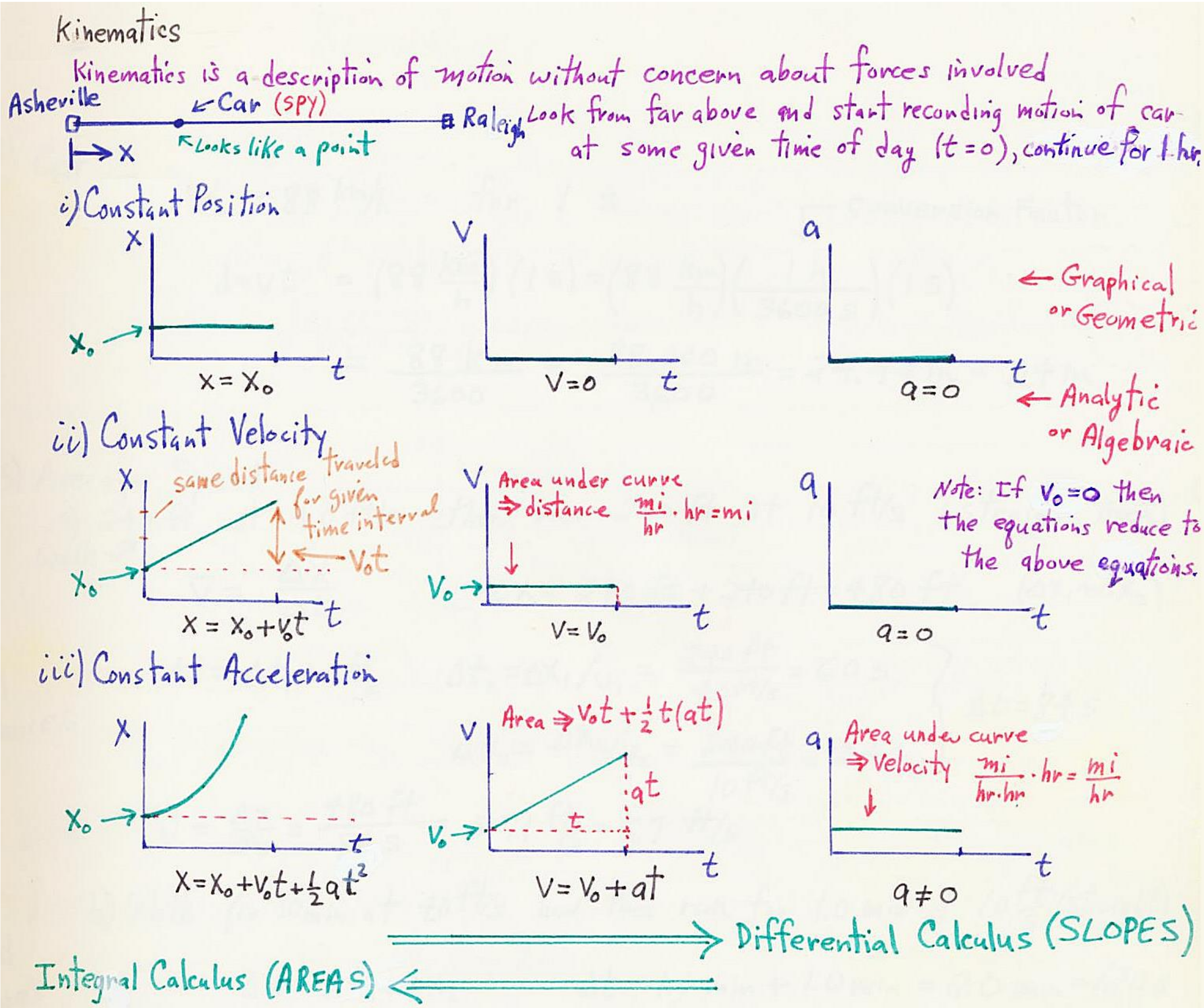
Theoretical Physics, Prof. Ruiz, UNCA Spring 2021 doctorphys.com
Chapter 0. Introductory Physics.

Intro 1. Linear Acceleration.



Moving from one graph to the next one on the right => differential calculus (slopes).
Moving from one graph to the next one on the left => integral calculus (areas).

$$x = x_o + v_o t + \frac{1}{2}at^2 \quad => \quad \frac{dx}{dt} = 0 + v_o + \frac{1}{2}a(2t) \quad => \quad v = v_o + at$$

$$x - x_o = v_o t + \frac{1}{2}at^2 \quad <= \quad \int_{x_o}^{x} \frac{dx}{dt}dt = \int_0^t v dt = \int_0^t (v_o + at)dt \quad <= \quad v = v_o + at$$

The same applies to the velocity and acceleration graphs.

Near Earth a = g = 9.8 m/s^2.

