an octave lower than that for the open pipe.

There are two secrets in understanding closed-pipe physics. First, if you close one end of an open pipe, you double the wavelength to 4L (lower the fundamental by an octave). Second, as we will see, the closed-pipe harmonics only includes the odd harmonics! These two important characteristics for closed pipes are stated below.

1. Close one end of an open pipe and the wavelength doubles to 4L.

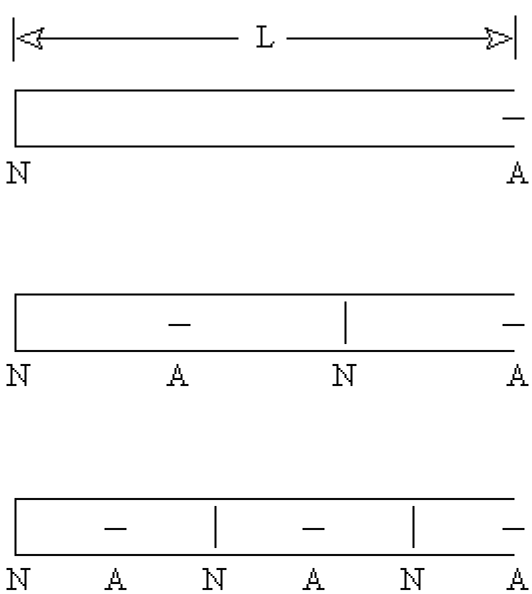
2. The harmonic series for closed pipes includes only the odd harmonics.

The second observation above becomes evident by studying Fig. H-10 below. For the first mode of vibration we have one quarter-wave fitted to L. Therefore, the wavelength is 4L. Do not worry about realizing an entire wavelength within the distance L. This only happens for H2 for the string or H2 for the open pipe.

Mode 1 for the closed pipe has one vertical line (at the wall) and one dash (at the open end). To get the next mode, we squeeze in another pair (the "A" and "N" inside the pipe in Fig. H-10). And herein lies the second secret. There are now three quarter waves. From left to right these are "N" to "A," "A" to "N," and "N" to "A." The squeezed wavelength is now 1/3 of what it was before. The frequency is triple. This is the third harmonic.

We skipped the second harmonic! The next mode adds another antinode-node pair resulting in 5 quarter-waves. Now, the wavelength is 1/5 of what we started with and the frequency is 5 times the fundamental. This gives us the fifth harmonic. We skipped the fourth harmonic. Can you work out the next case?

Fig. H-10. Standing Waves for Closed Pipe (Odd Harmonics Only).




First Mode: $(\lambda / 4) = L$

$\lambda = (4L)$ (First Harmonic, f)

Next Mode: $3(\lambda / 4) = L$

$\lambda = (4L)/3$ (Third Harmonic, 3f)

Next Mode: $5(\lambda / 4) = L$

$\lambda = (4L)/5$ (Fifth Harmonic, 5f)

The results for the first few harmonics for the closed pipe are found in Table H-2 below. Note the two basic secrets we discussed earlier. This enables us to generate Table H-2 from our previous table for the strings and open pipes. First, we note that when we close one end of an open pipe, the wavelength doubles to 4L. So we replace 2L (result for open pipe) by 4L (result for closed pipe) in our table. We always list "f" for the fundamental frequency by definition.

The fundamental is our reference frequency. So even though the closed pipe drops an octave relative to the original open pipe, we still use "f" for the fundamental. You might say we redefine what is meant by "f." This is an important convention. It is always most convenient to call the frequency of the first standing wave "f" and relate all overtones to "f."

Second, we note that the closed pipe only has odd harmonics. So we strike out all the even harmonics from our list and we are finished. We have Table H-2.

Table H-2. The First Few Harmonics for a Closed Pipe.

