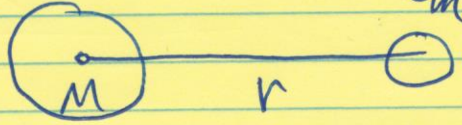
Note! $\vec{A}\cdot\vec{B} = AB\cos\theta$ (Dot Product)

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ (cross product) — $\hat{n}$ into the page

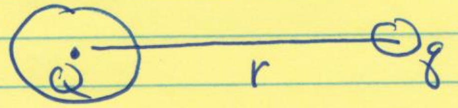
1/23/2020 D-1

D. Maxwell Equations

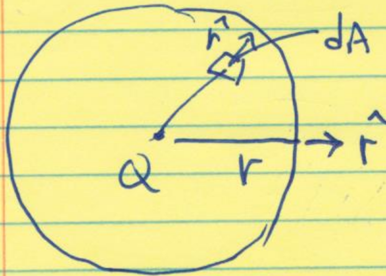
D1. Gauss's Law



$$F_G = \frac{GMm}{r^2}$$



$$F_E = \frac{kQq}{r^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$



$$\vec{E} = \frac{kQ}{r^2}\hat{r} \qquad V = \frac{4}{3}\pi r^3$$

$$d\vec{A} = \hat{r}\,dA \quad (4\pi r^2 dr) \qquad dV = \underbrace{4\pi r^2}_{dA} dr$$

$$\oiint \vec{E}\cdot d\vec{A} = \oiint \frac{kQ}{r^2} 4\pi r^2 \underbrace{\hat{r}\cdot\hat{r}}_{1}\, dA = 4\pi kQ \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

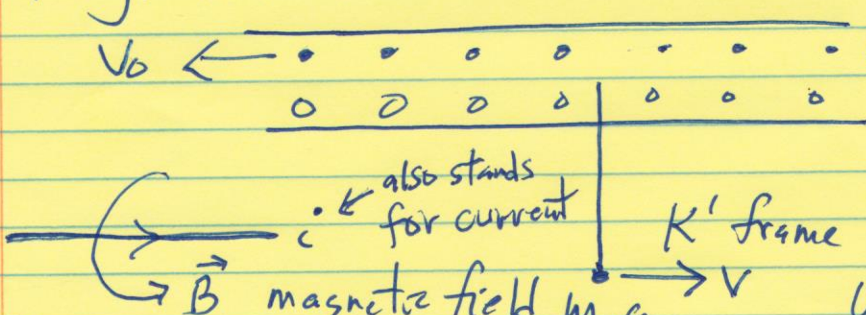
$$\boxed{\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}}$$ ← Q inside the volume

Line of Charge $\lambda = Q/L$

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{2\pi r}\frac{\lambda}{\varepsilon_0}}$$

D2. Magnetic Field



K' frame (from Prime frame)

$$E' = \frac{1}{2\pi r}\,\frac{\lambda'_- - \lambda'_+}{\varepsilon_0}$$

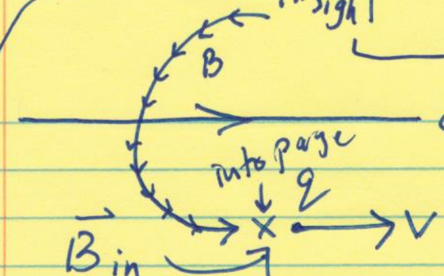
Will feel a sideways force

Skip details in Book

$$\vec{B} = \frac{\mu_0 i}{2\pi r}\hat{\theta} \quad \text{where } \mu_0 = \frac{1}{\varepsilon_0 c^2} \qquad \vec{F} = q\vec{v}\times\vec{B}$$

i also stands for current

Magic insight: Note $\mu_0 = \frac{1}{\varepsilon_0 c^2}$ — c → Special Relativity incorporated



$\vec{F} = q\vec{v} \times \vec{B}$

$q\vec{v} \times \vec{B}$: up towards wire

$\vec{B}$ in designated by X, a tail of an arrow →

$\oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$ $\quad \oiint \vec{B} \cdot d\vec{A} = 0$ $\quad \oint \vec{B} \cdot d\vec{l} = \mu_0 i$

$\vec{B}$ doesn't pierce through enclosed volume

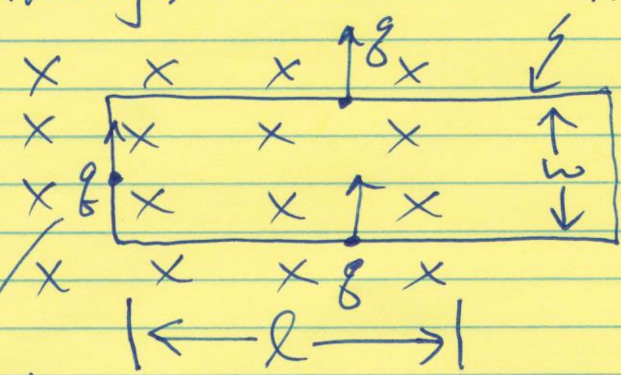
$\oint \vec{B} \cdot d\vec{l} = B 2\pi r$

$B = \frac{\mu_0}{2\pi r} i$

Constants of Electricity, Magnetism, and Speed of light!

Eqs. of E+M hint that c is a constant! → $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ Light is an electromagnetic wave! To be shown.

D3. Faraday's Law



Pull → $v = -\frac{dl}{dt}$

$\vec{F} = q\vec{v} \times \vec{B}$

gets current going clockwise

$F = qvB \equiv qE$

$$E = vB = -\frac{dl}{dt} B$$

↑$\vec{E}$ only gives contribution at left edge width w

$$\oint \vec{E} \cdot d\vec{l} = Ew$$

$$= -\frac{dl}{dt} B \cdot w$$

Could pull up

$$= -\frac{d(lw)}{dt} B = -\frac{dA}{dt} B$$

Could increase B instead of (pulling wire

$$= -\frac{d}{dt}(BA)$$

Φ_B magnetic flux

little hand waving here - not rigorous derivation.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

The 4 Maxwell Equations

Summary

No magnetic monopoles enclosed since no piercing out of volume

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} \quad \oint\!\!\oint \vec{B}\cdot d\vec{A} = 0 \quad \oint \vec{B}\cdot d\vec{l} = \mu_0 i \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

TO

D4. Displacement Current (Skip) Add $\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

Video assigned for homework derivation

Necessary for a super discovery.

LET THERE BE LIGHT! To be discussed.

Finally $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Maxwell equations 1865

contain the seed of Special Relativity

Special Relativity came late in 1905

Everything plugged into an electrical outlet or uses a battery comes out of E+M

Electrical Engineering - late 1800s

$\Rightarrow$ 20th Century

Toaster, Radio, Record Player, Tape Deck, CD player, Computer

21st $\rightarrow$ iPod, iPad, iphone