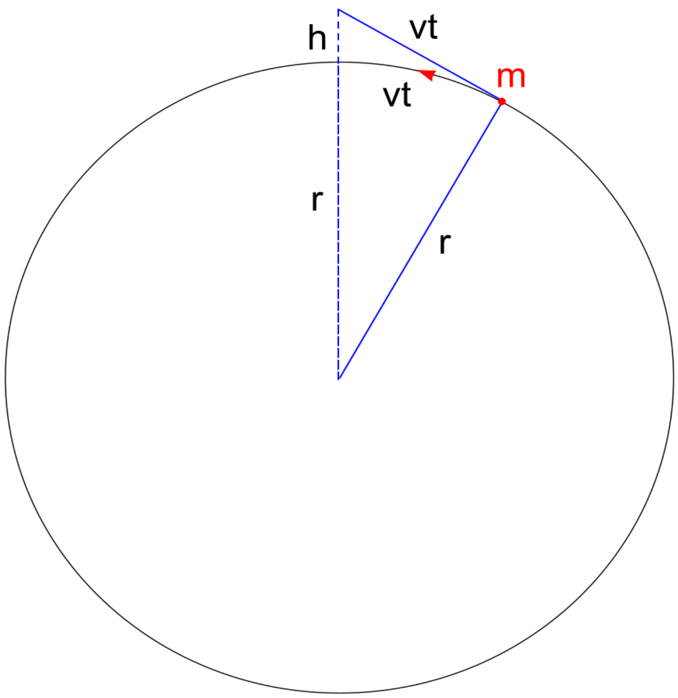
$x = x_o + v_o t + \frac{1}{2} a t^2$ (drop from rest where $d = x - x_0$) => $d = \frac{1}{2} g t^2$

Neglecting air resistance.

Time (s)	Distance (m)
0.5	1.2
1.0	4.9
1.5	11.0
2.0	19.6

Chimney Rock, North Carolina, July 20, 2014

Intro 2. Uniform Circular Motion.



A mass m is traveling along a circular path at constant speed v. The velocity though changes because the direction changes.

The mass falls a distance h during time t.

$$(r+h)^2 = r^2 + (vt)^2$$

$$r^2 + 2rh + h^2 = r^2 + v^2t^2$$

$$2rh + h^2 = v^2t^2$$

As t gets smaller and smaller,

$$h << r \, .$$

Then, $2rh = v^2t^2$ for t very short.

$$h = \frac{1}{2r}v^2t^2 \qquad h = \frac{1}{2}\frac{v^2}{r}t^2$$

From our acceleration formulas in the last section, we identify $a = \dfrac{v^2}{r}$.

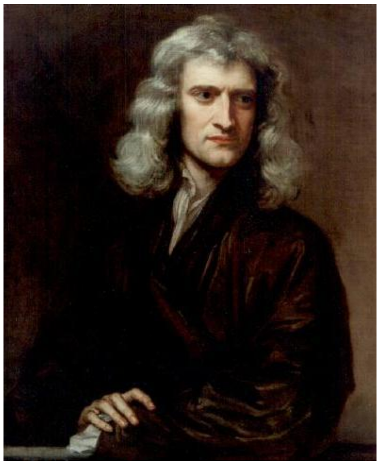
For the moon in orbit: $r = 384{,}000 \text{ km}$. The speed $v = \dfrac{2\pi r}{T}$, where $T = 27.3 \text{ days}$.

The period T used here is the one with respect to the fixed stars, the sidereal period.

$$T = 27.3 \text{ d}\left(\frac{24 \text{ h}}{1 \text{ d}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 2.3587 \times 10^6 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(384{,}000{,}000 \text{ m})}{2.3587 \times 10^6 \text{ s}^2} = 1022.9 \frac{\text{m}}{\text{s}}$$

$$a = \frac{v^2}{r} = \frac{(1022.9 \text{ m/s})^2}{3.84 \times 10^8 \text{ m}} = 2.72 \times 10^{-3} \frac{\text{m}}{\text{s}^2}$$



Intro 3. Newton's Second Law.

Sir Isaac Newton
1642-1727

Apply force F on a mass m and there is acceleration a.

$$F = ma$$

Since direction is important, one should use the vector form.

$$\vec{F} = m\vec{a}$$

Since acceleration is the change in velocity, i.e., $\vec{a} = \frac{d\vec{v}}{dt}$, we can write

$$\vec{F} = m\frac{d\vec{v}}{dt}.$$

In more general terms, when the mass is not constant we should write

$$\vec{F} = \frac{d(m\vec{v})}{dt}.$$

It is natural now to define a physical quantity called the momentum $\vec{p} = m\vec{v}$ so Newton's law can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt},$$