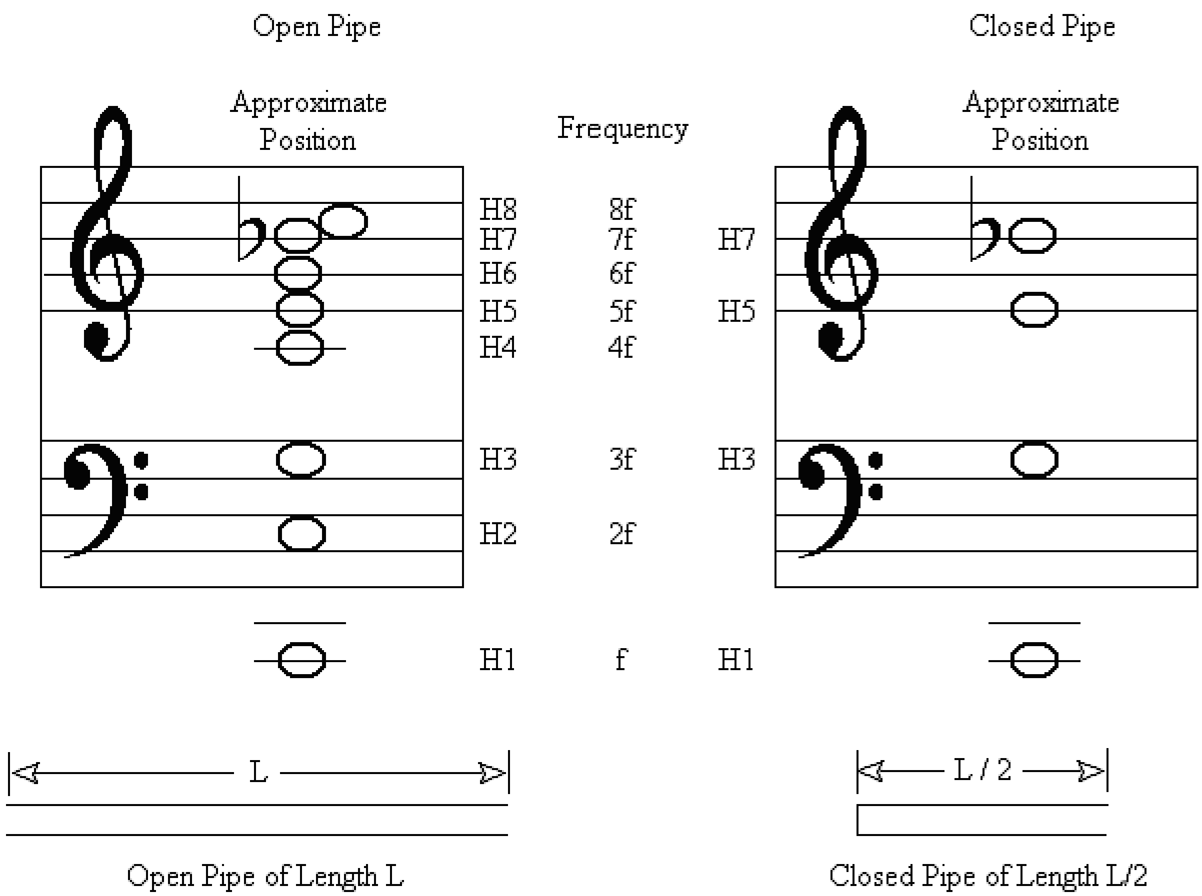
| Harmonic Number | Wave-Length | Frequency | Interval from First Harmonic |
|---|---|---|---|
| 1 | 4L | f | Unison |
| 2 | Not Present. | | |
| 3 | 4L / 3 | 3f | Octave + Fifth |
| 4 | Not Present. | | |
| 5 | 4L / 5 | 5f | Two Octaves + Third |
| 6 | Not Present. | | |
| 7 | 4L / 7 | 7f | (Two Octaves + Minor Seventh) |
| 8 | Not Present. | | |

We can find an open pipe and a closed pipe of different sizes so that each has the same fundamental. The closed pipe has to be one half as long. This offsets the drop in frequency we get by closing one end of an open pipe. Cutting a pipe in half doubles its frequency because we halve the wavelength by our cut. This compensates for the closed end. In summary, close one end of an open pipe and the frequency drops an octave. Now cut the closed pipe so that the new open end is half as far from the closed end. The closed pipe is one half its original size. This pushes the fundamental back up an octave. Fig, H-11 depicts the first few harmonics of an open pipe and a matched closed pipe.

As before, we point out that the notes on the musical staff are approximate. The scale in use today does not use perfect ratios except for the octave. Today's frequency assignment for the notes is called *equal temperament*. We will explain what this is in a later chapter. We also point out that the seventh harmonic is considerably approximate.

Fig. H-11. Harmonics for Matched Open and Closed Pipes.




Shortened length of closed pipe keeps the fundamental the same as the open pipe.

The fundamental in Fig. H-11 is about 60 Hz. This answer is based on the position of the fundamental on the musical staff. We will learn later how to determine the length of an open pipe necessary to produce a specific fundamental frequency. For now, we simply state the result. The open pipe needs to be almost 3 m. This is long. But then, this is a very low pitch. The closed pipe needs to be half this length.

--- End of Chapter H ---