Physics I with Calculus, Prof. Ruiz (Doc), UNC-Asheville (1978-2021), doctorphys on YouTube
Chapter D. Newton's Three Laws and Statics. **Prerequisite: Calculus I. Corequisite: Calculus II.**

D0. Introduction. Newton's Three Laws are introduced in this chapter. Then Statics is considered, an application of Newton's Laws when the total force on an object is zero. Statics is the study where the net force, i.e., total force, on an object is zero. Then we have no acceleration. Think of static equilibrium where the object is at rest relative to its surroundings. Examples include buildings and bridges since they are stationary. You can focus on any part or component of the bridge. Since the component does not move, we have static equilibrium. Below is the Walt Whitman Bridge that crosses over the Delaware River from Philadelphia, Pennsylvania (PA) to Gloucester City, New Jersey (NJ). You are looking at the New Jersey side in the photo. One exit heading left takes you into Fairview, South Camden, where I grew up. To the right of the bridge is Gloucester, NJ.

The Walt Whitman Bridge over the Delaware River, Philadelphia, PA to Gloucester City, NJ
Photo from a Kayak, looking towards New Jersey, June 11, 2010, Courtesy Jag9889, Wikimedia
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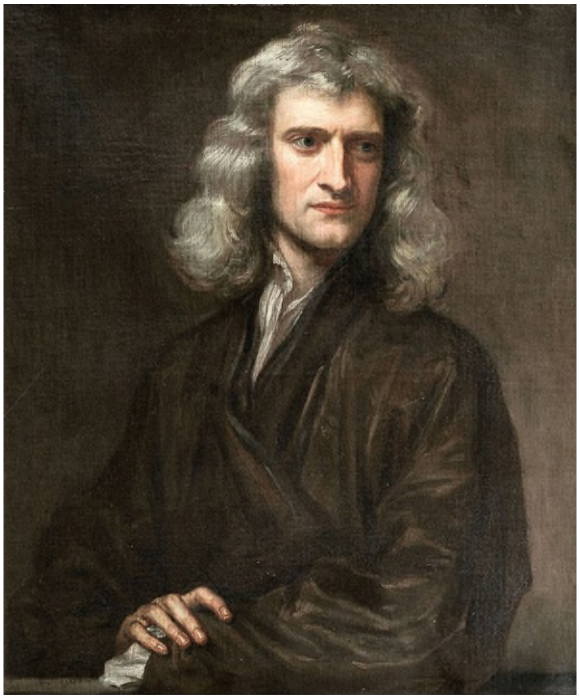
D1. Newton and the Scientific Revolution of the 16th-17th Centuries. We also take some time in this section to talk about what is science.



Portrait of Isaac Newton (1642-1727)
by Artist Godfrey Knellers
Public Domain, Wikipedia
Date: 1689
Medium: Oil on Canvas
Owner: The Earl of Portsmouth

As of the writing of this chapter in January 2022, the current Earl of Portsmouth is Quentin Wallop, the 10th Earl of Portsmouth (from 1984).

Newton and Einstein are generally considered to be the two best physicists of all time.

Newton was also a mathematician who discovered calculus, independent of Leibniz, the other discoverer. Newton also gave us his Universal Law of Gravitation, which we study later.

Wikipedia describes Newton as an "English mathematician, physicist, astronomer, alchemist, theologian, and author (described in his time as a 'natural philosopher')."

Newton's famous book, *Mathematical Principles of Natural Philosophy*, generally known as *The Principia*, was published in 1687. This masterpiece can be considered as the culmination of the Scientific Revolution of the 16th and 17th centuries. For about 2000 years, a dominant view of the universe was that the center was the Earth. The idea dates back to the ancient Greeks (c. 380 BCE).

Five European scientists played key roles in the revolution of the 16th and 17th centuries.

1. Nicolaus Copernicus (1473-1543), the Polish scientist, discovered that it was easier to describe the motion of planets if the Sun was at the center (heliocentric) instead of the Earth (geocentric).

2. Tycho Brahe (1546-1601), the Danish astronomer, spent decades gathering data before the invention of telescopes. His data was crucial in accurately plotting planetary orbits, that would be used later to add support to the heliocentric view.

3. Johannes Kepler (1571-1630), the German mathematician-astronomer, used mathematics with Brahe's data to arrive at three empirical laws of the heavens. We will study these later.

4. Galileo Galilei (1564-1642), the Italian astronomer, physicist, and engineer, was one of the inventors of the telescope that enabled him to discover four moons of Jupiter among other

discoveries. Showing that celestial bodies could orbit a planet indicated that everything did NOT go around the Earth.

5. Isaac Newton (1642-1727), the English physicist and more, discovered the laws of motion and gravitation that explained how the planets go around the Sun.

Today we know that that Sun and planets form the Solar System within a tremendously large universe.

During my teaching days I was asked about the Scientific Method from time to time. Many grade school textbooks talk about the Scientific Method as a hypothesis, then control variables, and experiments. Personally, as a theoretical physicist, I never did any of that. I develop for you below a description of the Scientific Method based on what the above five architects did during the Scientific Revolution of the 16-17th centuries.

Before doing so, we should point out that these European scientists built on the vast store of knowledge accumulated by many diverse groups such as Egyptians, the Chinese, Greeks, and Arabs. In fact, during the long period known as the "Dark Ages" in Western Europe, a time from the Fall of Rome to the Renaissance, the Arabs were major players in science and astronomy. Many of the names of stars we use today are Arabic from this time period.

