Theoretical physicists Genealogy

Bohr
↓
Wheeler
↓
Misner
↓
Ruiz
↓
you

Class I. Quantum Mechanics

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I1. Quantization 1900

1. Energy Max Planck $E_n = nhf$ $n = 0, 1, 2 \ldots$

   Harmonic Oscillator The QM result is $E_n = (n + \frac{1}{2})hf$

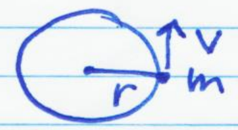
   ΔE_n same

   $\Delta E_n = hf$

2. Light Einstein 1905

   $E_{photon} = hf$

3. Angular Momentum Bohr 1913



   $L = pr = mvr$

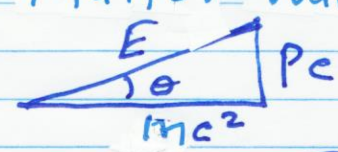
   $L = n\frac{h}{2\pi} \equiv n\hbar$

   No radiation unless electron drops to lower energy level.

   Gets you the Balmer Series and more.

Classial EM ⇒ shaking charge spiral in losing energy. Atoms should be unstable. You should not exist.

I2. Matter Waves



For light

$E = pc$ all energy is in pc max speed $\theta = 90°$

$E = pc = hf$

$\quad\ \ \hookrightarrow \lambda f$

$\Big\}$ $p\lambda = h$ $\boxed{\lambda = \frac{h}{p}}$

de Broglie → true also for matter

Matter			Waves
p	n	e⁻	γ
1913	1932	1897	1905 Photon ← Pure energy.

Bohr model ↗

$\underbrace{\text{p n e}^-}_{\text{particles}}$

Two Big Surprises

1. Light is particle also
2. Particles are waves also

I3. Wave Properties

$$\psi(x,t) = A\sin[k(x - vt)]$$

need for units of length to cancel

Let $t = 0$ $\psi(x, 0) = A\sin kx$ $k\lambda = 2\pi$

Wavelength

$$k = \frac{2\pi}{\lambda} \quad \text{Wave number}$$

Similarly $kvT = 2\pi$ $kv = \frac{2\pi}{T} = 2\pi f = \omega$

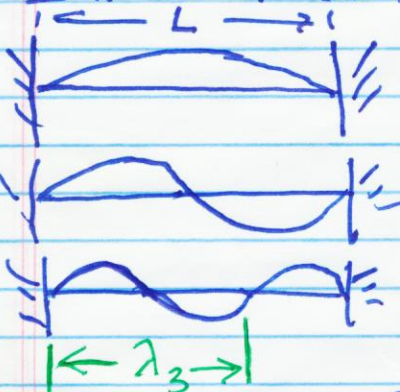
Period

angular velocity

Sometimes you see $\sin(kx - \omega t)$

← Harmonics

I4. "Back Door" to QM — Let There Be Music



$\lambda_3 = \frac{2L}{3}$ checks out

$\frac{2}{3}$ across

← half-wave

$n\frac{\lambda}{2} = L \qquad \lambda = \frac{2L}{n}$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$$

$$p = \frac{h}{\lambda} \Rightarrow E = \frac{1}{2m}\frac{h^2}{\lambda^2} = \frac{h^2}{2m\left(\frac{2L}{n}\right)^2}$$

$$E_n = \frac{n^2h^2}{8mL^2}$$

The Quantum mechanical result for a particle in a 1D Box!

Also written $E_n = \frac{n^2(2\pi\hbar)^2}{8mL^2} = \frac{n^2\hbar^2}{2mL^2}$

I5. Schrödinger Equation

We are in search of a differential equation.

Also called standing waves.

$\psi(x,t) = A\sin(kx)\cos(\omega t)$ describes the above waves.

$E = \frac{p^2}{2m} \qquad E = \hbar\omega \qquad p = \frac{h}{\lambda} = h\frac{k}{2\pi} = \hbar k$

$\hbar\omega = \frac{\hbar^2k^2}{2m}$

Can now make $\hbar\omega = \frac{\hbar^2k^2}{2m}$ work

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$$

$\left\{ \begin{array}{l} \frac{d}{dx}\sin kx = k\cos kx \\ \frac{d^2}{dx^2}\sin kx = -k^2\sin kx \end{array} \right.$

$$\frac{\partial\psi}{\partial t} = -\omega A\sin(kx)\sin(\omega t)$$

not equal to $-\omega\psi$

Use $\psi(x,t) = A\sin(kx)e^{-i\omega t}$ — like engineering

Then $\frac{\partial\psi}{\partial t} = -i\omega\psi$

Why we need $i = \sqrt{-1}$ in QM. To get $\hbar\omega\psi$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = i\hbar\frac{\partial\psi}{\partial t}$$

For $\frac{p^2}{2m} + V = E$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

1 Dim.

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E$$

Time independent form