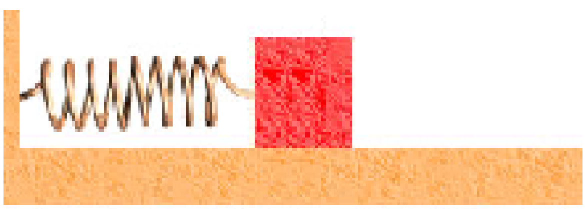
Theoretical Physics
Prof. Ruiz, UNC Asheville
Chapter U Homework. Green's Functions

U1. The Green's Function for Radioactive Decay. The differential equation for radioactive decay is $\frac{dn(t)}{dt} + \lambda n(t) = 0$, which comes from $\frac{dn(t)}{dt} = -\lambda n(t)$. Use your four-step procedure (delta function, Fourier transform, complex integration, Green's function) to show that the Green's function for the radioactive-decay differential equation is $G(t,0) = e^{-\lambda t}$.

U2. The Green's Function for the Damped Harmonic Oscillator.



Courtesy David M. Harrison
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The differential equation for the damped harmonic oscillator is

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$$, which comes from $F = -kx - bv = ma$ with

$\omega_0^2 = \frac{k}{m}$ and $\beta = \frac{b}{2m}$. For your specific problem $\alpha^2 = \omega_0^2 - \beta^2 > 0$.

Use your four-step procedure (delta function, Fourier transform, complex integration, Green's function) to show that the Green's function for the damped harmonic oscillator system is given by

$$G(t,0) = \frac{1}{\alpha} e^{-\beta t} \sin(\alpha t)$$.