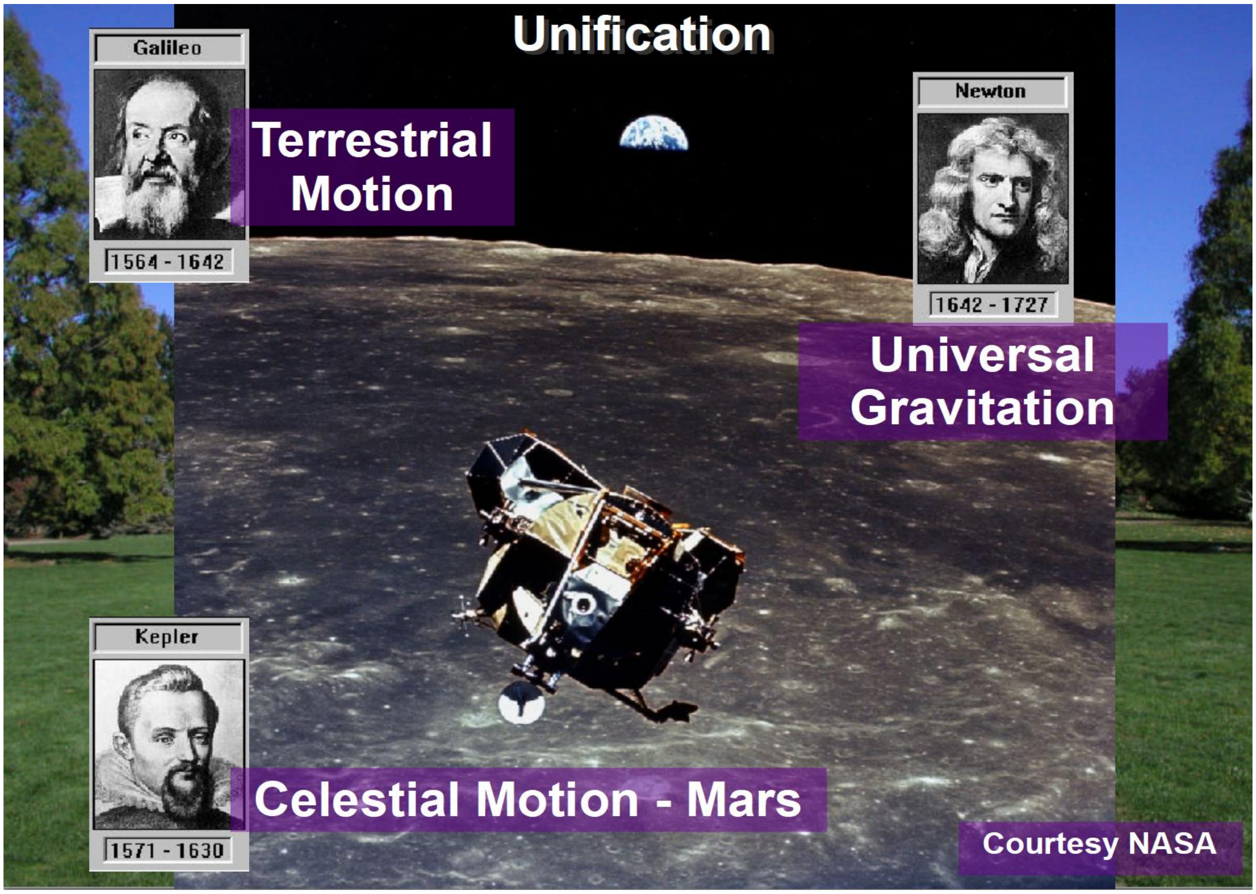
the most general form of Newton's Second Law.

Dimensional Analysis

$[F] = \text{newton} = \text{N}$ and $[ma] = [m][a] = \text{kg} \cdot \frac{\text{m}}{\text{s}^2}$ => $\text{N} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2}$

$$\frac{\text{m}}{\text{s}^2} = \frac{\text{N}}{\text{kg}}$$

Intro 4. Universal Law of Gravitation.



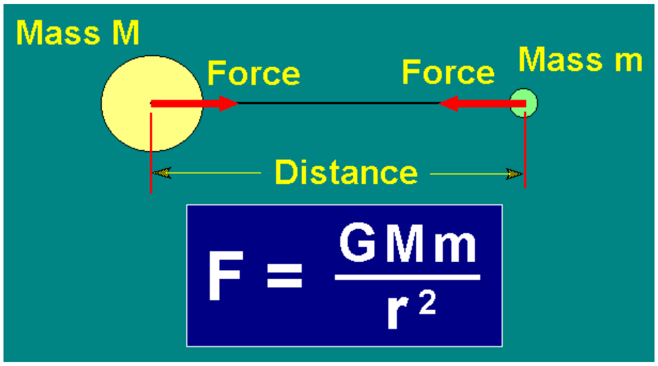
Terrestrial Motion: $d = \frac{1}{2} g t^2$ with $g = 9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2}$.

