Theoretical Physics (Prof. Ruiz/UNCA) Exam 2 M

Open Book/Notes/Internet (90 minutes including scan and email return).

Posted Syllabus Exam Time is 6:00 pm – 7:30 pm.

Exam emailed 5:45 pm and you can start as soon as you get it.

All work and related steps must be explicitly shown for full credit.

**[25] 1. Combinatorics.** There are 10 students taking a flexible course where 3 of the 10 are attending face to face, another 3 are attending online, and the remaining 4 are attending in a mixed more. How many different ways can this arrangement be realized?

[25] 2. Inverse Matrix. Find the inverse matrix for the Pauli combination

$$M = \sigma_{y} + \sigma_{z} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -i \\ i & -1 \end{bmatrix}.$$

Express your final answer in terms of  $\,M\,$  .

[25] 3. Matrix as Exponent. Find the simplest form for  $e^A$ , where  $A = i\theta\sigma_z$  and  $\sigma_z$  is the third Pauli matrix. Simplest form is defined by the least amount of characters needed to express the final answer. As an example, the function  $\cos\theta$  consists of 4 characters. A left or right parenthesis counts as one character.

[25] 4. Recurrence Relation. You are given the recurrence relation

$$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k \text{ for } f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots,$$
  
where  $k = 0, 1, 2, 3...$  and  $n = 0, 1, 2, 3...$ 

Find the specific polynomial f(x) where  $a_0 = 0$ ,  $a_1 = 1$ , and n = 5.