Theoretical Physics
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Exam 1

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1. Matrices. Calculate the commutator $\left[\sigma_x, \sigma_y\right] \equiv \sigma_x \sigma_y - \sigma_y \sigma_x$, where

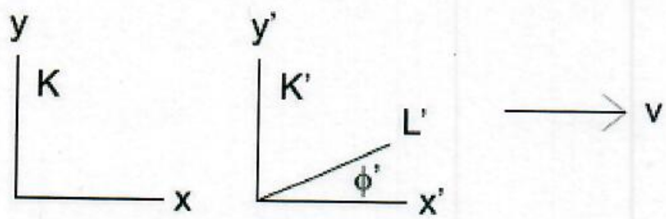
$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Express your final answer in terms of σ_x , σ_y , and σ_z .

2. Integral. Use a derivative trick with

$$\int_{-\infty}^{\infty} \frac{\cos mx}{x^2 + a^2}\, dx = \frac{\pi}{a} e^{-ma}$$ where $a > 0$, to evaluate $$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 1}\, dx.$$

3. Relativity. When the length of a rod along the x-axis in a moving frame K' is measured from the K frame, there is a Lorentz contraction of the form $L_x = L_x' \sqrt{1 - \frac{v^2}{c^2}}$. For a rod slanted in the K' frame at angle $\phi' = 30°$, what is the measured angle ϕ from the K frame for the velocity given below?



$$v = \frac{2\sqrt{2}}{3} c$$

4. Vector Calculus.

The vector $\vec{A} = x z \hat{i}$ and $\nabla \times \vec{A} = \frac{\partial \vec{B}}{\partial x}$, where the variables x, y, and z are all independent of each other. Find $\vec{B}$.