Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter J Homework. Spinors

HW-J1. Matrix Properties. You are given $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and the Pauli matrices

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \text{ and } \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Show that the matrix $M = \vec{\sigma} \cdot \vec{A} = \begin{bmatrix} A_z & A_x - iA_y \\ A_x + iA_y & -A_z \end{bmatrix}$. Then calculate each of the following six items:

$$Tr(M), \quad M^T, \quad M^*, \quad M^\dagger, \quad \det(M), \quad M^{-1}.$$

HW-J2. Pauli Matrices Identity. Given,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k},$$
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k},$$
$$\vec{\sigma} = \sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k},$$

show that

$$\vec{\sigma} \cdot \vec{A}\, \vec{\sigma} \cdot \vec{B} = \vec{A} \cdot \vec{B}\, I + i \vec{\sigma} \cdot (\vec{A} \times \vec{B}).$$

HW-J3. Find the eigenvalues and normalized eigenvectors for $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$.