Models – Copernicus: The Value of Models and Simplicity in Science

Observation – Brahe: The Importance of Observation and Gathering Data

Mathematics – Kepler: The Role of Mathematics in Science

Instrumentation – Galileo: The Use of Instrumentation to Extend the Senses

Unification – Newton: Synthesis and Unification in Science (Universal Law of Gravitation)

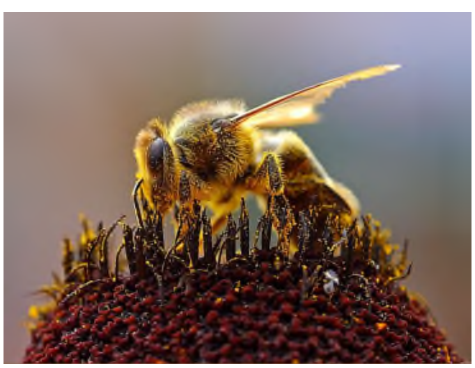
We are on safe grounds with the above descriptive components of the Method of Science, i.e., the Scientific Method.

Francis Bacon (1561-1626)
Detail of Portrait of Francis Bacon
by Paul van Somer I (1617)
Public Domain, Wikipedia

Let's briefly continue our discussion of the Method of Science. As we have stated, in many elementary science texts one can find a definition of the "scientific method" as making an hypothesis and testing it with a controlled experiment. However, there is no such thing as a single "scientific method" as commonly believed. But there is a "method of science." It is simply one where we use reason to combine theory (analysis) and experiment (observation). And we do this in many different ways as you will see throughout this course.

For most of history, the scientific way was not obvious. Francis Bacon was one of the early thinkers who struggled with articulating what we call science today. Consider Aphorism 95 from his *The New Organon* (1620), where Bacon criticizes the scientists of his day as being either too experimental ("empirics") or too theoretical ("dogmatists").

> Aphorism 95. "Those who have worked in science have either been empirics or dogmatists. The former are like ants, who only gather and use. The latter are like spiders, who spin webs out of their own materials. The bee, demonstrates a middle ground. The bee extracts substance from the flowers of the garden and field. It then transforms and digests it by a power of its own." Francis Bacon



Bee Collecting Pollen
Del Mar Fairgrounds, San Diego, California, USA
Courtesy Jon Sullivan, Wikipedia
Public Domain

Today, the blend of data and analysis, as exemplified by the bee analogy, is considered common sense, germane to so many fields of study. For the physical sciences, we gather observational data and apply our analytical reasoning to the physical world, while others may focus on areas such as human behavior, society, or even history.

The usual description of science with control variables that we alluded to above is far too restrictive and even misleading. The so-called "scientific method" as described by outsiders looking in at scientists really doesn't exist at all. If you ask ten scientists what is meant by the scientific method, you will probably get ten different responses. Isn't this true in other fields? For example, can you give a simple formula to describe the method of painting, photography, journalism or playing football?

Fictional Detective Sherlock Holmes
Illustration by Sidney Paget (1904)
Public Domain, Wikipedia
Holmes was created by Arthur Conan Doyle
Both Newton and Doyle were British and Knighted, earning the titles Sir Isaac Newton and Sir Arthur Conan Doyle

Science is an art, much like the art of being a detective trying to solve a mystery. You use anything at your disposal - observations, theoretical analysis, hunches, clues, measurements, testing samples, chance, luck, and yes, even "control variables." You do whatever you can - but the bottom line is that your work is based on reason and logic, supported by observation and data. Science must also be able to predict - and in the case of physics, with mathematical precision.

Science is also reproducible. In our crime analogy, this means that another detective brought in from halfway around the world and given the same mystery, should reach the same conclusion. For example, detectives should agree on the result, although each no doubt takes a different and unique approach in solving the crime.

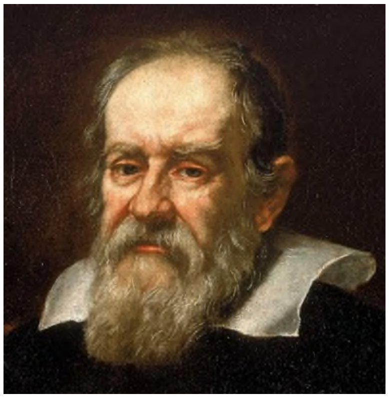
If they cannot reach the same conclusion in solving the mystery, then the observation and data are inconclusive or the analytical techniques are not powerful enough to work with the observed data. For example, one may have to wait for better DNA testing.

We will often experience connections of the interplay between theory (the basic laws, the analysis, the regularity, etc.) and experiment (observation, the demonstration, the data, etc.). In this way, the method of science is experienced from class to class over the period of our entire course in its rich multitude of varied forms. We will also see that thinking like a scientist serves us well in our many excursions outside of science.

Another important part of the art of science is the construction of models of great beauty. For example, in physics and astronomy, we search for principles that unify the laws of the physical universe. Some individuals work for their entire lives in this area, never doing a single experiment. They hope to unite known laws of nature, laws which are supported by experiment, into a coherent whole. The usual definition of the "scientific method" with its “control variables” criticized above misses out on this theoretical and inspiring holistic endeavor. In fact, the strict commonly-held view of the so-called "scientific method" leaves out your instructor as a scientist. His thesis work was on a theoretical quark model, attempting to unify relativity and quantum mechanics.

Newton achieved a goal of unification when he explained the terrestrial motion of a falling apple and the celestial motion of the planets with one Universal Law of Gravitation. Einstein popularized the quest for unification in more modern times with his attempt to unify the modern law of gravitation (his general theory of relativity) with electromagnetic theory. Today string theorists are hard at work in searching for a unified theory of forces. Inspiration comes from past success and thus we incorporate many historical developments in our course.

D2. Newton's Three Laws.

