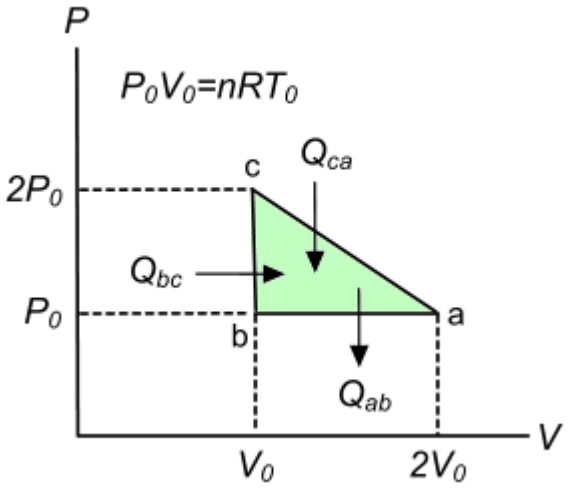
Theoretical Physics **Exam 2** **October 18, 2011**
Prof. Ruiz, UNC Asheville **Closed Book - No Calculator**

1. Engine. The total work W done by the engine during one "a-b-c" cycle is given by the shaded green region. The gas is ideal: PV = nRT and U = 3nRT/2.



a. Express W in terms of nRT_0.

b. Give a short sentence explaining why heat must flow into the system for the path b-c.

c. Calculate the total heat flowing in from b to c.

2. Statistical Mechanics. There are N = 26,000,000 particles, where some are in energy state $E_1 = 0$ and the rest are in state $E_2 = \varepsilon$. Give the occupation number n_1 for the 1st state when $\varepsilon = kT$, using the approximation $e = 2.718... \approx 2\frac{5}{7} = \frac{19}{7}$.

3. Commutator. In class we showed that $[x, p]\psi = i\hbar\psi$, i.e., $[x, p] = i\hbar$.

Express the commutator $\left[x^2, x\frac{d}{dx}\right]$ in simplest terms.

4. Dirac Four-Spinor. When a spin-1/2 is moving, the Dirac four spinor for the particle is given by

$$u(\vec{p}, s) = \sqrt{\frac{E + mc^2}{2E}}\begin{bmatrix} \phi(s) \\ \frac{c\vec{\sigma}\cdot\vec{p}}{E + mc^2}\phi(s) \end{bmatrix}, \text{ where } \phi(s) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}.$$

Express this four-spinor in simplest terms for the case where

$$E = 2,\ mc^2 = 1,\ c\vec{p} = \hat{i} + \hat{j} + \hat{k}, \text{ and } \phi(s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$