Celestial Motion (Planets): $T^2 = R^3$ (Kepler's Third Law with T in years and R in AU)

Earth Surface: $g = 9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2}$ **Moon in Orbit:** $a = \frac{v^2}{r} = 2.72 \times 10^{-3} \ \frac{\mathrm{m}}{\mathrm{s}^2}$

Ratio: $\frac{a}{g} = \frac{2.72 \times 10^{-3} \ \frac{\mathrm{m}}{\mathrm{s}^2}}{9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2}} = \frac{1}{3600}$

Distance to Moon: $R_{\text{Moon Orbit}} = 60 R_{\text{Earth Radius}}$ => Inverse Square Law.



$$F(r) = \frac{GMm}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$$

Terrestrial: $F(r) = \frac{GMm}{r^2} = ma$ => $a = g = \frac{GM_{\text{Earth}}}{R^2_{\text{Earth}}} = 9.8 \ \frac{\mathrm{m}}{\mathrm{s}^2}$

Celestial (Moon M and r): $\frac{GMm}{r^2} = ma$

$$\frac{GM}{r^2} = a \quad \Rightarrow \quad \frac{GM}{r^2} = \frac{v^2}{r}$$

Using $v = \frac{2\pi r}{T}$, then $\frac{GM}{r^2} = \frac{1}{r}(\frac{2\pi r}{T})^2$.

$$\frac{GM}{r^2} = \frac{1}{r}\frac{4\pi^2 r^2}{T^2} \quad \Rightarrow \quad \frac{GM}{r^2} = \frac{4\pi^2 r}{T^2} \quad \Rightarrow \quad \frac{1}{r^3} = \frac{4\pi^2}{GM}\frac{1}{T^2}$$

$$r^3 = \frac{GM}{4\pi^2}T^2$$

Check out the Planets: $T^2 = R^3$ (Kepler's Third Law with T in years and R in AU)

Michael J. Ruiz, "Kepler's Third Law Without a Calculator,"
The Physics Teacher **42**, 530 (2004).

Jupiter (R = 5 AU): $T = \sqrt{5^3} = \sqrt{125} \approx \sqrt{121} = 11 \text{ y}$. For R = 5.2 AU, T = 12 y.

Saturn (R = 10 AU): $T = \sqrt{10^3} \approx \sqrt{900} = 30 \text{ y}$. For R = 9.5 AU, T = 29 y.

Intro 5. Work and Kinetic Energy. Start with $F = ma$, which we can write as $F = m\frac{dv}{dt}$.

Since the momentum in classical physics is $p = mv$, we can also write $F = \frac{dp}{dt}$. This last form is actually the best form for Newton's second law since it allows for rocket problems where mass is shot out the back and thus m changes as well as velocity v. Of course in its formal form, it is a vector equation $\vec{F} = \frac{d\vec{p}}{dt}$.

Let's push or pull a mass from rest in outer space (no friction) through some distance x. The work you do is defined as the product of the force component along the direction of motion times the distance. Think about this definition.

Work = Force x Distance = Fd

If you were paying workers to push boxes of appliances on rollers across a warehouse floor, doesn't it make sense to pay them depending on how much force they apply times the distance? Would you pay anything if someone was applying a force to a refrigerator against the wall going nowhere? Then the distance would be zero and force times distance would be zero, i.e., no work. What about the zombie who walks around with hands extended applying zero force to the air? The zombie pushes nothing. So force times distance is zero due to the zero force. What about someone sitting down on the job (F = 0, d = 0)?

To allow for non-constant situations we use the calculus definition of applying a force F for an infinitesimal distance dx and do an integral. We will take the force aligned with x and write

$$W = \int F(x)dx \, .$$

Lets apply a force to a mass in outer space from rest and then let go of it. What is the work we do?

$$W = \int F\,dx = \int ma\,dx = \int m\frac{dv}{dt}\,dx = \int m\,\frac{dv}{dx}\frac{dx}{dt}\,dx = \int_0^v mv\,dv = \frac{1}{2}mv^2$$

Do you like our tricks with the chain rule, differentials, and changing the integration variable? This result is called the work-energy theorem. You do work on the mass and this work goes into energy in the form of motion. So we define the energy due to the work we did as the kinetic energy since the mass is now moving through space.

$$E = \frac{1}{2}mv^2 \, .$$

If you apply a force on an already moving object, the work you do is expressed as a difference of kinetic energies. See below for the more general case of the work-energy theorem.

$$W = \int_{x_1}^{x_2} F\, dx = \int_{v_1}^{v_2} mv\, dv = \frac{1}{2} m{v_2}^2 - \frac{1}{2} m{v_1}^2$$

Intro 6. Potential Energy.

a) Gravity Near Earth: Pick up a rock a height h and drop it. Gravity does work on the rock.

Choose up as positive. As the rock falls the force is then down, i.e., in the negative direction.

$$W = \int F\, dz = \int -mg\, dz = -\int_h^0 mgdz = \int_0^h mgdz = mgz\Big|_0^h = mgh = \frac{1}{2} mv^2$$

We can call mgh the potential energy, energy there as potential before dropping the rock.

Note that the speed right before it hits the ground is $v = \sqrt{2gh}$.

This result is a shortcut equivalent to formulas in Section A1:

$h = \frac{1}{2} gt^2$ and $v = gt$.

