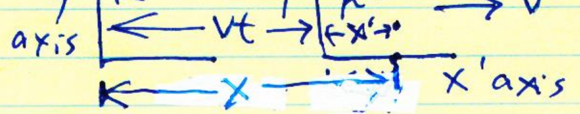
1/21/2020 C1

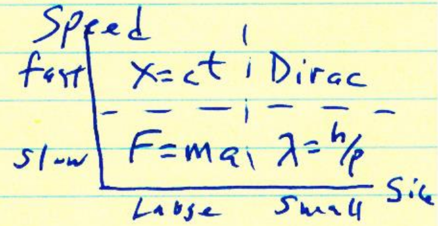
C. Relativity

C1. Galilean Transformation



$x' = x - vt$

$t' = t$



See figure on Page C1

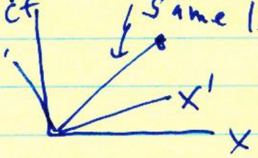
C2. Lorentz Transformation + SR
C3.

At $t'=t=0$ light $x = ct$ $x' = ct'$ ← same →

$$x^2 - c^2t^2 = x'^2 - c^2t'^2$$

Rotation Matrix $x' = x\cos\theta + ict\sin\theta$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$



$t' = -x\sin\theta + ict\cos\theta$

Landau + Lifshitz (1962)
student
Starting ~1930s
10 volumes of
Theoretical Physics

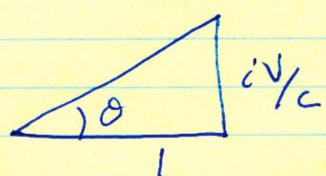
Stay put in K' frame so

$$\Delta x' = 0 = \Delta x\cos\theta + ic\Delta t\sin\theta$$

$$-\Delta x\cos\theta = ic\Delta t\sin\theta$$

$$\underbrace{\frac{\Delta x}{\Delta t}}_{v} = -ic\tan\theta$$

$$\boxed{\tan\theta = iv/c}$$



Don't worry about imaginary length Wow!

$$\text{hypotenuse} = \sqrt{1 - v^2/c^2}$$

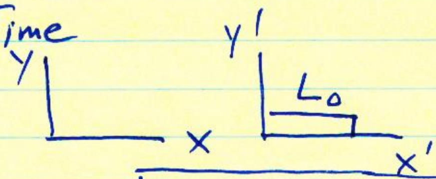
$$x' = \frac{x}{\sqrt{1-v^2/c^2}} + ict\,\frac{iv}{c}\,\frac{1}{\sqrt{1-v^2/c^2}}$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1-v^2/c^2}}}$$

$$\boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}}}$$

$$\frac{1}{\sqrt{1-v^2/c^2}} \equiv \gamma$$

C4. Space & Time



$$\underbrace{\Delta x'}_{L_o} = \gamma(\underbrace{\Delta x}_{L} - \underbrace{v\Delta t}_{\to 0})$$

simultaneously measure end to end

$$\boxed{L = L_o\sqrt{1-v^2/c^2}}$$ Lorentz Contraction

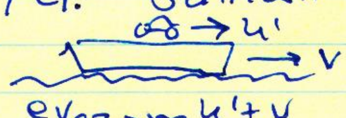
$$\Delta t = (\Delta t' + \underbrace{\Delta x' v/c^2}_{\to 0})\gamma$$ Why?

$$\boxed{T = \frac{T_o}{\sqrt{1-v^2/c^2}}}$$ Time Dilation

C5. Velocity Addition

Practice Problem

PC1. Galilean — Show if $u = \frac{\Delta x}{\Delta t}$, $u' = \frac{\Delta x'}{\Delta t'}$



$$\boxed{u = u' + V}$$

Important to do as many practice problems as possible. Sometime they appear on exams.

PC2. Relativistic Velocity Addition

Show $$\boxed{u = \frac{u'+V}{1+\frac{u'V}{c^2}}}$$

for $u' \ll c$, $v \ll c$ $\quad u \simeq u' + V$

for $u' = c$, $V = c$

$$u = \frac{c+c}{1+\frac{c\cdot c}{c^2}} = \frac{2c}{1+1} = c$$

PC3.
$$u_y = \frac{u_y'\sqrt{1-v^2/c^2}}{1+u_x' V/c^2}$$

C6. Four Vectors

$$X^\mu = (ct, x, y, z) \qquad dx^\mu = (c\,dt, dx, dy, dz)$$
$$X_\mu = (ct, -x, -y, z)$$
$$X^\mu X_\mu = c^2t^2 - x^2 - y^2 - z^2$$

$$ds^2 \equiv c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt'^2 - dx'^2 - dy'^2 - dz'^2$$

Watch in pocket
You can't move away from yourself

$dt' = d\tau$

$dx' = dy' = dz' = 0$

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$
$$= c^2\left[1 - \frac{v^2}{c^2}\right] dt^2$$

Note:
$$d\tau = \frac{dt}{\gamma}$$

$$d\tau = \sqrt{1 - v^2/c^2}\, dt$$

Time Dilation
$$dt = \frac{d\tau}{\sqrt{1 - v^2/c^2}}$$

1st Four Velocity $u^\mu \equiv \dfrac{dx^\mu}{d\tau} = \gamma(c, \vec{v})$

$x^\mu = (ct, \vec{r})$

$$p^\mu = m\frac{dx^\mu}{d\tau} = \gamma(mc, m\vec{v})$$

$$x^\mu x_\mu = c^2 t^2 - r^2 = \tau^2$$

$$p^\mu p_\mu = \frac{(m^2c^2 - m^2v^2)}{\left[\sqrt{1 - v^2/c^2}\right]^2} = m^2c^2$$

C7. Work + Energy Skip $E = mc^2$ Will assign a video as homework.

Cute

C8. Triangle

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) \quad \vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

$$p^\mu p_\mu = \frac{E^2}{c^2} - p^2 = m^2c^2$$

$$E^2 = m^2c^4 + p^2c^2$$