Theoretical Physics
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Chapter M Homework. The Method of Frobenius

HW-M1. Sines and Cosines. You know that the basic solutions to the differential equation

$$y'' + y = 0$$

are the even function $\cos x$ and the odd function $\sin x$.

Use the Method of Frobenius to solve this differential equation. Find the recurrence relation. Note that your recurrence relation for the power series gives a_{k+2} in terms of a_k, which means you can break these into an even series and odd series. Since you have a second order differential equation, you have two free constants a_0 and a_1. Set $a_0 = 1$ and $a_1 = 0$ to get your even power series. See if your even power series is the same as the Taylor Series for $\cos x$. Then set $a_0 = 0$ and $a_1 = 1$ to get your odd power series. See if your odd power series is the same as the Taylor Series for $\sin x$.

HW-M2. The Laguerre Differential Equation. The Laguerre differential equation is given below.

$$xy'' + (1-x)y' + ny = 0$$

Use the Method of Frobenius to solve this differential equation up to the point of the recursion relation. Show that your recurrence relation for the power series terminates if n = 0, 1, 2, 3, etc.

HW-M3. Laguerre Polynomials. Use your recurrence relation from HW-M2 to find the Laguerre polynomials L₀(x), L₁(x), L₂(x), and L₃(x), where you choose $a_0 = n!$ in each case.