	
Galileo Galilei (1564-1642)	Isaac Newton (1642-1727)

Newton's First Law is due to Galileo. Galileo discovered that **a body persists in its state of rest or uniform motion in a straight line unless a net force acts on it.**

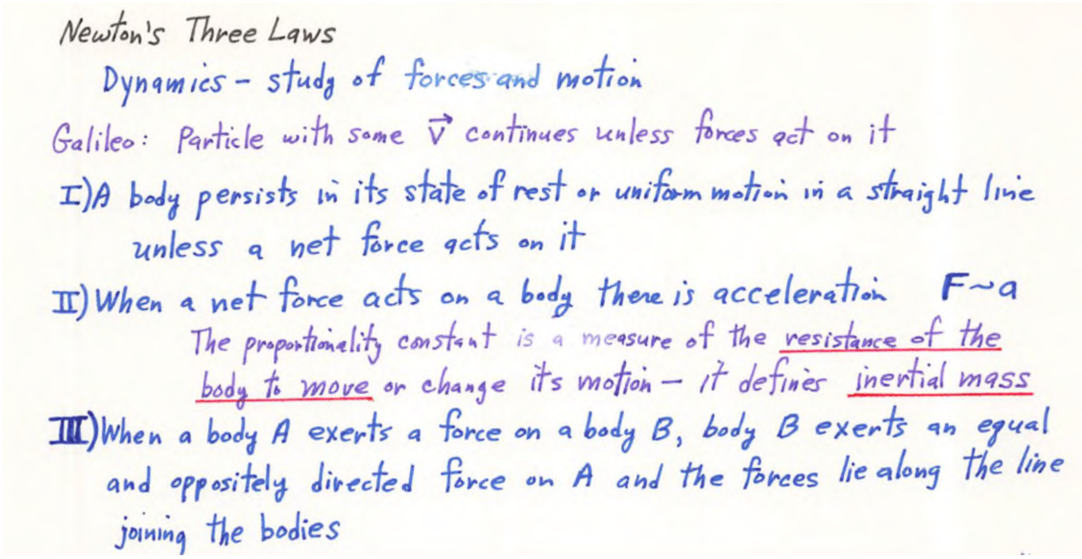
Before Galileo it was thought that you needed some kind of force to keep an object moving. This false idea was called the "theory of impetus" and it takes back to Aristotle (c. 350 BCE). You can see why they came up with that wrong idea. On Earth, everything appears to come to rest unless you apply forces. This fact is due to forces of resistance. If you eliminate resistance by watching an object travel in outer space far away from gravitational effects of celestial objects, the object continues on at constant linear velocity. Galileo figured this out. The mass wants to keep doing what it is doing, whether that is to stay at rest or keep moving. This feature is called inertia. Galileo's Law of Inertia becomes Newton's First Law. Newton's Three Laws are:

1) A body persists in its state of rest or uniform motion in a straight line unless a net force acts on it.

2) When a net force acts on a body, there is acceleration proportional to the force ($F \sim a$).

> The proportionality constant is a measure of the resistance of the body to move or change its motion – it defines inertial mass (m). We can write Newton's Second Law as $F = ma$.

3) When a body A exerts a force on a body B, body B exerts an equal and oppositely directed force on A and the forces lie along the line joining the bodies. See my vintage lecture notes below.



Newton's Three Laws

Dynamics – study of forces and motion

Galileo: Particle with some $\vec{v}$ continues unless forces act on it

I) A body persists in its state of rest or uniform motion in a straight line unless a net force acts on it

II) When a net force acts on a body there is acceleration $F \sim a$
The proportionality constant is a measure of the resistance of the body to move or change its motion – it defines inertial mass

III) When a body A exerts a force on a body B, body B exerts an equal and oppositely directed force on A and the forces lie along the line joining the bodies

Notes. Newton Laws, doctorphys c. 1980.

Let's give nicknames to Newton's Three Laws.

Newton's First Law – The Law of Inertia
Newton's Second Law – The Law of Force, Mass, and Acceleration or simply $F = ma$
Newton's Third Law – The Law of Action and Reaction

For the law of inertia, the more stuff you have, the more inertia there is. We call stuff mass in physics. So, more mass means more inertia, i.e., more reluctance for an object to change its motion. Later we will talk about a subtle distinction between inertial mass and gravitational mass.

For the second law, think of a force as a push or a pull. Stand on a scale and the Earth's gravity is pulling you down, which causes you to press against the scale and get a weight reading.

For the third law, I once saw this description of force action-reaction pairs in a high school text: "You cannot touch without being touched." If you touch someone, they are touching you back with an equal and opposite force.

If you touch a keypad like I am doing now, the keypad pushes back on a finger with an equal and opposite force.

A couple of last comments. First, since acceleration is a vector, F = ma, means that force will also be a vector. So we can write

$$\vec{F} = m\vec{a}\,.$$

Second, although $\vec{F} = m\vec{a}$ or $F = ma$ is the common way we refer to Newton's Second Law, it is technically not complete. Here is why. Write the acceleration as the derivative $\vec{a} = \frac{d\vec{v}}{dt}$. Then, Newton's Second Law is

$$\vec{F} = m\frac{d\vec{v}}{dt}\,.$$

Should that mass factor be moved inside the derivative? YES! For the complete law, we need to move that mass m into the derivative to get

$$\vec{F} = \frac{d(m\vec{v})}{dt}\,.$$

This more general expression allows for changes in mass, i.e., rocket problems where mass is ejected and the mass of the moving rocket decreases. Consider a rocket in one dimension, losing mass since it is ejecting some out the back in the opposite direction of its motion.