Solve for v and you get $t = \sqrt{\frac{2h}{g}}$ and $v = gt = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$, the same result!

Let $h = h_1 - h_2 > 0$, where 1 is above 2.

Then $mgh = \frac{1}{2} mv^2 \rightarrow mgh_1 - mgh_2 = \frac{1}{2} m{v_2}^2 - \frac{1}{2} m{v_1}^2$

$$\frac{1}{2} m{v_1}^2 + mgh_1 = \frac{1}{2} m{v_2}^2 + mgh_2$$

Conservation of energy: $E = \frac{1}{2} mv^2 + mgh = \text{const}$

Potential Energy: $U = mgh$

Some think of the potential energy by picking up the mass: $W = \int_0^h mgdz = mgh$.

b) Gravity Far from Earth: Drop a Rock from infinity (zero reference) to r.

Choose up as positive. The force is then down, i.e., in the negative direction.

$$W = \int F(r)dr = \int_\infty^r -\frac{GMm}{r^2}\,dr = \frac{GMm}{r}\bigg|_\infty^r = \frac{GMm}{r}$$

$$W = \frac{GMm}{r} = \frac{1}{2}mv^2$$

Drop from r_1 to r_2 where $r_1 > r_2$.

$$W = \int F(r)dr = \int_{r_1}^{r_2} -\frac{GMm}{r^2}\,dr = \frac{GMm}{r}\bigg|_{r_1}^{r_2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

$$W = \int_{v_1}^{v_2} mv\,dv = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2}$$

$$\frac{GMm}{r_2} - \frac{GMm}{r_1} = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2}$$

$$\frac{1}{2}mv_1^{\,2} - \frac{GMm}{r_1} = \frac{1}{2}mv_2^{\,2} - \frac{GMm}{r_2}$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \text{const}$$

Potential Energy: $U = -\frac{GMm}{r}$

$$\boxed{E = \frac{1}{2}mv^2 + U = \text{const}}$$

Intro 7. Hooke's Law.

Portrait Thought to be Robert Hooke (1635-1703)

Ideal Spring: force proportional to stretch or compression.

$$F = -kx$$

Stretch and let go.

$$W = \int_x^0 F\,dx = \int_x^0 -kx\,dx = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

If a mass m is attached and you let go, then $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$,

when the mass reaches the center. All the energy is kinetic at that point.

Newton's Second Law $F = ma$ with Hooke's Law $F = -kx$ leads to

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

$$a = -\omega_0^2 x$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

The constants A and B depend on the initial conditions.

For the case where you pull back and release: $x(0) = A$.

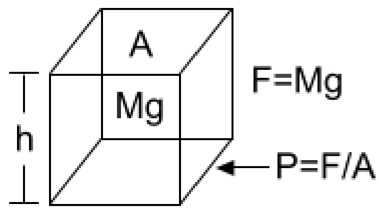
Note that $v(t) = \frac{dx(t)}{dt} = -\omega_0 A\sin(\omega_0 t) + \omega_0 B\cos(\omega_0 t)$ and $v(0) = \omega_0 B$.

Intro 8. Pascal

1. Pascal's Law (Pascal's Principle)

Basic definitions. Mass m is measured in kilograms. Weight is a force given by W = mg. Pressure if force per unit area.

Blaise Pascal (1623-1662). Pascal's Law.
Courtesy School of Mathematics and Statistics
University of St. Andrews, Scotland



$$P = \frac{F}{A} = \frac{Mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h$$

Include atmospheric pressure pressing down from above: $P \rightarrow P_0 + \rho g h$.

Pascal's Law gives the pressure due to the weight of a fluid or gas at a depth h.

Units:1 pascal = 1 newton per square meter, i.e., $\mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2}$.

Atmospheric pressure under normal conditions: $14.7 \frac{\mathrm{lb}}{\mathrm{in}^2} = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$.

Consider 10 meters below water. The density of water is 1000 kg/m^3.

$$\rho g h = 1000 \ \frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9.8 \frac{\mathrm{N}}{\mathrm{kg}} \cdot 10 \text{ m} \approx 10^5 \frac{\mathrm{N}}{\mathrm{m}^2} = 10^5 \text{ Pa} = 1 \text{ atm}$$

Going underwater every 10 meters picks up an atmosphere of pressure.

2. Pascal’s Triangle

0^{th}									1							
1^{st}								1		1						
2^{nd}							1		2		1					
3^{rd}						1		3		3		1				
4^{th}					1		4		6		4		1			
5^{th}				1		5		10		10		5		1		
6^{th}			1		6		15		20		15		6		1	
7^{th}		1		7		21		35		35		21		7		1

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Apples and Bananas Courtesy absfreepic.com

$(a+b)^n = ?$ Let a = apple and b = banana.

Number of ways to pick n apples => 1, giving a^n, i.e., $a^n b^0$ (indicating no bananas).

Number of ways to pick 1 banana => n, giving $na^{n-1}b^1$ (note number total fruit is always n).

