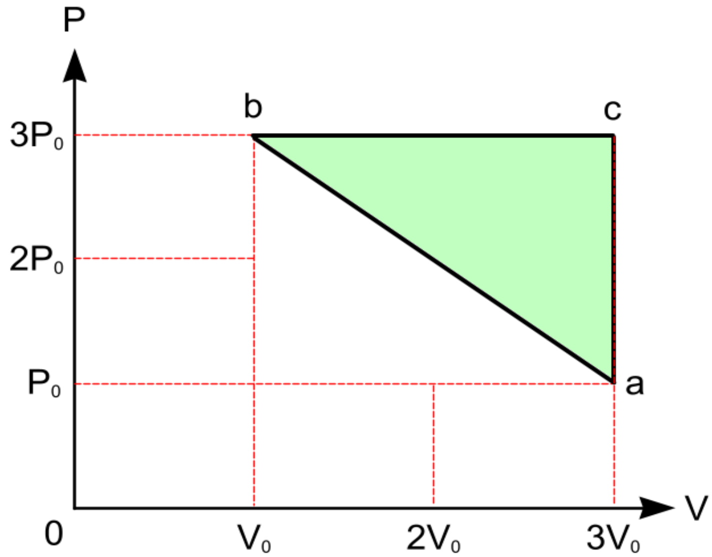
Theoretical Physics **Exam 2** **March 24, 2020**
Prof. Ruiz, UNCA **Closed Book/Notes but Calculator is Permitted**

1. Engine. Use $PV = nRT$, $P_0V_0 = nRT_0$, and take the number of moles to be such that $nR = 1\ \frac{\text{joules}}{\text{kelvins}} = 1\ \frac{J}{K}$. Take the temperature $T_0 = 300\text{K}$.



a. Show that $T_a = 900\text{K}$.

b. Show that $T_b = 900\text{K}$.

c. Show that $T_c = 2700\text{K}$.

d. Show that $\Delta Q_{a\to b} = -1200\ \text{J}$

e. Show that $\Delta Q_{b\to c} = 4500\ \text{J}$

f. Show that $\Delta Q_{c\to a} = -2700\,\text{J}$.

g. Calculate $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}}$.

2. Stat Mechanics. The energy levels are 0, ε, 2ε, 3ε, 4ε, 5ε, …, with ε = kT. Use the identity $S = 1 + r + r^2 + r^3 + r^4 ... = \frac{1}{1-r}$, where r < 1, to show that the partition function $Z = \sum_{n=0}^{\infty} e^{-\frac{E_n}{kT}}$ is 1.58. Find the occupation probability in each of these levels: 0, ε, 2ε, 3ε, 4ε.

3. Quantum Mechanics. The time independent Schrödinger equation in one dimension can be written as $H\psi = E\psi$ where $H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$. Let $V(x) = \frac{1}{2}m\omega^2 x^2$. Then consider special units where $\hbar = m = \omega = 1$. Show that $\psi(x) = \frac{1}{\sqrt{\pi}} e^{-\frac{x^2}{2}}$ is an eigenfunction, i.e., $H\psi = E\psi$, and find the eigenvalue E.

4. Eigenvectors and Eigenvalues. Find the normalized eigenvectors and eigenvalues for the matrix operator $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$. A vector $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ is normalized if $c_1^* c_1 + c_2^* c_2 = 1$.