$$F = \frac{d(mv)}{dt} = \frac{dm}{dt}\cdot v + m\frac{dv}{dt}$$

Note that the mass of the rocket is decreasing due its losing mass:

$$\frac{dm}{dt} < 0\,.$$

Let $\frac{dm}{dt} \equiv -\alpha$, where $\alpha > 0$ to insure that $\frac{dm}{dt} < 0$. Then

$$F = \frac{dm}{dt}\cdot v + m\frac{dv}{dt} = -\alpha v + m\frac{dv}{dt}$$

and

$$F + \alpha v = m\frac{dv}{dt}\,.$$

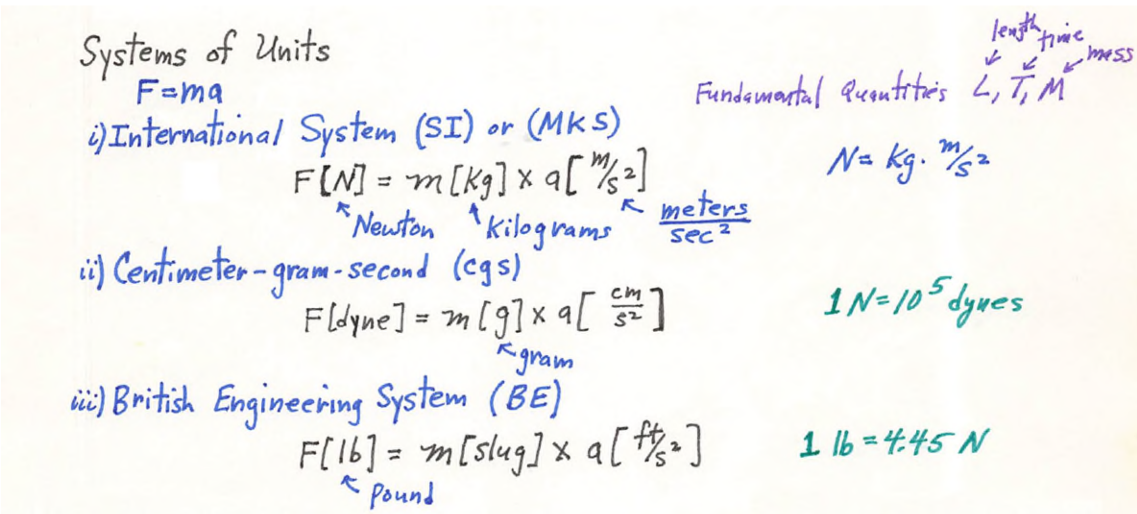
The ejecting mass adds to the externally applied force to make the rocket go even faster. In the usual scenario the external force is negative due to gravity. Therefore, the ejecting mass is the only thing pushing the rocket up via Newton's Third Law of Action and Reaction.

Later, we will define $m\vec{v}$ as the momentum $\vec{p}$. Newton's Second Law is then

$$\vec{F} = \frac{d\vec{p}}{dt},$$

which states that the net force on an object is equal to the change in momentum of the object.

D3. Systems of Units. Three common systems of units are shown below for force, mass, and acceleration. The scientific standard is the International System of Units, abbreviated SI for the French *Système international*. The long version of the international system is *Système international d'unités* (international system of units).



The systems are defined by the "Big Three" basic units: length (distance), time, and mass.

System	Length	Time	Mass
Metric (MKS)	meter (m)	second (s)	kilogram (kg)
Metric (cgs)	centimeter (cm)	second (s)	gram (g)
British (BE)	foot (ft)	second (s)	slug (no abbreviation)

For Newton's Second Law, the units for force, mass, and acceleration are given in the next table. Note that lowercase is used when you spell out the newton but we use N for the abbreviation.

System	Force	Mass	Acceleration
Metric (MKS)	newton (N)	kilogram (kg)	meters/second^2 (m/s^2)
Metric (cgs)	dyne (dyn)	gram (g)	centimeters/second^2 (cm/s^2)
British (BE)	pound (lb)	slug (no abbr.)	feet/second^2 (ft/s^2)

The British system is also referred to as the FPS or fps for foot-pound-second (length, weight, time).

Conversions. Note that the force units are defined in the respective systems when the mass and acceleration in those systems have a value of 1 with their appropriate units.

$$1\text{ N} = 1\text{ kg}\cdot 1\ \frac{\text{m}}{\text{s}^2} \qquad 1\text{ dyn} = 1\text{ g}\cdot 1\ \frac{\text{cm}}{\text{s}^2} \qquad 1\text{ lb} = 1\text{ slug}\cdot 1\ \frac{\text{ft}}{\text{s}^2}$$

$$\text{N} = \text{ kg}\cdot \ \frac{\text{m}}{\text{s}^2} \qquad \text{dyn} = \text{ g}\cdot \ \frac{\text{cm}}{\text{s}^2} \qquad \text{lb} = \text{slug}\cdot \ \frac{\text{ft}}{\text{s}^2}$$

We see another example of deriving new physical quantities from the basic three: length, time, and mass. In this case, the new physical quantity is force.

We can easily get the conversion from newtons to dynes.

$$1\text{ N} = 1\text{ kg}\cdot 1\ \frac{\text{m}}{\text{s}^2} = 1000\text{ g}\cdot 100\ \frac{\text{cm}}{\text{s}^2} = 100{,}000\text{ g}\cdot \frac{\text{cm}}{\text{s}^2} = 100{,}000\text{ dyn}$$

Therefore, we have the following conversions.

$$1\text{ N} = 100{,}000\text{ dyn}$$

$$1\text{ newton} = 100{,}000\text{ dynes}$$

$$\boxed{1\text{ newton} = 10^5\text{ dynes}}$$

$$\boxed{1\text{ dyne} = 10^{-5}\text{ newton}}$$

The conversion to the British system is trickier. My favorite way is to first consider the conversion of pounds to kilogram · weight, which we discussed in our first chapter. For describing weight, it is better to say kilogram · weight (kg · wt) or kilogram · force rather than simply kilogram since the kilogram is technically a measure of mass, while pounds refer to weight. By saying kilogram · weight or kilogram · force we are referring to the weight, i.e., force, of one kilogram on Earth.

