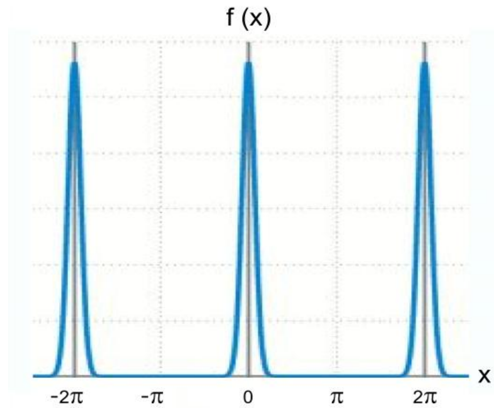
Theoretical Physics | **Exam 3** | **April 14, 2020**
Prof. Ruiz, UNC Asheville | **Closed Book - No Calculator**

[15] Q1. Fourier Series. Find the Fourier Series for the periodic wave $f(x)$ shown at the right, where you take the peaks to be delta functions. Write out the first 5 terms of the Fourier series in their simplest forms.



[10] Q2. Fourier Transform. Now remove all the delta functions in the figure of Q1 except for the one at 2π. Find the Fourier transform $F(k)$ for your delta function at 2π. Give the result for $F(1)$, i.e., when $k = 1$.

[15] Q3. Laplace Transform. Use the derivative trick with integrals to show that the Laplace transform of t is $\frac{1}{s^2}$, i.e., $L\{t\} = F(s) = \frac{1}{s^2}$. You can use this result in Q4 below.

[15] Q4. Laplace Transform of a Differential Equation. A car starts from rest at $x(0) = x_0$ and accelerates with constant acceleration $a > 0$. The differential equation that describes this scenario is $\frac{dx}{dt} = at$. Take the Laplace transform of the differential equation and obtain the Laplace transform of $x(t)$, which you can call $X(s)$. Now find $x(t)$ from the Laplace Table, where you find $L\{t^n\} = \frac{n!}{s^{n+1}}$. Use this table information to obtain the solution $x(t)$.

[15] Q5. Convolution. Show that the "derivative trick" can be used to express the convolution of a radioactive-waste dumping function $f(t) = t$ and impulse radioactive-decay response $g(t) = e^{-\lambda t}$ as $f(t) * g(t) = e^{-\lambda t} \frac{d}{d\lambda}\left[\frac{e^{\lambda t} - 1}{\lambda}\right]$.

[10] Q6. Cauchy-Riemann. Show that $z^2 = (x + iy)^2$ is analytic.

[20] Q7. Complex Integration. Using complex integration, evaluate the definite integral $I = \int_{-\infty}^{\infty} \frac{dx}{x^4 + 4}$. If you have trouble, you may for half credit evaluate $I = \int_{-\infty}^{\infty} \frac{dx}{x^2 + 4}$ instead.