Number of ways to pick 2 bananas => n(n-1). But we do not care about the order.

So divide by 2 => $\frac{n \cdot (n-1)}{1 \cdot 2} a^{n-2} b^2$, writing out 1 times 2 in the denominator.

Number of ways to pick 3 bananas => n(n-1)(n-2) .

But we do not care about the order. How many ways can you arrange the 3 bananas?

Answer is 6: 123, 132, 213, 231, 312, 321.

How many backs to pick 1 out of 3 = > 3. Then 2 ways to get the next one => $3\cdot 2\cdot 1 = 6$.

Conclusion: $\dfrac{n\cdot(n-1)(n-2)}{1\cdot 2\cdot 3}a^{n-3}b^{3}$.

The r^th term, where $r \le n$ =>

$$\binom{n}{r}a^{n-r}b^{r} = \frac{n\cdot(n-1)(n-2)...(n-r+1)}{1\cdot 2\cdot 3...\cdot r}a^{n-r}b^{r}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

But it is easy to easy to remember
r factors on top counting backwards from n
and r factors on bottom counting upward from 1.

Check $\binom{n}{r} = \binom{4}{2} = \dfrac{4!}{2!(4-2)!} = \dfrac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1(2\cdot 1)} = \dfrac{4\cdot 3}{2} = 6$

The easy way: $\binom{n}{r} = \binom{4}{2} = \dfrac{4\cdot 3}{1\cdot 2} = 6$

Note that the Pascal Triangle gives $(a+b)^0 = 1$ and $(a+b)^1 = a+b$.

Just remember that when you try to find n and r, be sure to count from zero.

$$C^0_0$$

$$C^1_0 \quad C^1_1$$

$$C^2_0 \quad C^2_1 \quad C^2_2$$

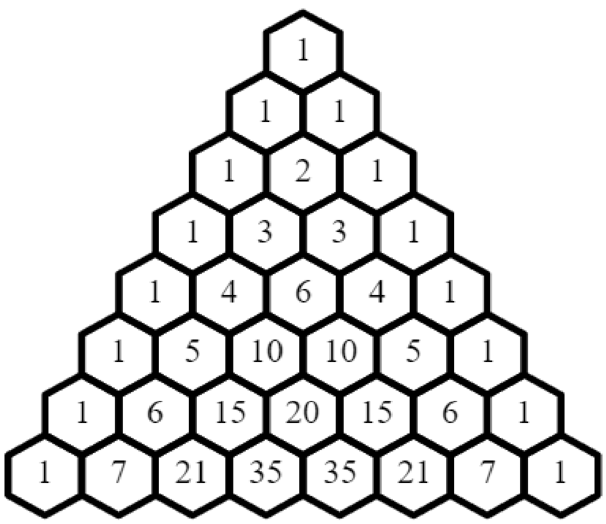
$$C^3_0 \quad C^3_1 \quad C^3_2 \quad C^3_3$$

$$C^4_0 \quad C^4_1 \quad C^4_2 \quad C^4_3 \quad C^4_4$$

$$C^5_0 \quad C^5_1 \quad C^5_2 \quad C^5_3 \quad C^5_4 \quad C^5_5$$

$$C^6_0 \quad C^6_1 \quad C^6_2 \quad C^6_3 \quad C^6_4 \quad C^6_5 \quad C^6_6$$

$$C^7_0 \quad C^7_1 \quad C^7_2 \quad C^7_3 \quad C^7_4 \quad C^7_5 \quad C^7_6 \quad C^7_7$$



Figures Courtesy GeoGebra

3. Pascal's Wager. After Pascal died, the publication of his philosophical writings earned him an esteemed position among philosophers. In his famous "Wager" he argues that you are better off living a life as if there is a Creator, an argument cast in terms of probabilities. My late colleague Philosopher Deryl Howard introduced the wager in our UNCA Team-Taught Core Humanities Program in the 1980s – in the third course: *HUM 324 The Modern World*. I do not know if they still include it as I stopped teaching in the program around the year 2000.

Intro 9. The Method of Undetermined Multipliers.

Joseph-Louis Lagrange (1736-1813)
Courtesy www.scientific-web.com

Before using the Lagrange Method of Undetermined Multipliers, let's first do a max-min problem using the standard approach. Find the largest area you can enclose with a fixed amount of fence given by L = 2x + 2y.

Let A = A(x,y). Then find A as A(x) only and set dA/dx = 0. Solve for x. You will get two solutions: x = 0 and x = L/4. The second one is your square with x = y = L/4 as expected.