To two significant figures,

$$1.0\text{ kg}\cdot\text{wt} = 2.2\text{ lb}\,.$$

To six significant figures

$$1.00000\text{ kg}\cdot\text{wt} = 2.20462\text{ lb}\,.$$

The force on 1 kg in newtons is $F = ma = mg$, where $m = 1\text{ kg}$, but what should we use for g? Doesn't g vary from place to place on Earth? We do not want the conversion to depend on location. So we have to agree on some standard value for g that we can use to define the conversion. A standard value for the acceleration of gravity has been agreed upon. It is **EXACTLY**

$$g = 9.80665 \frac{\mathrm{m}}{\mathrm{s}^2} .$$

This assignment was made in 1901. They were aiming at a value that could apply to a latitude of 45° at sea level. It is not really an average, but some erroneously think so. Now we can understand why some textbooks use

$$g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$$

for the acceleration due to gravity. However, many texts go with

$$g = 9.8 \frac{\mathrm{m}}{\mathrm{s}^2} ,$$

as we often do in our course. With the standard value for g, the force on one kilogram is

$$F = \ 1.00000 \text{ kg} \cdot \frac{9.80665 \text{ m}}{\mathrm{s}^2} ,$$

$$F = 9.80665 \text{ newtons} .$$

But this force is also the kilogram · weight (kg · wt) or kilogram · force, which is 2.20462 lb.

$$F = 9.80665 \text{ newtons} = 1.00000 \text{ kg} \cdot \text{wt} = 2.20462 \text{ lb}$$

Therefore,

$$1 \text{ lb} = \frac{9.80665}{2.20462} \text{ newtons} = 4.44823 \text{ newtons}$$

$$1 \text{ lb} = 4.44823 \text{ N}$$

$$1 \text{ newton} = \frac{2.20462}{9.80665} \text{ lb} = 0.225 \text{ lb}$$

To three significant figures: $\boxed{1 \text{ lb} = 4.45 \text{ N}}$ $\boxed{1 \text{ N} = 0.225 \text{ lb}}$

Next we find the conversion between the slug and kilogram.

$$F = mg$$

We need the accurate value for g in the British system. So we do a conversion.

$$g = \frac{9.80665\ \text{m}}{\text{s}^2} = \frac{9.80665\ \text{m}}{\text{s}^2} \cdot \frac{3.280840\ \text{ft}}{1\ \text{m}} = 32.1740 \frac{\text{ft}}{\text{s}^2}$$

Next we use Newton's Second Law where everything is 1 so that gives 1 pound.

$$F = 1.00000\ \text{slug} \cdot 1.00000 \frac{\text{ft}}{\text{s}^2} = 1.00000\ \text{lb}$$

Solve for the slug.

$$1\ \text{slug} = 1 \frac{\text{lb}}{\text{ft/s}^2}$$

Use the conversion from lb to N.

$$1\ \text{slug} = \frac{4.44823\ \text{N}}{1\ \text{ft/s}^2}$$

Write out the explicit units for the newton N, i.e., $1\ \text{lb} = 4.44823\ \text{N}$.

$$1\ \text{slug} = \frac{4.44823\ \text{kg} \cdot \frac{\text{m}}{\text{s}^2}}{1\ \text{ft/s}^2}$$

Convert meters to feet.

$$1\ \text{slug} = \frac{4.44823\ \text{kg} \cdot \frac{\text{m}}{\text{s}^2}}{1\ \text{ft/s}^2} \cdot \frac{3.280840\ \text{ft}}{1\ \text{m}}$$

$$1\ \text{slug} = 14.5939 \frac{\text{kg} \cdot \frac{\text{m}}{\text{s}^2}}{\text{ft/s}^2} \cdot \frac{\text{ft}}{\text{m}}$$

$$1\ \text{slug} = 14.5939\ \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{ft}} \cdot \frac{\text{ft}}{\text{m}}$$

$$1 \text{ slug} = 14.5939 \text{ kg}$$

$$1 \text{ kg} = \frac{1 \text{ slug}}{14.5939} = 0.06852 \text{ slug}$$

To three significant figures, the conversions are given below.

$$\boxed{1 \text{ slug} = 14.6 \text{ kg}}$$

$$\boxed{1 \text{ kg} = 0.0685 \text{ slug}}$$

Summary.

$$\boxed{1 \text{ newton} = 10^5 \text{ dynes}} \qquad \boxed{1 \text{ dyne} = 10^{-5} \text{ newton}}$$

$$\boxed{1 \text{ lb} = 4.45 \text{ N}} \qquad \boxed{1 \text{ N} = 0.225 \text{ lb}}$$

$$\boxed{1 \text{ slug} = 14.6 \text{ kg}} \qquad \boxed{1 \text{ kg} = 0.0685 \text{ kg}}$$

NOTE. When you consider the full British System you find some subtle differences when you compare definitions to the system used in the United States. The British version is called the Imperial System of Units or Imperial System for short. You can also encounter the equivalent name Imperial Units. The US system is called US Customary Units or the US Customary System. The US Customary System became official in 1832 and it is based on an early form of British units. Likewise, the Imperial System is based on earlier definitions of British units. All systems of units evolve over time, including the Metric System.

