Theoretical Physics

A. Taylor Series, Rotation, Groups

A1. Taylor Series

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots a_n x^n + \dots$$

$$f(0) = a_0$$

$$f'(0) \equiv f^{(1)}(x) = a_1$$

$$f^{(2)}(0) = 2a_2$$

$$f^{(3)}(0) = 3 \cdot 2\, a_3$$

$$\vdots$$

$$f^{(n)}(0) = n!\, a_n \qquad a_n = \frac{f^{(n)}(0)}{n!}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \qquad \text{Maclaurin Series}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \qquad \text{Taylor Series}$$

A2. Taylor Expansion

Speed	Large	Small
Fast	RM	RQM
Slow	CM	QM

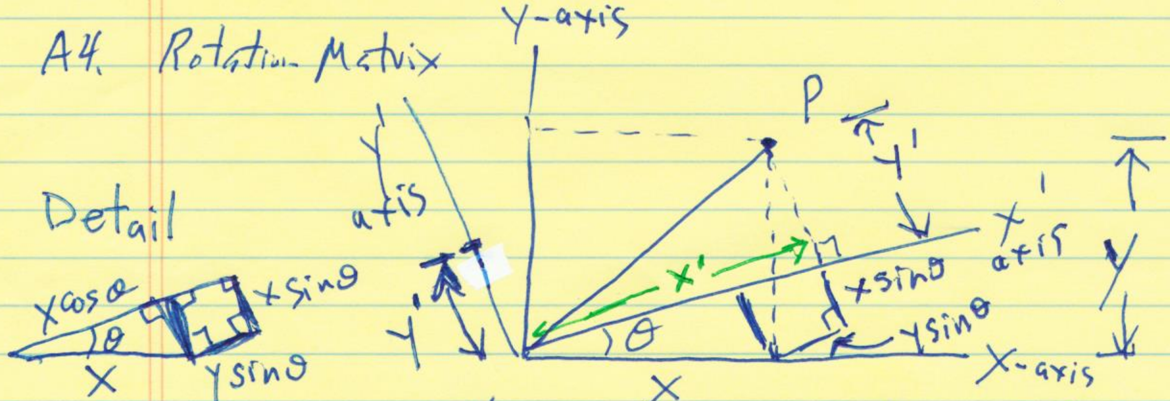
Size

A3. Matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

A4. Rotation Matrix



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R(\theta)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x' = x\cos\theta + y\sin\theta$$

$$y' = y\cos\theta - x\sin\theta$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

A5. Trig Identities

$$R(\alpha+\beta) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} = \begin{bmatrix} \cos(\alpha+\beta) & \cdots \\ \cdots & \cdots \end{bmatrix}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \cos\alpha\sin\beta + \sin\alpha\cos\beta$$

A6. Visualization of a Trig Identity

See Text

A7. Groups Set $S = \{ \&, \boxdot, 3, 8, \pi \}$

$G = \{a, b, c, d \ldots\}$ set with binary operation $a \cdot b$

1. Closure: for $a, b \in G$, then $a \cdot b \in G$
2. Association: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
3. Identity: $I \in G$ such that $a \cdot I = a$ technically right (side) identity
4. Inverse: $a^{-1} \in G$ such that $a \cdot a^{-1} = I$

Group is Abelian if $a \cdot b = b \cdot a$

$I \cdot a = b$?

$I \cdot \underbrace{a \cdot a^{-1}}_{I} = b \cdot a^{-1}$

$I = b \cdot a^{-1} \Rightarrow b = a$ $\qquad I \cdot a = a$ Left side works too

$a^{-1} \cdot a = I$?

left side works too

$a^{-1} \cdot a = b$

$a^{-1} \cdot \underbrace{a \cdot a^{-1}}_{I} = b \cdot a^{-1}$

$\underbrace{a^{-1} I} = b \cdot a^{-1}$

$a^{-1} = b \cdot a^{-1} \Rightarrow b = I \qquad a^{-1} \cdot a = I$

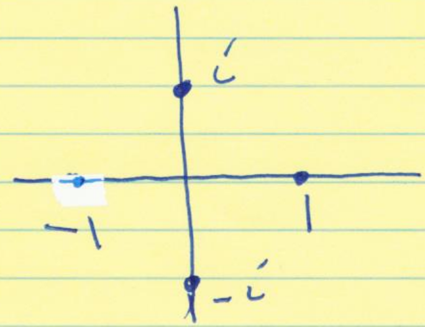
A8. Military Group $G = \{A, B, L, R\}$

	A	B	L	R
A	A	B	L	R
B	B	A	R	L
L	L	R	B	A
R	R	L	A	B

$i = \sqrt{-1}$ means face left
rotate 90°
counterclockwise

$G = \{1, -1, i, -i\}$

	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1



Isomorphism

SAT i^{100} ⇒ 100 left faces   4 gets you back
25 full rotations ⇒ $i^{100} = 1$
$i^{101} = i$ etc.