Maximize the Area using the Lagrange Method of Undetermined Multipliers. You will do the same problem now by considering

$$dA(x,y) = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy = 0 .$$

Before we continue, let's talk about partial derivatives in case you have not had them yet in calculus. If you had them in math, you can skip this part. But you might want to skim anyway since there are some nice visualizations included below. For a function of one variable $f(x)$ you can write

$$df = \frac{df}{dx}dx ,$$

which looks perhaps too obvious. We are really stating a simple fact like $\Delta f = \frac{\Delta f}{\Delta x}\Delta x$ in differential form. When you have two variables $f(x,y)$, we can write the delta of the function as coming from a change in each variable:

$$df(x,y) = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy\ ,$$

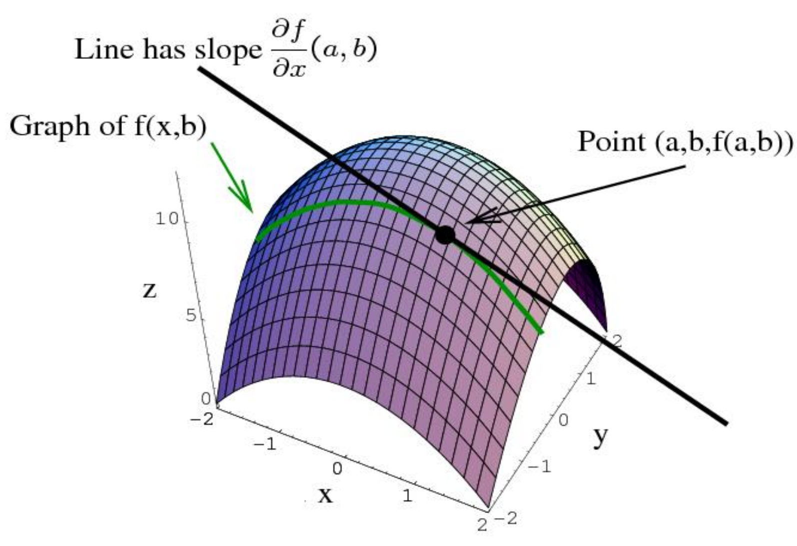
where $\frac{\partial f}{\partial x}$ is a partial derivative. You take y to be a constant and take the derivative with respect to x. Then, for $\frac{\partial f}{\partial y}$ you take x to be constant and take the derivative with respect to y. Here is an example. If $f = x^2 y$, then

$$\frac{\partial f}{\partial x} = 2xy \quad \text{and} \quad \frac{\partial f}{\partial y} = x^2\ .$$

Formal definitions for the partial derivatives are given below.

$$\frac{\partial f}{\partial x} = \lim_{x \to \infty} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial f}{\partial y} = \lim_{y \to \infty} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

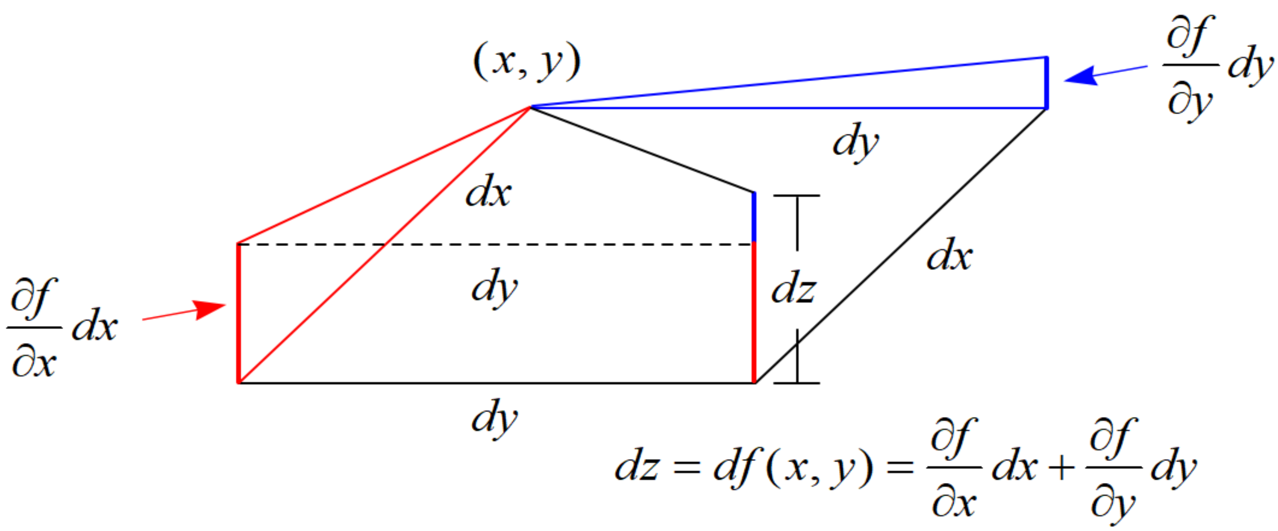


Here is a visualization courtesy Duane Q. Nykamp, found at mathinsight.org. The function $z = f(x,y)$ can be graphed as a surface. The partial derivative $\frac{\partial f}{\partial x}$ is the slope along the x-direction, while the partial derivative $\frac{\partial f}{\partial y}$ is the slope along the y-direction.

A total rise in z-height for a slanted direction where there is a dx and a dy is then given by adding the two rises in height together.

$$dz = df(x, y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

See the figure below for a visualization.