As an example of evolving units, more Metric prefixes have been added over the years. Refer to our table in Chapter A, where we encountered this phenomenon. Another example is the redefinition of the meter. The meter was first defined as one ten-millionth of the distance from the North Pole to the Equation along the shortest arc, a portion of a great circle. And a bar of that length was kept in France, from which copies were made.

Then in 1983 it was changed to ever so slightly so that the speed of light is exactly

$$c = 299{,}792{,}458 \ \frac{\text{m}}{\text{s}}.$$

Since then you do not need a reference bar. You simple use light! Light will travel exactly one meter in

$$\frac{1}{299{,}792{,}458} \text{ s}.$$

D4. Weight vs Mass. We can now get a better handle on the difference between weight and mass through the use of Newton's Second Law.

Weight is the force on a body due to gravity.

With Newton's Second Law we can write W, representing weight, for the force and g for the acceleration.

$$F = ma$$

$$W = mg$$

You can see that the weight depends on g, while the mass m does not. The mass m represents how much stuff you have and the weight is the force at a planet's surface with acceleration g. The mass is the same no matter where it is, but the weight is different, depending on the celestial body.

Consider the Moon and Earth:

$$W_{\text{Moon}} = mg_{\text{Moon}} = m \cdot 1.625 \ \frac{\text{m}}{\text{s}^2} \quad \text{and} \quad W_{\text{Earth}} = mg_{\text{Earth}} = m \cdot 9.807 \ \frac{\text{m}}{\text{s}^2} \, .$$

The relative weight on the Moon compared to Earth is

$$\frac{W_{\text{Moon}}}{W_{\text{Earth}}} = \frac{mg_{\text{Moon}}}{mg_{\text{Earth}}} = \frac{g_{\text{Moon}}}{g_{\text{Earth}}} = \frac{1.625 \ \frac{\text{m}}{\text{s}^2}}{9.807 \ \frac{\text{m}}{\text{s}^2}} = 0.166 = \frac{1}{6} \, .$$

So if you weigh 180 lb (800 N) on Earth, you would weigh 30 lb (about 130 N) on the Moon, not counting for the space suit. Note that the mass m cancels out in the above equation. The 1/6 rule applies to everyone, i.e., all masses.

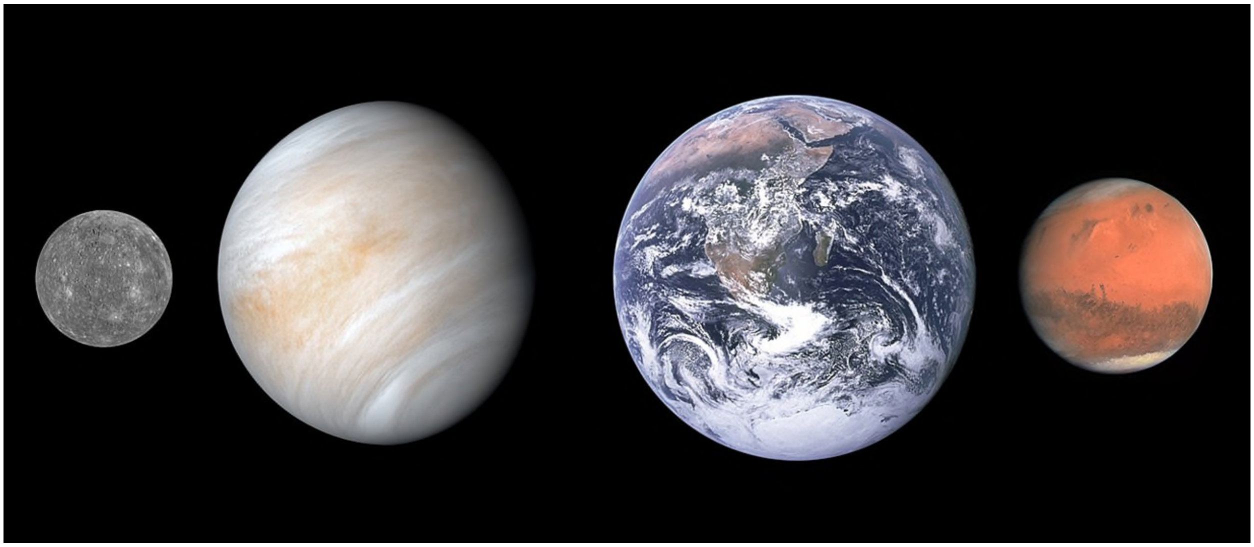
Below are the relative weights on the terrestrial planets and Earth for a constant mass m. We leave out the gas giants because you really can't walk on them.

Terrestrial Planet	g (m/s^2)	g/g_{Earth}
Mercury	3.7	0.38
Venus	8.9	0.91
Earth	9.8	1.00
Mars	3.7	0.38

Data Posted by NASA

Note the same listing for Mercury and Mars.

Later we will see how the mass of the planet and its radius play roles in the formula that give the planet's gravitational acceleration near its surface.



The Terrestrial Planets. Left to Right: Mercury, Venus, Earth, Mars
Courtesy NASA, Public Domain, Found on Wikimedia

D5. Statics. Statics is the study of bodies where the total or net forces are zero. Newton's Second Law is then

$$\vec{F} = m\vec{a} = 0 \,.$$

The convention for a vector of length zero is to just write down 0 and not worry about the vector sign.

