Class K. The Pauli Equation

K1. Measurement

$$H\psi_n = E_n \psi_n$$

← eigenfunction; ← Eigenvalue

$$H = \underbrace{-\frac{\hbar^2}{2m}\nabla^2}_{\frac{p^2}{2m}} + V$$

$p \to -i\hbar\nabla$

$E \to i\hbar\frac{\partial}{\partial t}$

$[x, p_y] = 0$ AND

K2. Heisenberg Uncertainty Relation

operators

$$p = -i\hbar\frac{d}{dx}$$

$$x(p\psi) = x\left[-i\hbar\frac{d}{dx}\right]\psi = -i\hbar x\frac{d\psi}{dx}$$

$$p(x\psi) = \left[-i\hbar\frac{d}{dx}\right](x\psi) = -i\hbar\psi - i\hbar x\frac{d\psi}{dx}$$

for $\frac{\partial}{\partial y}(x\psi)$ This term would be zero.

(Schrödinger Differential equation

$P(x)\,dx = \psi^*\psi\,dx$

Probability distribution

Max Born

$$(xp)\psi - (px)\psi = i\hbar\psi$$

Can't measure x & p at same time.

$$[x, p] = i\hbar \quad \text{Do not commute.}$$

If they commuted You would have a common state - same eigenvectors for each operator & can have the 2 measurements simultaneously.

K3. Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

use cyclic yzx — cyclic xyz

$$L_x = y p_z - z p_y$$

$$L_y = z p_x - x p_z$$

$$L_z = x p_y - y p_x$$

cyclic zxy

$$[L_x, L_y] = [y p_z - z p_y,\ z p_x - x p_z]$$

$$= [y p_z, z p_x] - [y p_z, x p_z] - [z p_y, z p_x] + [z p_y, x p_z]$$

$$= y p_x \underbrace{[p_z, z]}_{-i\hbar} - yx\underbrace{[p_z, p_z]}_{0} - p_y p_x \underbrace{[z, z]}_{0} + x p_y \underbrace{[z, p_z]}_{i\hbar}$$

Lie Algebra

ORBITAL

$$[L_x, L_y] = i\hbar L_z \quad \text{since } L_z = x p_y - y p_x$$

Practice Problem

But the cyclic trick implies this result.

$$[L_j, L_k] = i\hbar\,\varepsilon_{jkl} L_l$$

But Wait!

$$[\sigma_j, \sigma_k] = 2i\,\varepsilon_{jkl}\,\sigma_l$$

← Spin Angular Momentum

We are on to something big!

We can fix this.

Orbital Angular momentum $[L_j, L_k] = i\hbar \varepsilon_{jkl} L_l$ / $\left[\frac{\sigma_j}{2}, \frac{\sigma_k}{2}\right] = \frac{2i}{2}\varepsilon_{jkl}\frac{\sigma_l}{2}$ Divide by 4

We have another type of angular momentum!

Intrinsic angular momentum. $\vec{S} = \frac{\hbar}{2}\vec{\sigma} \Rightarrow [S_j, S_k] = i\hbar\varepsilon_{jkl} S_l$

→ The Spin of the electron!

$$S_z\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}1\\0\end{pmatrix}$$

One-half $\hbar$ ← eigenvalue

K4. K5. Skip Mostly *

K6. Pauli Equation   Put in Spin by hand

Promote ψ to $\psi = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}$

Promote $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$ Time Independent form $V = V(\vec{r})$ no time dependence.

$$-\frac{\hbar^2}{2m}\nabla^2\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} + V\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} = E\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}$$

where $V = \begin{bmatrix}V_{11} & V_{12}\\ V_{21} & V_{22}\end{bmatrix}$

$r = \sqrt{x^2+y^2+z^2}$

$\tan\phi = y/x$

$\phi = \tan^{-1}\frac{y}{x}$

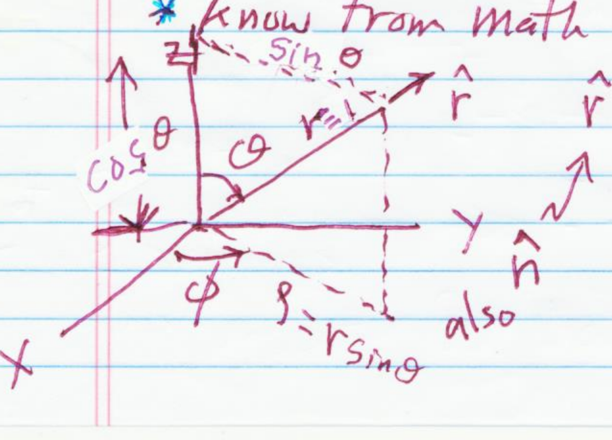
Operator $H = -\frac{\hbar^2}{2m}\nabla^2 + V$ becomes

$$H = -\frac{\hbar^2}{2m}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\nabla^2 + \begin{bmatrix}V_{11} & V_{12}\\ V_{21} & V_{22}\end{bmatrix}$$

$\theta = \cos^{-1}\frac{z}{r}$

To go from (x,y,z) to (r,θ,ϕ).

* know from math class



$\hat{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$

taking $r=1$ from the general

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

(r,θ,ϕ) spherical coordinates