If the dx and dy were independent, we could set each partial to zero. In the standard approach you get rid of the y and write an equation where the derivative of A with respect to x is set to zero. Here is another way. Use

$$dA(x, y) - \lambda dL = 0 \text{ and } \left[\frac{\partial A}{\partial x} - \lambda \frac{\partial L}{\partial x}\right] dx + \left[\frac{\partial A}{\partial y} - \lambda \frac{\partial L}{\partial y}\right] dy = 0 .$$

Though dx and dy are not independent, we pick λ so

$$\frac{\partial A}{\partial y} - \lambda \frac{\partial L}{\partial y} = 0$$

Since the dx can be thought of as an arbitrary differential, we have

$$\frac{\partial A}{\partial x} - \lambda \frac{\partial L}{\partial x} = 0$$

The beauty of the method is we can write the pair

$$\frac{\partial A}{\partial x} - \lambda \frac{\partial L}{\partial x} = 0 \quad \text{and} \quad \frac{\partial A}{\partial y} - \lambda \frac{\partial L}{\partial y} = 0 .$$

The price we pay is that we introduced λ, which must be found. But that is worth the deal. Use

$$A = xy \text{ and } L = 2x + 2y .$$

$$\frac{\partial A}{\partial x} - \lambda \frac{\partial L}{\partial x} = 0 \quad => \quad y - \lambda \cdot 2 = 0 \quad => \quad y = 2\lambda$$

$$\frac{\partial A}{\partial y} - \lambda \frac{\partial L}{\partial y} = 0 \quad => \quad x - \lambda \cdot 2 = 0 \quad => \quad x = 2\lambda$$

Therefore, $x = y = 2\lambda$.

$$L = 2x + 2y \quad => \quad L = 2x + 2x \quad => \quad x = \frac{L}{4}$$

With $x = y = \frac{L}{4}$ we have a square, which we know from the previous result.

Intro 10. The Ideal Gas Law.

Boyle's Law. Pressure of a gas is inversely related to volume at constant temperature.
A process at constant temperature is called isothermal.

$$P \sim \frac{1}{V} \text{ and } P_1 V_1 = P_2 V_2 \text{ as constant temperature.}$$

When temperature is used with the equations in this section, the Kelvin scale is used, i.e., T = Celsius Temperature − 273.

Robert Boyle (1627-1691). Images from the School of Mathematics and Statistics, Univ. of St. Andrews, Scotland.

Charles's Law. Volume of a gas is proportional to temperature at constant pressure.
A process at constant pressure is called isobaric.

$$V \sim T \text{ and } \frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ as constant pressure}$$

and we use the absolute Kelvin temperature scale. Consider this as a definition for the absolute temperature scale: as you cool the gas down at constant pressure, the volume shrinks to zero as temperature goes to zero.

Jacques Charles (1746-1823)

Gay-Lussac's Law. Pressure of a gas is proportional to temperature at constant volume.
A process at constant volume is called isochoric or isometric.

$$P \sim T \text{ and } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ at constant volume.}$$

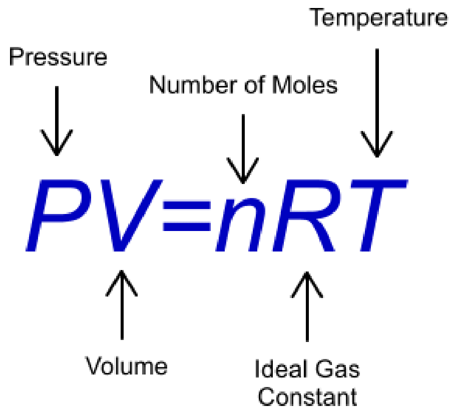
Think of this as an alternative definition for temperature on the absolute scale. As you lower the pressure at constant volume, the temperature lowers, both heading towards zero.

Joseph Gay-Lussac (1778-1850)

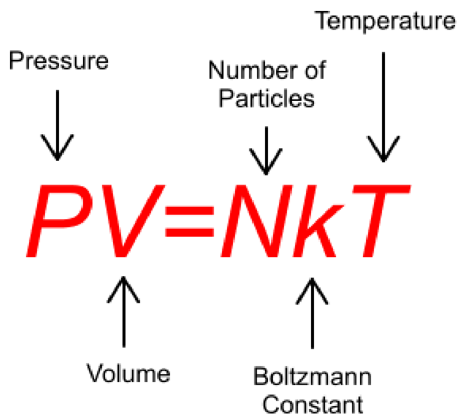
We can incorporate all these laws in the form:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

The chemists write the ideal gas law as



The physicists often like to write



The definition of the mole is

$$n = \frac{N}{N_A}$$, where $N_A = 6.022x10^{23}$ is called Avogadro's number.

Since $PV = nRT = NkT = nN_AkT$, the physicist's Boltzmann constant is related to the chemist's idea gas constant as

$$k = \frac{R}{N_A}.$$