When the acceleration is zero:

$$\vec{a} = 0 \,,$$

we have indication that the velocity is a constant,

$$\vec{v} = \vec{v_0} \,.$$

We can always put ourselves in the reference frame of the body so that the body will be at rest with respect to us. Then,

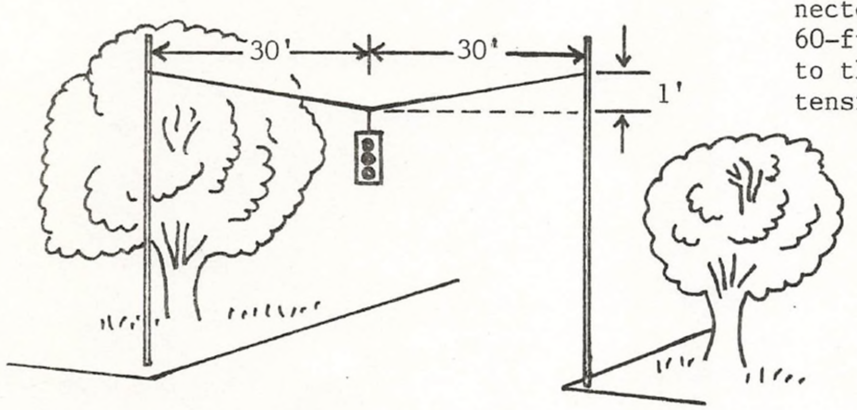
$$\vec{v_0} = 0 \,.$$

Let's do a couple of statics problems.

D6. Statics: The Traffic Light.

Problem

A 50-lb traffic light, connected in the center of a 60-ft cable sags 1 ft (refer to the figure). Find the tension in the cable.

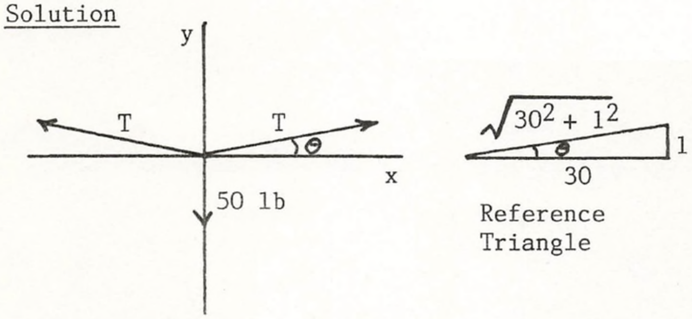


Note that figures are not usually drawn to scale since the dimensions are explicitly stated in the figures.

The first step is to draw a force diagram where the body of interest is at the origin. The best point to pick here is the point where the traffic light short cable attaches to the two slanted cables. Since that juncture point and all other points do not move, the net force at that point is zero.

Due to the symmetry, the tension (force) in each slanted cable is the same, which we label T in the figure below. The weight of the traffic light is 50 lb (220 N) downward, which we also include in the figure. We pick the usual positive directions: to the right for positive x and up for positive y. Note the 30’ = 30 ft (9.14 m) and 1’ = 1 ft (0.305 m) labels in the figure. We will need these.

Solution



Since $\vec{F} = 0$ is a vector equation,

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 ,$$

i.e., the sum of the forces in the x direction equals zero and the sum of the forces in the y direction equals zero. Or you can say that the sum of forces to the right equals the sum of forces to the left and the sum of forces up equals the sum of forces down. You can pick your preference.

Picking right as positive and up as positive, the equations of static equilibrium are:

$$\sum F_x = T\cos\theta - T\cos\theta = 0 ,$$

$$\sum F_y = 2T\sin\theta - 50 = 0 .$$

The x-equation tells us nothing. However, we should appreciate it because we could have labeled the left cable and right cable with different tensions. We would have then started with

$$\sum F_x = T_{\text{right}}\cos\theta - T_{\text{left}}\cos\theta = 0$$

and this equation would tell us that $T_{\text{right}}\cos\theta = T_{\text{left}}\cos\theta$ and $T_{\text{right}} = T_{\text{left}} = T$. We skipped this step because we used symmetry to start out with $T_{\text{right}} = T_{\text{left}} = T$. But the equations of equilibrium are our friends. They guide us to the solution and would have told us that $T_{\text{right}} = T_{\text{left}} = T$ if we didn't know it.

The y equilibrium equation will now lead to the answer.

$$2T\sin\theta = 50$$

Using the reference triangle, $\sin\theta = \dfrac{1}{\sqrt{30^2+1^2}}$ and we obtain

$$2T\frac{1}{\sqrt{30^2+1^2}} = 50$$

$$2T\frac{1}{\sqrt{901}} = 50$$

$$T = \frac{50\cdot\sqrt{901}}{2}$$

$$T = \frac{50\cdot 30.017}{2} = 750.4 \text{ lb}$$

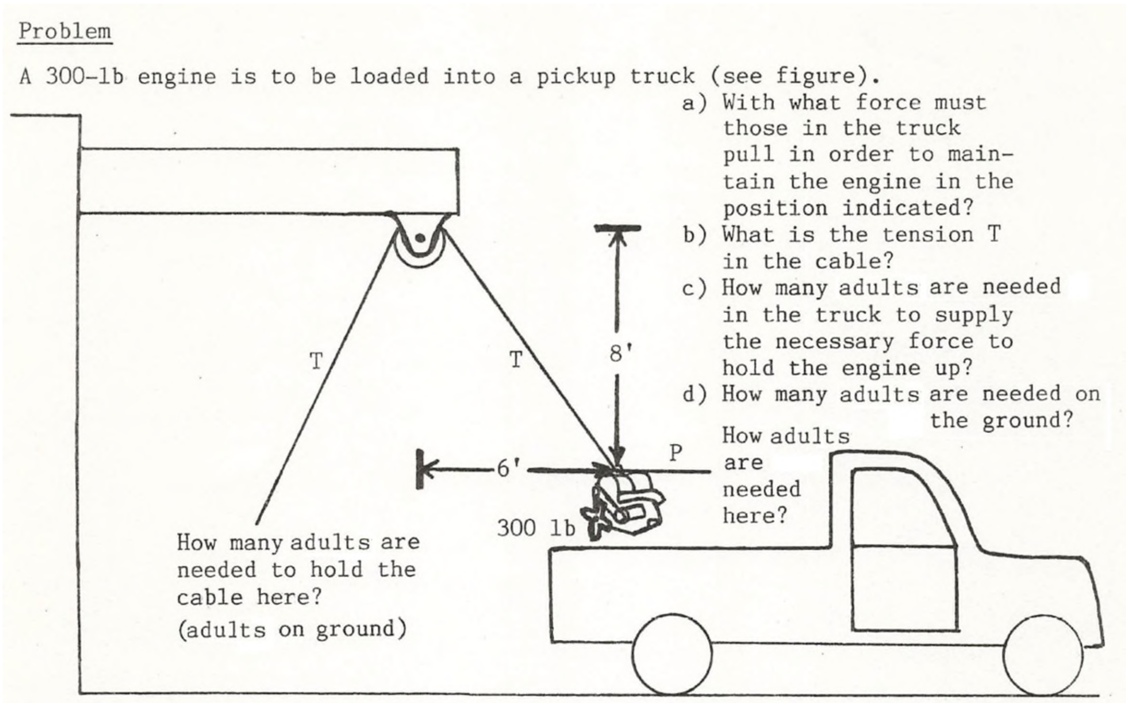
We do not want more than two significant figures given the input numbers. We are assuming that the data given in the problem are good to two significant figures. If I were making this problem today, I would include the decimal points and use metric. Then, to two significant figures

$$T = 750 \text{ lb} ,$$

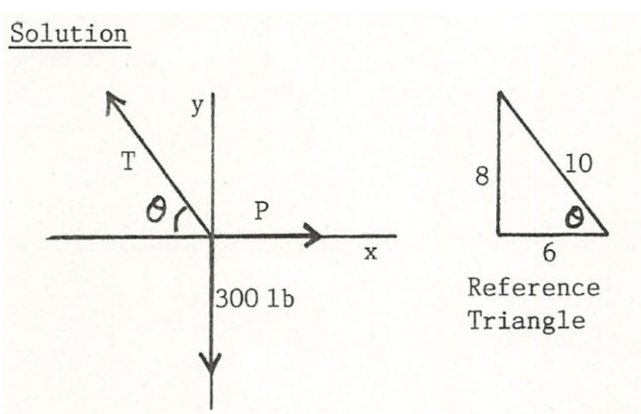
and in newtons

$$T = 750 \text{ lb}\cdot\frac{4.45 \text{ N}}{\text{lb}} = 3300 \text{ N}$$

D7. Statics: Lifting a Car Engine. In the figure below an engine is being loaded into a truck. At the moment, there is static equilibrium. The engine weighs 300.0 lb (1334 N). Folks in the truck are pulling to the right with force P and folks on the ground are grabbing a cable pulling with force T. The cable has no friction at the pulley so that the force tension T is the same everywhere throughout the cable. You are to find the forces T and P. Then you are to estimate how many people you will need in the truck and on the ground so that no one gets hurt.



It seems logical here to pick the engine as the point to analyze for the equilibrium.



Note that the length dimensions 8 ft (2.4 m) and 6 ft (1.8 m) from the figure are used to construct a triangle to determine the angle in the force diagram at the left.

Equilibrium in the x-direction: $T\cos\theta = P$

Equilibrium in y-direction: $T\sin\theta = 300 \text{ lb}$

Use the y equation and reference triangle:

$$T = \frac{300}{\sin\theta} = \frac{300}{8/10} = \frac{10}{8}\cdot 300 = \frac{5}{4}\cdot 300 = 5\cdot 75 = 375 \text{ lb} \text{ (1670 N)}$$

The x-equation gives P.

$$P = T\cos\theta = 375\cdot\frac{6}{10} = 375\cdot\frac{3}{5} = 75\cdot 3 = 225\text{ lb}\ \ (1000\text{ N})$$

I am going to assume adults that are not too strong. For the adults in the truck needing P = 225 lb (1000 N), I would feel comfortable with 3 or 4 people. For the folks on the ground needing T = 375 lb (1670 N), I would like 4 or 5.

I occasionally like to think philosophically. So I notice that we are using the I-ARC model: Inquiry, Apply, Reflect, and Communicate. There is a question, then we apply our physics, reflect on our answer, and communicate with proper units and significant figures. I did not mention explicitly reflecting, but those numbers that we got in each case seemed to be reasonable values.

Here are some specific steps for equilibrium problems that you might find helpful.

1) Sketch the figure and include the given data along with the question.
2) Draw a force diagram where the object or point of interest is at the origin.
3) Write down the equilibrium equations in each direction (x and y) or (x, y, and z) for 3D.
4) Solve the equations.

Always report your answers with proper units and significant figures.

A typical engineering professor likes you to box off your final answer. And always be neat. Do work first on scrap paper. Then copy to prepare your report. Include a nice diagram at the start, list all the given data, and state the question. Then proceed to show each step clearly. Occasionally include some phrases, i.e., written words, to make the steps even clearer.

I will have to compliment here engineering students. After teaching both physics (PHYS) and engineering (ENGR) courses, I have to say that on average, the engineering students do much neater work. Their homework is fun to grade. I think their early taking mechanical drawing along with strict expectations by engineering faculty lead to the superior communication.